

Full extent of the Particle Spectrum

This is realized by considering papers such as UFT 249, UFT 247  
 The semi-classical derivation is essentially the minimal prescription:

$$E \rightarrow \bar{E} - e\phi \quad - (1)$$

$$p \rightarrow p - e\mathbf{A} \quad - (2)$$

electromagnetism. So the Dirac energy equation becomes:

$$(\bar{E} - e\phi)^2 = c^2 (p - e\mathbf{A})^2 + m^2 c^4 \quad - (3)$$

After quantization, Eq. (3) leads to:

$$(H_1 + H_2 + H_3)\psi = \bar{E}\psi \quad - (4)$$

where:

$$H_1 = mc^2 + e\phi \quad - (5)$$

$$H_2 = \frac{1}{2m} \underline{\sigma} \cdot (-i\hbar \underline{\nabla} - e\mathbf{A}) \underline{\sigma} \cdot (-i\hbar \underline{\nabla} - e\mathbf{A}) \quad - (6)$$

$$H_3 = \frac{1}{2m} \left( \underline{\sigma} \cdot (-i\hbar \underline{\nabla} - e\mathbf{A}) e\phi \underline{\sigma} \cdot (-i\hbar \underline{\nabla} - e\mathbf{A}) \right) \quad - (7)$$

Considering initially the interaction between a proton and neutron is Mikowski spacetime, where:

$$n(r) = \frac{1}{r} \quad - (8)$$

the electromagnetic momentum  $e\mathbf{A}$  is replaced by the momentum  $\mathbf{v}$  of the strong field, and  $e\phi$  is replaced by the potential of interaction between the strong field and neutron. So eq. (4) becomes:

$$(H_1 + H_2 + H_3)\psi = \bar{E}\psi \quad - (9)$$

where:

$$H_1 = mc^2 + U \quad - (10)$$

$$H_2 = \frac{1}{2m} \underline{\sigma} \cdot (-i\hbar \underline{\nabla} - \mathbf{v}) \underline{\sigma} \cdot (-i\hbar \underline{\nabla} - \mathbf{v}) \quad - (11)$$

2)

and:

$$H_3 = \frac{1}{2m} \underline{\sigma} \cdot (-i\hbar \underline{\nabla} - \underline{q}) U \underline{\sigma} \cdot (-i\hbar \underline{\nabla} - \underline{q}) - (12)$$

At this stage, the strong force between the proton and neutron is being treated as classical, but there is already a lot of structure in the eqns. This structure parallels that of the semiclassical interaction between the electromagnetic field and the electron in the Dirac theory: the g factor of two, the half integral spin, the Zeeman effect and spin orbit coupling.

The expectation value of eq. (10) is:

$$E = \langle H_1 \rangle = mc^2 + \langle U \rangle - (13)$$

where

$$U = \frac{q_1 q_2}{4\pi \epsilon_0 r^2} - (14)$$

$$\text{so } E = \langle U \rangle = \int \psi^* \frac{q_1 q_2}{4\pi \epsilon_0 r^2} \psi d\tau - (15)$$

and produces quantized energy levels and a mass spectrum defined by:

$$m = \frac{E}{c^2} - (16)$$

A mass spectrum appears even on the semiclassical level, and even from the simplest Hamiltonian,  $H_1$ . These masses can be observed in the collision of a proton and atom, specifically from the repulsive potential (14) between the proton and the protons of the atomic nucleus. The particles released in the collision are observed as neutrons.

Note carefully that on the Minkowski level there is no string force, because in the theory the string force is produced by  $m(r) \neq 0; \frac{dm(r)}{dr} \neq 0$  - (17)

Next consider the Hamiltonian (11) of the Minkowski level. It can be developed as:

$$H_2 \psi = \frac{1}{2m} \left( -\hbar^2 \nabla^2 \psi + \underline{v}^2 \psi + i\hbar \underline{\nabla} \cdot (\underline{v} \psi) - \hbar \underline{\sigma} \cdot \underline{\nabla} \times (\underline{v} \psi) + i\hbar \underline{v} \cdot \underline{\nabla} \psi - \hbar \underline{\sigma} \cdot \underline{v} \times \underline{\nabla} \psi \right) = E \psi. \quad - (18)$$

So many new energy levels and masses are produced from the expectation values:

$$E = \langle H_2 \rangle \quad - (19)$$

and there are energy levels produced by the  $\underline{v}$  dependent terms:

$$E = \frac{1}{2m} \int \psi^* \left( \underline{v}^2 \psi + i\hbar \underline{\nabla} \cdot (\underline{v} \psi) - \hbar \underline{\sigma} \cdot \underline{\nabla} \times (\underline{v} \psi) + i\hbar \underline{v} \cdot \underline{\nabla} \psi - \hbar \underline{\sigma} \cdot \underline{v} \times \underline{\nabla} \psi \right) d\tau \quad - (20)$$

Finally describe energy levels produced in Minkowski space by the Hamiltonian  $H_3$

$$E = \langle H_3 \rangle = \int \psi^* H_3 \psi d\tau \quad - (21)$$

In order to describe the interaction between a proton and neutron in a fully self consistent way, the string force is needed as described in Eq. (47) of YFT428:

$$4) H = m(r)^{1/2} (mc^2 + U) + H_1 + H_2 + H_3 + H_4 \quad (22)$$

where:  $H_1 = \frac{1}{2m} \frac{\sigma \cdot p}{m(r)^{1/2}} \frac{1}{m(r)^{1/2}} \frac{\sigma \cdot p}{m(r)^{1/2}} \quad (23)$  (24)

$$H_2 = -\frac{1}{2m} \left( \frac{\sigma \cdot \nabla}{m(r)^{1/2}} \frac{1}{m(r)^{1/2}} \frac{\sigma \cdot p}{m(r)^{1/2}} + \frac{\sigma \cdot p}{m(r)^{1/2}} \frac{1}{m(r)^{1/2}} \frac{\sigma \cdot \nabla}{m(r)^{1/2}} \right)$$

$$H_3 = \frac{1}{2m} \frac{\sigma \cdot \nabla}{m(r)^{1/2}} \frac{1}{m(r)^{1/2}} \frac{\sigma \cdot \nabla}{m(r)^{1/2}} \quad (25)$$

$$H_4 = \frac{1}{2m} \frac{\sigma \cdot (p - \nabla)}{2mc^2 m(r)^{1/2}} \frac{U}{m(r)^{1/2}} \frac{\sigma \cdot (p - \nabla)}{m(r)^{1/2}} \quad (26)$$

The potential  $U$  in these equations is the sum of the potential of the  $m$  force and the potential of the Coulombic repulsion. The string force is identified with the  $m$  force:

$$F = -\frac{dm(r)}{dr} \frac{m^{1/2}(r)}{2m(r) - r \frac{dm(r)}{dr}} \quad E \quad (27)$$

where:  $E^2 = p^2 c^2 + m(r)^2 m^2 c^4 \quad (28)$

is the energy of the  $m$  force.

In eqs. (23) to (26),  $m$  is the mass of the proton or neutron and  $\nabla$  is the vector potential of the string field, i.e. its momentum. In eq. (28),  $m$  is the mass of the pion being considered. The energy from eq. (28)

$$is \quad E = \frac{p^2 c^2}{E + m(r)^{1/2} mc^2} + m(r)^{1/2} mc^2 \quad (29)$$

In the low velocity approximation:

$$E = \frac{p^2}{2m(r)^{1/2}} + m(r)^{1/2} mc^2 \quad - (30)$$

The expectation value of energy are:

$$\langle E \rangle = -\frac{\hbar^2}{2m} \int \psi^* \nabla^2 \left( \frac{\psi}{m(r)^{1/2}} \right) d\tau \quad - (31)$$

$$+ mc^2 \int \psi^* m(r)^{1/2} \psi d\tau$$

and this gives a mass spectrum. If  $m$  is considered to be the mean mass of the pion, it is known from experiment that eq. (31) should give the pions.

### Quark Structure of the Proton and Neutron

From an equation such as (23), a particle such as a proton of mass  $m$  gives energy levels:

$$E = \langle H_1 \rangle = -\frac{\hbar^2}{2m} \int \sigma \cdot \nabla \left( \frac{1}{m(r)^{1/2}} \sigma \cdot \nabla \psi \right) \quad - (32)$$

which is a mass spectrum. This can be identified with quark masses inside the proton.

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