425(3): New Equations of Motion for m Theory from the Hamilton Canonical Equations.

The Hamilton canonical equations are:

$$\dot{r} = \frac{\partial H}{\partial p} \quad -(1)$$

and

$$\dot{p} = -\frac{\partial H}{\partial r} \quad -(2)$$

where

$$H = H(q, p, t) \quad -(3)$$

so the independent canonical variables are q and p. They can be combined with the Euler Lagrange equations:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial q} \quad -(4)$$

where

$$\mathcal{L} = \mathcal{L}(q, \dot{q}, t) \quad -(5)$$

so the independent Lagrange variables are q and $\dot{q}$. If the Lagrange variables are chosen to be r and p, then

$$\dot{p} = \frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} = -\frac{\partial H}{\partial r} \quad -(6)$$

and

$$\dot{p} = \frac{\partial \mathcal{L}}{\partial r} = -\frac{\partial H}{\partial r} \quad -(7)$$

As shown in Note 425(2), eq. (7) gives in m theory:

$$\frac{\partial}{\partial r_1}\left(m(r_1)\gamma m c^2\right) = \frac{\partial}{\partial r_1}\left(\frac{mc^2}{\gamma}\right) \quad -(8)$$

i.e.

$$\frac{\partial}{\partial r_1}\left(\gamma m(r_1)\right) = \frac{\partial}{\partial r_1}\left(\frac{1}{\gamma}\right) \quad -(9)$$

As shown in Note 425(2), eq. (9) leads to:

$$\frac{dm(r_1)}{dr_1} = \frac{-2L^2 x}{\gamma^2 c^2 m^2 r_1^3}, \quad -(10)$$

$$x = 1 + \gamma^2 m(r_1) \quad -(10a)$$

) i.e.

$$\frac{dm(r_1)}{dr_1} = -\frac{2r_1\dot{\phi}^2 x}{c^2} \quad -(11)$$

using the conservation of motion:

$$L = \gamma m r_1^2 \dot{\phi} \quad -(12)$$

which is the angular momentum.

Another possible solution of eq. (9) is:

$$\gamma m(r_1) = \frac{1}{\gamma} \quad -(13)$$

so

$$m(r_1) = \frac{1}{\gamma^2} = m(r_1) - \frac{v_1^2}{c^2} \quad -(14)$$

in which case:

$$v_1^2 = \dot{r}_1^2 + r_1^2\dot{\phi}^2 = 0 \quad -(15)$$

i.e.

$$\dot{r}_1 = \mp r_1 \dot{\phi} \quad -(16)$$

In general the orbital velocity v_1 is not zero, so the general solution for non-zero v_1 is eq. (10).

The second Hamilton equation gives:

$$\dot{r}_1 = \frac{\partial H}{\partial p_1} \quad -(17)$$

where

$$p_1 = \gamma m v_N \quad -(18)$$

where v_N is the Newtonian velocity. The canonical variables are r_1 and p_1 so this gives the equation defining the fact that p_1 and r_1 are independent:

3)
$$\frac{\partial p_1}{\partial r_1} = 0 \quad -(19)$$

It is also possible to define the vector Hamilton equations:

$$\underline{p}_1 = -\frac{\partial H}{\partial \underline{r}_1} \quad -(20)$$

$$\dot{\underline{r}}_1 = \frac{\partial H}{\partial \underline{p}_1} \quad -(21)$$

and

$$\frac{\partial \underline{p}_1}{\partial \underline{r}_1} = 0 \quad -(21)$$

Eq. (20) is equivalent to:

$$\underline{p}_1 = -\underline{\nabla} H \quad -(22)$$

$$= -\frac{\partial H}{\partial r_1}\underline{e}_r - \frac{1}{r_1}\frac{\partial H}{\partial \phi}\underline{e}_\phi$$

$$= m\gamma \dot{\underline{r}}_1$$

$$= m\gamma\left(\dot{r}_1 \underline{e}_r + r_1 \dot{\phi}\, \underline{e}_\phi\right)$$

It follows that:

$$m\gamma \dot{r}_1 = -\frac{\partial H}{\partial r_1} \quad -(23)$$

and

$$m\gamma r_1 \dot{\phi} = -\frac{1}{r_1}\frac{\partial H}{\partial \phi} \quad -(24)$$

Eqs. (23) and (24) give new dynamical information.

Eq. (24) gives:

$$L = m\gamma r_1^2 \dot{\phi} = -\frac{\partial H}{\partial \phi} \quad -(25)$$

where L is the angular momentum. The L and ϕ are conjugate variables

) The definition of the problem is based on the fundamental:

$$\mathcal{L} = p_1 \dot{q}_1 - H \quad -(26)$$

in generalized conjugate coordinates p_1 and q_1.

In UFT 424 the following choice was made:

$$\boxed{\begin{aligned} p_1 &= \gamma m v_1 \quad -(27) \\ \dot{q}_1 &= v_1 \quad -(28) \end{aligned}}$$

where v_1 is defined by:

$$\gamma = \left(m(r_1) - \frac{v_1^2}{c^2} \right)^{-1/2} \quad -(29)$$

From eqs. (26) to (28):

$$\mathcal{L} = \gamma m v_1^2 - H \quad -(30)$$

$$= \frac{p_1^2 c^2}{\gamma m c^2} - H$$

From eq. (27):

$$p_1^2 c^2 = \gamma^2 m^2 c^4 \frac{v_1^2}{c^2} \quad -(31)$$

From eq. (29):

$$\frac{1}{\gamma^2} = m(r_1) - \frac{v_1^2}{c^2} \quad -(32)$$

From eqs. (31) and (32):

$$p_1^2 c^2 = m^2 c^4 \left(\gamma^2 m(r_1) - 1 \right) \quad -(33)$$

5) Define the hamiltonian as:

$$H = E + U \quad -(34)$$

where E is the total relativistic energy:

$$E = m(r_1)\gamma mc^2 \quad -(35)$$

and it follows that:

$$\boxed{E^2 = m(r_1)\left(p_1^2c^2 + m^2c^4\right)} \quad -(36)$$

This is the Einstein energy equation in m space. In flat space it reduces to:

$$E^2 = p^2c^2 + m^2c^4 \quad -(37)$$

From eq. (30) and (33):

$$\boxed{\mathcal{L} = m(r_1)\gamma mc^2 - \frac{mc^2}{\gamma} - H} \quad -(38)$$

so

$$p_1 \dot{q}_1 := p_1 \dot{r}_1 = m(r_1)\gamma mc^2 - \frac{mc^2}{\gamma} \quad -(39)$$

In Lagrangian dynamics:

$$\dot{p}_1 = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}_1} = \frac{\partial \mathcal{L}}{\partial r_1} \quad -(40)$$

In Hamiltonian dynamics:

$$\dot{p}_1 = -\frac{\partial H}{\partial r_1} \quad -(41)$$

It follows that:

$$\boxed{\frac{\partial \mathcal{L}}{\partial r_1} = -\frac{\partial H}{\partial r_1}} \quad -(42)$$

and

$$\boxed{\frac{\partial}{\partial r_1}\left(p_1 \dot{r}_1\right) = 0} \quad -(43)$$

Fm eq. (39) and (43):

$$\frac{\partial}{\partial r_1}\left(m(r_1)\gamma m c^2 - \frac{mc^2}{\gamma}\right) = 0 \quad -(44)$$

nd using:

$$\mathcal{L} = -\frac{mc^2}{\gamma} + \frac{mMG}{r_1} \quad -(45)$$

nd

$$H = m(r_1)\gamma m c^2 - \frac{mMG}{r_1} \quad -(46)$$

eq. (42) gives:

$$\frac{\partial}{\partial r_1}\left(m(r_1)\gamma m c^2 - \frac{mc^2}{\gamma}\right) = 0 \quad -(47)$$

hich is eq. (44), Q.E.D.

So the system of equations is rigorously self consistent.

Eqs. (42) and (43) both give:

$$\boxed{\frac{\partial}{\partial r_1}\left(\gamma m(r_1)\right) = \frac{\partial}{\partial r_1}\left(\frac{1}{\gamma}\right)} \quad -(48)$$

The second Hamilton equation gives:

$$\boxed{v_1 = \dot{r}_1 = \frac{\partial H}{\partial p_1}} \quad -(49)$$

) The static solution of Eq. (48) is:

$$\gamma m(r_1) = \frac{1}{\gamma} \quad -(50)$$

i.e.

$$\frac{1}{\gamma^2} = m(r_1) \quad -(51)$$

Comparing eqs. (32) and (51):

$$v_1 = 0 \quad -(52)$$

So this is the solution for the rest particle.

The hamiltonian for the rest particle is:

$$H = \gamma m(r_1) mc^2 - \frac{mMG}{r_1} \quad -(53)$$

where

$$\gamma = \frac{1}{m(r_1)^{1/2}} \quad -(54)$$

so

$$H = m(r_1)^{1/2} mc^2 - \frac{mMG}{r_1} \quad -(55)$$

$$= E_0 + U$$

where the rest energy is:

$$\boxed{E_0 = m(r_1)^{1/2} mc^2} \quad -(56)$$

which is the rest energy from eq. (36), Q.E.D.

So this solution is rigorously self consistent and describes a particle m interacting with a particle M, both particles being at rest. The rest energy in m space is eq. (56), as derived in previous UFT papers. Clearly, m(r_1) in eq. (56) must be static.

5) In general, the solution of Eq. (48) is:

$$m(r_1)\frac{\partial \gamma}{\partial r_1} + \gamma \frac{\partial m(r_1)}{\partial r_1} = \frac{\partial}{\partial r_1}\left(\frac{1}{\gamma}\right) \quad -(57)$$

where

$$\gamma = \left(m(r_1) - \frac{v_1^2}{c^2}\right)^{-1/2} \quad -(58)$$

This can be worked out by computer algebra to eliminate human error. We have:

$$\frac{\partial}{\partial r_1}\left(\frac{1}{\gamma}\right) = -\frac{1}{2}\left(\frac{\partial m(r_1)}{\partial r_1} - \frac{1}{c^2}\frac{\partial v_1^2}{\partial r_1}\right)\gamma \quad -(59)$$

and

$$\frac{\partial \gamma}{\partial r_1} = -\frac{1}{2}\left(\frac{\partial m(r_1)}{\partial r_1} - \frac{1}{c^2}\frac{\partial v_1^2}{\partial r_1}\right)\gamma^3 \quad -(60)$$

In the inertial system:

$$v_1 = \dot{r}_1 \quad -(61)$$

and in the plane polar system

$$v_1^2 = \dot{r}_1^2 + r_1^2\dot{\phi}^2 \quad -(62)$$

In the inertial system:

$$\frac{\partial v_1^2}{\partial r_1} = 0 \quad -(63)$$

so

$$\frac{\partial}{\partial r_1}\left(\frac{1}{\gamma}\right) = -\frac{1}{2}\gamma\frac{\partial m(r_1)}{\partial r_1} \quad -(64)$$

and

$$\frac{\partial \gamma}{\partial r_1} = -\frac{1}{2}\gamma^3\frac{\partial m(r_1)}{\partial r_1} \quad -(65)$$

so

$$-\frac{1}{2}\gamma^3 m(r_1) + \gamma = -\frac{\gamma}{2} \quad -(66)$$

) and

$$\boxed{n(r_1) = \frac{1}{2}\frac{v_1^2}{c^2}} \quad -(67)$$

Therefore the hamiltonian in the vertical system is

$$\boxed{H = \frac{1}{2} m v_1^2 - \frac{m M G}{r_1}} \quad -(68)$$

In the (r, ϕ) frame:

$$H = \frac{1}{2}\frac{m}{n(r)} v^2 - n(r)^{1/2}\frac{m M G}{r} \quad -(69)$$

and the classical result is regained as:

$$n(r) \rightarrow 1 \quad -(70)$$

i.e.

$$H = \frac{1}{2} m v^2 - \frac{m M G}{r} \quad -(71)$$

Q.E.D.

Eq. (68) is a remarkable result of combining the Lagrangian and Hamiltonian dynamics.

The first Evans Eckardt equation can be applied to Eq. (68):

$$\frac{dH}{dt} = \frac{d}{dt}\left(\frac{1}{2} m v_1^2 - \frac{m M G}{r_1}\right) = 0 \quad -(72)$$

Now use:

$$\frac{d}{dt} v_1^2 = 2 v_1 \frac{d v_1}{dt} \quad -(73)$$

and

$$\frac{d}{dt}\left(-\frac{m M G}{r_1}\right) = \frac{m M G}{r_1^2} v_1 \quad -(74)$$

and

$$v_1 = \dot{r}_1 \quad -(75)$$

to find:

$$\boxed{F = m\dot{v}_1 = -\frac{mMG}{r_1^2}} \quad -(76)$$

This reduces to the classical result:

$$F = m\dot{v} = -\frac{mMG}{r^2} \quad -(77)$$

as

$$n(r_1) \longrightarrow 1 \quad -(78)$$

In the (r, ϕ) coordinate system:

$$F = m\frac{d}{dt}\left(\frac{v}{n(r)^{1/2}}\right) = -n(r)\frac{mMG}{r^2} \quad -(79)$$

$$= \frac{m}{n(r)^{1/2}}\dot{v} + mv\frac{d}{dt}\left(\frac{1}{n(r)^{1/2}}\right)$$

This leads to a new definition of vacuum force:

$$\boxed{F(\text{vac}) = mv\frac{d}{dt}\left(\frac{1}{n(r)^{1/2}}\right)} \quad -(80)$$

To transform eq. (68) to plane polar coordinates use

$$v_1^2 = \dot{r}_1^2 + r_1^2\dot{\phi}^2 \quad -(81)$$

so

$$H = \frac{1}{2}m\left(\dot{r}_1^2 + r_1^2\dot{\phi}^2\right) - \frac{mMG}{r_1} \quad -(82)$$

The angular momentum is:

$$L = \gamma m r_1^2\dot{\phi} \quad -(83)$$

11) from Lagrangian dynamics. So:

$$L^2 = \gamma^2 m^2 r_1^4 \dot{\phi}^2 \quad -(84)$$

and

$$r_1^2 \dot{\phi}^2 = \frac{L^2}{\gamma^2 m^2 r_1^2} \quad -(85)$$

The hamiltonian is therefore: $\quad -(86)$

$$H = \frac{1}{2} m \left(r_1^2 + \frac{L^2}{\gamma^2 m^2 r_1^2} \right) - \frac{mMG}{r_1}$$

Finally use

$$\gamma^2 = \frac{3}{n(r_1)} \quad -(87)$$

from eq. (66) to find: $\quad -(88)$

$$H = \frac{1}{2} m \left(r_1^2 + \frac{L^2}{3n(r_1) m^2 r_1^2} \right) - \frac{mMG}{r_1}$$

As flat spacetime is approached:

$$n(r_1) \rightarrow 1 \quad -(89)$$

and

$$\frac{1}{\gamma^2} = \frac{1}{3} = n(r_1) - \frac{v_1^2}{c^2} \quad -(90)$$

$$= 1 - \frac{v_1^2}{c^2} \quad -(91)$$

So

$$\boxed{\frac{v_1^2}{c^2} \rightarrow \frac{2}{3}} \quad -(92)$$

Finally consider the solution found in the previous note:

$$\frac{dn(r_1)}{dr_1} = 2\frac{r_1\dot{\phi}^2}{c^2}\left(\frac{1+n(r_1)\gamma^2}{\gamma^2 n(r_1)-1}\right) \quad -(93)$$

in plane polar coordinates. This solution allows a link to be made between $dn(r_1)/dr_1$ and $n(r_1)$. We have:

$$r_1\dot{\phi}^2 = \frac{L^2}{\gamma^2 m^2 r_1^3} \quad -(94)$$

Eq. (93) has been checked by computer algebra. In general:

$$\gamma^2 = \left(n(r_1) - \frac{v_1^2}{c^2}\right)^{-1} \quad -(95)$$

where

$$v_1^2 = \dot{r}_1^2 + r_1^2\dot{\phi}^2 \quad -(96)$$

So in general eq. (93) is a very complicated differential equation, which does not have an analytical solution. However it does show that in order for $dn(r_1)/dr_1$ to be non zero, the angular momentum L must be non zero. Therefore if:

$$\frac{dn(r_1)}{dr_1} = 0 \quad -(97)$$

then

$$L = 0 \quad -(98)$$

one possibility, indicating that:

$$v_1 = \dot{r}_1 \quad -(99)$$

and the system is inertial. Another possibility is:

$$1 + n(r_1)\gamma^2 = 0 \quad -(100)$$

$$n(r_1) = v_1^2/c^2 \quad -(101)$$