425(1): Self Consistent Results from the Lagrangian and Hamiltonian Methods

The lagrangian of m theory can be written as:

$$\mathcal{L} = -mc^2\left(m(r_1) - \frac{1}{c^2}\left(\dot{r}_1^2 + r_1^2\dot{\phi}^2\right)\right)^{1/2} + \frac{mMG}{r_1} \quad -(1)$$

where

$$v_1^2 = \dot{r}_1^2 + r_1^2\dot{\phi}^2 \quad -(2)$$

in the coordinate system (r_1, ϕ). In an inertial system:

$$v_1 = \dot{r}_1 \quad -(3)$$

and the Euler Lagrange equation is:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial v_1} = \frac{\partial \mathcal{L}}{\partial r_1} \quad -(4)$$

It follows that:

$$\frac{d}{dt}(\gamma m v_1) = -\frac{mc^2}{2}\gamma\frac{dm(r_1)}{dr_1} - \frac{mMG}{r_1^2} \quad -(5)$$

The magnitude of the vacuum force is:

$$F(vac) = -\frac{mc^2}{2}\gamma\frac{dm(r_1)}{dr_1} \quad -(6)$$

where

$$\gamma = \left(m(r_1) - \frac{v_1^2}{c^2}\right)^{-1/2} \quad -(7)$$

The Euler Lagrange equation:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \frac{\partial \mathcal{L}}{\partial \phi} \quad -(8)$$

gives the second Evans Eckardt equation:

2)

$$\frac{dL}{dt} = 0 \quad -(9)$$

where

$$L = \gamma m(r) r_1^2 \dot{\phi} \quad -(10)$$

is the angular momentum.

The Hamiltonian is:

$$H = m(r_1)\gamma mc^2 - \frac{mMG}{r_1} \quad -(11)$$

and is related to the lagrangian:

$$\mathcal{L} = -\frac{mc^2}{\gamma} + \frac{mMG}{r_1} \quad -(12)$$

by

$$H = \frac{p_1^2 c^2}{\gamma mc^2} - \mathcal{L} \quad -(13)$$

and the first principles of Lagrangian and Hamiltonian dynamics. For self consistency, eqs. (11) to (13) shows that:

$$\frac{dm(r_1)}{dt} = 0. \quad -(14)$$

The Hamiltonian is a constant of motion, so:

$$\frac{dH}{dt} = 0 \quad -(15)$$

It follows that:

$$mc^2 \frac{d}{dt}\left(m(r_1)\gamma\right) = \frac{d}{dt}\left(\frac{mMG}{r_1}\right) \quad -(16)$$

$$= -\frac{mMG}{r_1^2}\frac{dr_1}{dt} = -\frac{mMG}{r_1^2} v_1$$

) Using eq. (14):

$$mc^2 n(r_1) \frac{d\gamma}{dt} = -\frac{mMG}{r_1^2} v_1 = mc^2 n(r_1) \frac{d\gamma}{dv_1} \frac{dv_1}{dt} \quad -(17)$$

Now use:

$$\frac{d\gamma}{dv_1} = \gamma^3 \frac{v_1}{c^2} \quad -(18)$$

to find that:

$$m n(r_1) \gamma^3 \frac{dv_1}{dt} . v_1 = -\frac{mMG}{r_1^2} v_1 \quad -(19)$$

So

$$\boxed{n(r_1) \gamma^3 \frac{dv_1}{dt} = -\frac{MG}{r_1^2}} \quad -(20)$$

Eq. (20) is based on:

$$\frac{dn(r_1)}{dt} = 0 \quad -(21)$$

and

$$\frac{dn(r_1)}{dv_1} = 0 \quad -(22)$$

Eqs. (21) and (22) also follow from the fact that the infinitesimal line element:

$$ds^2 = c^2 d\tau^2 = n(r) c^2 dt^2 - \frac{dr^2}{n(r)} - r^2 d\phi^2 \quad -(23)$$

is a steady state line element. Combining eqs. (5) and (20):

4)

$$\frac{d}{dt}\left(\gamma m v_1\right) = m n(r_1)\gamma^3 \frac{dv_1}{dt} - \frac{mc^2}{2}\gamma \frac{dn(v_1)}{dv_1} \quad -(24)$$

In special relativity this equation reduces to:

$$\frac{d}{dt}\left(\gamma v\right) = \gamma^3 \frac{dv}{dt} \quad -(25)$$

which is a well known equation of special relativity,

Q.E.D. Eq. (25) is proven as follows:

$$\frac{d}{dt}\left(\gamma v\right) = \frac{d}{dv}\left(\gamma v\right)\frac{dv}{dt} = \left(\gamma + v\frac{d\gamma}{dv}\right)\frac{dv}{dt}$$

$$= \left(\gamma + \frac{v^2}{c^2}\gamma^3\right)\frac{dv}{dt}$$

$$= \gamma^3\frac{dv}{dt}\left(\frac{1}{\gamma^2} + \frac{v^2}{c^2}\right)$$

$$= \gamma^3\frac{dv}{dt}\left(1 - \frac{v^2}{c^2} + \frac{v^2}{c^2}\right)$$

$$= \gamma^3\frac{dv}{dt} \quad -(26)$$

Q.E.D.

In order to transform eq. (24) to the plane polar system (r_1, ϕ) use eq. (2)