430(3): The Casimir Force in ECE Theory

The Casimir force is given by eq. (19) of the previous note:

$$\langle F \rangle = \frac{\hbar^2}{2m} \int \psi^* \nabla^2 \left(\frac{dm(r)}{dr} \cdot \frac{1}{\left(2m(r) - r\frac{dm(r)}{dr}\right)} \right) \psi \, d\tau \quad -(1)$$

where m is an effective mass and where ψ is the wavefunction of the Casimir effect:

$$\psi_n(X, Y, Z, t) = \exp\left(-i\left(\omega_n t - \kappa_X X - \kappa_Y Y\right)\right) \times \sin(\kappa_n Z) \quad -(2)$$

given in:

www.cs.mcgill.ca/~rwest/ -(3)

Here κ_X and κ_Y are wavefunctions parallel to the capacitor plate of the Casimir effect. The wave numbers are defined by:

$$\kappa_n = \frac{n\pi}{r} \quad -(4)$$

Let

$$f(r) = \nabla^2 \left(\frac{dm(r)}{dr} \cdot \frac{1}{\left(2m(r) - r\frac{dm(r)}{dr}\right)} \right) \quad -(5)$$

then:

$$\langle F \rangle = \frac{\hbar^2}{2m} \int \psi^* f(r) \psi \, d\tau \quad -(6)$$

$$= \frac{\hbar^2}{2m} \int f(r) \psi^* \psi \, d\tau$$

From eq. (2):

$$\psi_n^* \psi_n = \sin^2(\kappa_n Z) \quad -(7)$$

so the Casimir force for mode n is:

2)

$$\langle F_n \rangle = \frac{\hbar^2}{2m} \int f(r) \sin^2(k_n Z)\, d\tau \quad -(8)$$

$$= \frac{\hbar^2}{2m} \int f(r) \sin^2\left(n\pi \frac{Z}{r}\right) d\tau$$

The total Casimir force for all n modes is:

$$\langle F \rangle = \sum_n \langle F_n \rangle \quad -(9)$$

$$= \frac{\hbar^2}{2m} \int f(r) \sum_n \sin^2\left(n\pi \frac{Z}{r}\right) d\tau$$

and the Casimir force per unit area A is:

$$\frac{\langle F \rangle}{A} = \frac{\hbar^2}{2mA} \int f(r) \sum_n \sin^2\left(n\pi \frac{Z_1}{r}\right) d\tau \quad -(10)$$

Here A is the area of the plates between which the Casimir force is measured.

If there is only one mode of radiation, i.e. monochromatic radiation, then:

$$n = 1 \quad -(11)$$

and

$$\frac{\langle F \rangle}{A} = \frac{\hbar^2}{2mA} \int f(r) \sin^2\left(\pi \frac{Z_1}{r}\right) d\tau \quad -(12)$$

In spherical polar coordinates:

$$\cos\phi = \frac{Z_1}{r} \quad -(13)$$

so the Casimir force per area is:

3)

$$\frac{\langle F \rangle}{A} = \frac{\hbar^2}{2mA} \int f(r) \sin^2(\pi \cos\phi)\, d\tau \quad -(14)$$

For n modes of radiation:

$$\frac{\langle F \rangle}{A} = \frac{\hbar^2}{2mA} \int f(r) \sum_n \sin^2(n\pi \cos\phi)\, d\tau \quad -(15)$$

This is a hybrid theory that still retains elements of quantum electrodynamics from the wavefunction (2).

The Casimir effect can be developed on the classical level using eq. (39) of UFT428:

$$H = \frac{1}{m(r)^{1/2}} \frac{p^2}{2m} \left(1 + \frac{U_0}{2mc^2}\right) + m(r)^{1/2} \left(U_0 + mc^2\right) \quad -(16)$$

$$= E + U$$

where the total relativistic energy is:

$$E = \frac{1}{m(r)^{1/2}} \frac{p^2}{2m} \left(1 + \frac{U_0}{2mc^2}\right) + m(r)^{1/2} mc^2 \quad -(17)$$

and

$$U = m(r)^{1/2} U_0 \quad -(18)$$

Here

$$E_0 = m(r)^{1/2} mc^2 \quad -(19)$$

is the rest energy, and

$$U_0 = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(20)$$

4) is the Coulombic potential between two charges. The Casimir force can be considered classically as eq. (14) of UFT430(2):

$$F = -\frac{dm(r)}{dr}\left(\frac{m(r)^{1/2}}{2m(r) - r\frac{dm(r)}{dr}}\right)E \quad -(21)$$

where E is defined in general by eq. (17). The relevant E is obtained by removing the rest energy from consideration, so:

$$E_0 = E - m(r)^{1/2}mc^2 \quad -(22)$$

and in the limit:

$$U_0 << 2mc^2 \quad -(23)$$

we obtain:

$$E_0 = \frac{1}{m(r)^{1/2}}\frac{p^2}{2m} \quad -(24)$$

The Casimir force is therefore

$$F_0 = -\frac{dm(r)}{dr}\left(\frac{m(r)^{1/2}}{2m(r) - r\frac{dm(r)}{dr}}\right)E_0$$

$$= -\frac{dm(r)}{dr}\left(2m(r) - r\frac{dm(r)}{dr}\right)^{-1} \quad -(25)$$

i.e.

$$F_0 = -\frac{dm(r)}{dr}\,\frac{p^2/(2m)}{2m(r) - r\frac{dm(r)}{dr}} \quad -(26)$$

5) Eq. (26) is an entirely new type of force in physics, it is a force between the particle of mass m and a capacitor plate, and is neither gravitational nor electromagnetic. It is due purely to the nature of n space. It is an attractive force as measured experimentally using two very accurately engineered plates.

The units of the force in eq. (26) are:

$$F_0 = \text{kgm } m^2 s^{-2} / m = \text{kg } m s^{-2} \quad -(27)$$

The force can be expressed as:

$$\boxed{F_0 = -f(r)\, T} \quad -(28)$$

where

$$f(r) = \frac{dm(r)/dr}{2m(r) - r\frac{dm(r)}{dr}} \quad -(29)$$

and

$$T = \frac{1}{2} m v^2 \quad -(30)$$

Eq. (28) is the first classical description of the Casimir effect, and is a clear demonstration of the ability of n space to produce an experimentally observable force, the well known Casimir force. In an n space with property:

$$2n(r) = r\frac{dn(r)}{dr} \quad -(31)$$

the Casimir force is amplified to infinity.