

```
(%i1) kill(all);
(%o0) done
```

```
(%i1) depends([p,v],t);
(%o1) [p(t),v(t)]
```

□ **1 Eq. (26)**

```
(%i2) gamma: 1/sqrt(1-v^2/c^2);
```

```
(%o2) 
$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

```

```
(%i3) p: gamma*m*v;
```

```
(%o3) 
$$\frac{m v}{\sqrt{1-\frac{v^2}{c^2}}}$$

```

```
(%i4) diff(p,v);
```

```
(%o4) 
$$\frac{m}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{m v^2}{c^2 \left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

```

Test

```
(%i5) 1/gamma^2+v^2/c^2;
```

```
(%o5) 1
```

□ **2 Eq. (28)**

```
(%i6) H: m*c^2*gamma-m*M*G/r^2;
```

```
(%o6) 
$$\frac{c^2 m}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{G M m}{r^2}$$

```

```
(%i7) diff(H,v);
```

```
(%o7) 
$$\frac{m v}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

```

□ **3 Eq. (60)**

⌈ (%i8) gamma_1: (m(r1)-(r1_d^2+r1^2*phi_d^2)/c^2)^(-1/2);

(%o8)
$$\frac{1}{\sqrt{m(r1) - \frac{r1_d^2 + \varphi_d^2 r1^2}{c^2}}}$$

⌈ (%i9) E60: diff(1/gamma_1,r1);

(%o9)
$$\frac{\frac{d}{d r1} m(r1) - \frac{2 \varphi_d^2 r1}{c^2}}{2 \sqrt{m(r1) - \frac{r1_d^2 + \varphi_d^2 r1^2}{c^2}}}$$

□ **4 Eq. (61)**

⌈ (%i10) E61: diff(gamma_1,r1);

(%o10)
$$-\frac{\frac{d}{d r1} m(r1) - \frac{2 \varphi_d^2 r1}{c^2}}{2 \left(m(r1) - \frac{r1_d^2 + \varphi_d^2 r1^2}{c^2} \right)^{3/2}}$$

□ **5 Eq. (66)**

⌈ from (57):

⌈ (%i11) E57: gamma_1*diff(m(r1),r1) + m(r1)*diff(gamma_1,r1) = di

(%o11)
$$\frac{\frac{d}{d r1} m(r1)}{\sqrt{m(r1) - \frac{r1_d^2 + \varphi_d^2 r1^2}{c^2}}} - \frac{m(r1) \left(\frac{d}{d r1} m(r1) - \frac{2 \varphi_d^2 r1}{c^2} \right)}{2 \left(m(r1) - \frac{r1_d^2 + \varphi_d^2 r1^2}{c^2} \right)^{3/2}} =$$

$$\frac{\frac{d}{d r1} m(r1) - \frac{2 \varphi_d^2 r1}{c^2}}{2 \sqrt{m(r1) - \frac{r1_d^2 + \varphi_d^2 r1^2}{c^2}}}$$

⌈ Re-insert gamma_1

```
(%i12) E57a: ratsubst(Gamma_1, gamma_1, E57);
```

$$\begin{aligned}
 (\%o12) \quad & - \frac{(\Gamma_1^3 c^2 m(r1) - 2 \Gamma_1 c^2) \left(\frac{d}{d r1} m(r1) \right) - 2 \Gamma_1^3 \varphi_d^2 r1 m(r1)}{2 c^2} = \\
 & \frac{\Gamma_1 c^2 \left(\frac{d}{d r1} m(r1) \right) - 2 \Gamma_1 \varphi_d^2 r1}{2 c^2}
 \end{aligned}$$

```
(%i13) E57b: solve(E57a, diff(m(r1), r1));
```

$$(\%o13) \quad \left[\frac{d}{d r1} m(r1) = \frac{2 \Gamma_1^2 \varphi_d^2 r1 m(r1) + 2 \varphi_d^2 r1}{\Gamma_1^2 c^2 m(r1) - c^2} \right]$$

```
(%i14) factor(%);
```

$$(\%o14) \quad \left[\frac{d}{d r1} m(r1) = \frac{2 \varphi_d^2 r1 (\Gamma_1^2 m(r1) + 1)}{c^2 (\Gamma_1^2 m(r1) - 1)} \right]$$