+29(1): Anomalous g Factor of the Electron in n Theory

In Dirac theory the g factor of the electron is calculated from the Zeeman term:

$$H_2 \psi = i \frac{e\hbar}{2m} \underline{\sigma} \cdot \underline{\nabla} \, \underline{\sigma} \cdot \underline{A} \, \psi + \cdots \quad -(1)$$

Using Pauli algebra:

$$\underline{\sigma} \cdot \underline{\nabla} \, \underline{\sigma} \cdot \underline{A} = \underline{\nabla} \cdot \underline{A} + i\underline{\sigma} \cdot \underline{\nabla} \times \underline{A} \quad -(2)$$

So

$$H_2 \psi = - \frac{e\hbar}{2m} \underline{\sigma} \cdot \underline{\nabla} \times \underline{A} \, \psi + \cdots \quad -(3)$$

where the Bohr magneton is:

$$g_B = \frac{e\hbar}{2m} \quad -(4)$$

The spin angular momentum operator is defined by:

$$\underline{S} = \frac{\hbar}{2} \underline{\sigma} \quad -(5)$$

So:

$$H_2 \psi = 2 \left(\frac{e}{2m} \right) \underline{\sigma} \cdot \underline{S} \, \psi \quad -(6)$$

$$:= g \left(\frac{e}{2m} \right) \underline{\sigma} \cdot \underline{S} \, \psi$$

where:

$$g = 2 \quad -(7)$$

is the g factor of the electron in Dirac theory.

Finally we:

$$\underline{S} \psi = \hbar m_s \psi \quad -(8)$$

So

$$H_2 \psi = 2 g_B \hbar m_s \psi \quad -(9)$$

) whee

$$m_s = \pm \frac{1}{2} \quad -(10)$$

In n theory:

$$H_2 \psi = \frac{ie\hbar}{2m} \, \underline{\sigma} \cdot \underline{\nabla} \, \frac{1}{m(r)^{1/2}} \, \underline{\sigma} \cdot \underline{A} \, \psi \quad -(11)$$

$$-(12)$$

$$= \frac{ie\hbar}{2m} \left(\frac{1}{m(r)^{1/2}} \, \underline{\sigma} \cdot \underline{\nabla} \, \underline{\sigma} \cdot \underline{A} + \underline{\sigma} \cdot \underline{\nabla} \left(\frac{1}{m(r)^{1/2}} \right) \underline{\sigma} \cdot \underline{A} \right) \psi + \cdots$$

$$= -\frac{1}{m(r)^{1/2}} \left(\frac{e\hbar}{2m} \right) \underline{\sigma} \cdot \underline{\nabla} \times \underline{A} \, \psi + \frac{ie\hbar}{2m} \underline{\sigma} \cdot \underline{\nabla} \left(\frac{1}{m(r)^{1/2}} \right) \underline{\sigma} \cdot \underline{A} \, \psi$$

$$-(13)$$

Now use the Pauli algebra:

$$\underline{\sigma} \cdot \underline{B} \, \underline{\sigma} \cdot \underline{A} = \underline{B} \cdot \underline{A} + i \underline{\sigma} \cdot \underline{B} \times \underline{A} \quad -(14)$$

to find that the real and physical part of the
Hamiltonian is:

$$-(15)$$

$$H_2 \psi = -\frac{e\hbar}{2m \, m(r)^{1/2}} \, \underline{\sigma} \cdot \underline{\nabla} \times \underline{A} \, \psi - \frac{e\hbar}{2m} \underline{\sigma} \cdot \underline{\nabla} \left(\frac{1}{m(r)^{1/2}} \right) \times \underline{A} \psi$$

$$= -\left(\frac{2}{m(r)^{1/2}} \right) \left(\frac{e}{2m} \right) \underline{S} \cdot \underline{B} \, \psi - 2 \left(\frac{e}{2m} \right) \underline{S} \cdot \underline{\nabla} \left(\frac{1}{m(r)^{1/2}} \right) \times \underline{A} \psi$$

$$= -\left(\frac{2}{m(r)^{1/2}} \right) \left(\frac{e\hbar}{2m} \right) m_s \, B_z \, \psi$$

$$-(15)$$

$$- 2 \left(\frac{e}{2m} \right) \underline{S} \cdot \underline{\nabla} \left(\frac{1}{m(r)^{1/2}} \right) \times \underline{A} \, \psi$$

3) Therefore from the first term in Eq. (15), the anomalous g factor of the electron is:

$$g = \frac{2}{n(r)^{1/2}} \quad -(16)$$

Experimentally:

$$g = 2.002319314 \quad -(17)$$

So

$$n(r)^{1/2} = 0.9988417 \quad -(18)$$

$$n(r) = 0.99942068 \quad -(19)$$

The <u>n theory</u> gives a <u>satisfactory description</u> of the <u>g factor of the electron to any accuracy</u>.