```
(%i1)  kill(all);
(%o0)  done

(%i1)  depends([p,v],t);
(%o1)  [p(t),v(t)]
```

1 Eq.(26)

```
(%i2)  gamma: 1/sqrt(1-v^2/c^2);
```

$$(\text{gamma})\quad \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

```
(%i3)  p: gamma*m*v;
```

$$(\text{p})\quad \frac{m\,v}{\sqrt{1-\frac{v^2}{c^2}}}$$

```
(%i4)  diff(p,v);
```

$$(\%o4)\quad \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}+\frac{m\,v^2}{c^2\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

Test

```
(%i5)  1/gamma^2+v^2/c^2;
(%o5)  1
```

2 Eq.(28)

```
(%i6)  H: m*c^2*gamma-m*M*G/r^2;
```

$$(\text{H})\quad \frac{c^2\,m}{\sqrt{1-\frac{v^2}{c^2}}}-\frac{G\,M\,m}{r^2}$$

```
(%i7)  diff(H,v);
```

$$(\%o7)\quad \frac{m\,v}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

3 Eq.(60)

```
(%i8)  gamma_1: (m(r1)-(r1_d^2+r1^2*phi_d^2)/c^2)^(-1/2);
```

$$(\text{gamma\_1})\quad \frac{1}{\sqrt{\text{m}(r1)-\frac{r1_d{}^2+\varphi_d{}^2\,r1^2}{c^2}}}$$

```
(%i9)  E60: diff(1/gamma_1,r1);
```

$$(\text{E60})\quad \frac{\frac{d}{d\,r1}\,\text{m}(r1)-\frac{2\,\varphi_d{}^2\,r1}{c^2}}{2\sqrt{\text{m}(r1)-\frac{r1_d{}^2+\varphi_d{}^2\,r1^2}{c^2}}}$$

4 Eq.(61)

```
(%i10)  E61: diff(gamma_1,r1);
```

$$(\text{E61})\quad -\frac{\frac{d}{d\,r1}\,\text{m}(r1)-\frac{2\,\varphi_d{}^2\,r1}{c^2}}{2\left(\text{m}(r1)-\frac{r1_d{}^2+\varphi_d{}^2\,r1^2}{c^2}\right)^{3/2}}$$

5 Eq.(66)

from (57):

(%i11) `E57: gamma_1*diff(m(r1),r1) + m(r1)*diff(gamma_1,r1) = diff(1/gamma_1,r1);`

(E57)
$$\frac{\frac{d}{d\,r1}m(r1)}{\sqrt{m(r1)-\frac{r1_d{}^2+\varphi_d{}^2\,r1^2}{c^2}}}-\frac{m(r1)\left(\frac{d}{d\,r1}m(r1)-\frac{2\,\varphi_d{}^2\,r1}{c^2}\right)}{2\left(m(r1)-\frac{r1_d{}^2+\varphi_d{}^2\,r1^2}{c^2}\right)^{3/2}}=\frac{\frac{d}{d\,r1}m(r1)-\frac{2\,\varphi_d{}^2\,r1}{c^2}}{2\sqrt{m(r1)-\frac{r1_d{}^2+\varphi_d{}^2\,r1^2}{c^2}}}$$

Re-insert gamma_1

(%i12) `E57a: ratsubst(Gamma_1, gamma_1, E57);`

(E57a)
$$-\frac{(\Gamma_1{}^3\,c^2\,m(r1)-2\,\Gamma_1\,c^2)\left(\frac{d}{d\,r1}m(r1)\right)-2\,\Gamma_1{}^3\,\varphi_d{}^2\,r1\,m(r1)}{2\,c^2}=\frac{\Gamma_1\,c^2\left(\frac{d}{d\,r1}m(r1)\right)-2\,\Gamma_1\,\varphi_d{}^2\,r1}{2\,c^2}$$

(%i13) `E57b: solve(E57a, diff(m(r1),r1));`

(E57b)
$$\left[\frac{d}{d\,r1}m(r1)=\frac{2\,\Gamma_1{}^2\,\varphi_d{}^2\,r1\,m(r1)+2\,\varphi_d{}^2\,r1}{\Gamma_1{}^2\,c^2\,m(r1)-c^2}\right]$$

(%i14) `E57c: factor(E57b);`

(E57c)
$$\left[\frac{d}{d\,r1}m(r1)=\frac{2\,\varphi_d{}^2\,r1\,(\Gamma_1{}^2\,m(r1)+1)}{c^2\,(\Gamma_1{}^2\,m(r1)-1)}\right]$$

6 Eq.(70)

(%i15) `E68: L = 'gamma_1*m*r1^2*phi_d;`

(E68)
$$L=gamma_1\,m\,\varphi_d\,r1^2$$

(%i16) `E69: E68^2/('gamma_1^2*r1^3*m^2);`

(E69)
$$\frac{L^2}{gamma_1{}^2\,m^2\,r1^3}=\varphi_d{}^2\,r1$$

(%i17) `factor(ratsubst(lhs(E69), r1*phi_d^2, E57b));`

(%o17)
$$\left[\frac{d}{d\,r1}m(r1)=\frac{2\,L^2\,(\Gamma_1{}^2\,m(r1)+1)}{c^2\,gamma_1{}^2\,m^2\,r1^3\,(\Gamma_1{}^2\,m(r1)-1)}\right]$$

7 Strategies for solving Eq.(66/67)

(%i18) `E57d: first(E57c)*c^2*(Gamma_1^2*m(r1)-1);`

(E57d)
$$c^2\,(\Gamma_1{}^2\,m(r1)-1)\left(\frac{d}{d\,r1}m(r1)\right)=2\,\varphi_d{}^2\,r1\,(\Gamma_1{}^2\,m(r1)+1)$$

(%i19) `E57e: ode2(E57d, m(r1), r1);`

(E57e)
$$-\frac{2\,c^2\log(\Gamma_1{}^2\,m(r1)+1)-\Gamma_1{}^2\,c^2\,m(r1)}{2\,\Gamma_1{}^2\,\varphi_d{}^2}=\frac{r1^2}{2}+\%c$$

(%i20) `E57f: solve(E57e, m(r1));`

(E57f)
$$\left[m(r1)=\frac{2\,c^2\log(\Gamma_1{}^2\,m(r1)+1)+\Gamma_1{}^2\,\varphi_d{}^2\,r1^2+2\,\%c\,\Gamma_1{}^2\,\varphi_d{}^2}{\Gamma_1{}^2\,c^2}\right]$$

(%i21) `expand(%);`

(%o21)
$$\left[m(r1)=\frac{2\log(\Gamma_1{}^2\,m(r1)+1)}{\Gamma_1{}^2}+\frac{\varphi_d{}^2\,r1^2}{c^2}+\frac{2\,\%c\,\varphi_d{}^2}{c^2}\right]$$

Alternatively:

(%i22) `E57g: solve(E57d, m(r1));`

(E57g)
$$\left[m(r1)=\frac{c^2\left(\frac{d}{d\,r1}m(r1)\right)+2\,\varphi_d{}^2\,r1}{\Gamma_1{}^2\,c^2\left(\frac{d}{d\,r1}m(r1)\right)-2\,\Gamma_1{}^2\,\varphi_d{}^2\,r1}\right]$$

(%i23) `E57h: solve(E57d, diff(m(r1),r1));`

(E57h)
$$\left[\frac{d}{d\,r1}m(r1)=\frac{2\,\Gamma_1{}^2\,\varphi_d{}^2\,r1\,m(r1)+2\,\varphi_d{}^2\,r1}{\Gamma_1{}^2\,c^2\,m(r1)-c^2}\right]$$

Constant m(r1)

(%i24)
```
m(r1) := 1;
```
(%o24)
$$m(r1) := 1$$

(%i25)
```
ev(E57e);
```
(%o25)
$$-\frac{2\log\left(\Gamma_1^2+1\right)c^2-\Gamma_1^2 c^2}{2\,\Gamma_1^2\,\varphi_d^2}=\frac{r1^2}{2}+\%c$$

(%i26)
```
solve(%, Gamma_1);
```
(%o26)
$$\left[\Gamma_1=-\frac{\sqrt{2}\,\%i\,\sqrt{\log\left(\Gamma_1^2+1\right)}\,c}{\sqrt{\varphi_d^2\,r1^2+2\,\%c\,\varphi_d^2-c^2}},\ \Gamma_1=\frac{\sqrt{2}\,\%i\,\sqrt{\log\left(\Gamma_1^2+1\right)}\,c}{\sqrt{\varphi_d^2\,r1^2+2\,\%c\,\varphi_d^2-c^2}}\right]$$

(%i27)
```
E67i: ev(E57g);
```
(E67i)
$$\left[1=-\frac{1}{\Gamma_1^2}\right]$$

(%i28)
```
solve(%, Gamma_1);
```
(%o28)
$$\left[\Gamma_1=-\%i,\ \Gamma_1=\%i\right]$$

(%i29)
```
ev(E57h);
```
(%o29)
$$\left[0=\frac{2\,\Gamma_1^2\,\varphi_d^2\,r1+2\,\varphi_d^2\,r1}{\Gamma_1^2\,c^2-c^2}\right]$$

(%i30)
```
solve(%, Gamma_1);
```
(%o30)
$$\left[\Gamma_1=-\%i,\ \Gamma_1=\%i\right]$$