431(5): Development of the Woods Saxon Potential

The Woods Saxon potential was developed in UFT 226 to UFT 230 and is:

$$U = -\frac{U_0}{1+\exp\left(\frac{r-R_0}{a_N}\right)} \quad -(1)$$

The region

$$r < R \quad -(2)$$

defines the interior of a fused entity such as $^{64}Ni + p$. Here a_N is the surface thickness of the nucleus. In UFT 227 - 229 this potential was developed in great detail, and in the ~~the~~ attractive force between neutrons and protons both inside and outside the fused entity. The attractive force due to (1) is:

$$F = -\frac{\partial U}{\partial r}$$

$$= -\frac{U_0}{\left(1+\exp\left(\frac{r-R}{a_N}\right)\right)^2}\left(1+\frac{a_0}{a_N}\exp\left(\frac{r-R}{a_N}\right)\right) \quad -(3)$$

where a_0 is a constant with units of metres which must be introduced to keep the units correct. The force (3) is identified with the m force:

$$F = -\frac{dm(r)}{dr}\,\frac{m(r)\,mc^2}{2m(r)-r\frac{dm(r)}{dr}} \quad -(4)$$

By comparison of eqs. (3) and (4), it is clear that when:

$$2m(r) = r\frac{dm(r)}{dr} \quad -(5)$$

ie:

$$a_o \longrightarrow \infty \quad -(6)$$

and

$$\frac{a_N}{a_o} \longrightarrow 0 \quad -(7)$$

so the normalized surface thickness a_N / a_o goes to zero.

In UFT 227 to UFT 229, when a nucleus consisting of Z protons interacted with one containing Z_2 protons, the region $r < R$ defined the interior of the fused entity modelled as a sphere of radius R. The attractive force is counterbalanced by a repulsive force between protons. In the region $r < R$ this is:

$$U_c = \frac{Z_1 Z_2 e^2}{R}\left(3 - \left(\frac{r}{R}\right)^2\right) \quad -(8)$$

In the region $r > R$ the repulsive force between ^{64}Ni and p is:

$$U_c = \frac{Z_1 Z_2 e^2}{r} \quad -(9)$$

The total potential is:

$$U_{total} = U + U_c \quad -(10)$$

If the proton is moving and ^{64}Ni is static,

... then:

$$F = -\frac{dm(r)}{dr}\left(\frac{m(r)}{2m(r) - r\frac{dm(r)}{dr}}\right)^{1/2} E \quad -(11)$$

where

$$E^2 = p^2c^2 + m(r)m^2c^4 \quad -(12)$$

The proto wave is then defined by Schrodinger quantization to give the d'Alembert equation:

$$\left(\Box + m(r)\left(\frac{mc}{\hbar}\right)^2\right)\phi = 0 \quad -(13)$$

The quantization proceeds with:

$$E^2\phi = -\hbar^2\frac{\partial^2\phi}{\partial t^2} \quad -(14)$$

$$p^2\phi = -\hbar^2\nabla^2\phi \quad -(15)$$

so eq. (13) follows, where the d'Alembertian is

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \quad -(16)$$

and where:

$$R = m(r)\left(\frac{mc}{\hbar}\right)^2 = q_a^{\nu}\,\partial^{\mu}\left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}\right) \quad -(17)$$

The proton is the proto wave defined by:

$$(\Box + R)\phi = 0 \quad -(18)$$