134(3) : Planck Quantization in n Space

Consider as in UFT415 the infinitesimal line element the stationary metric representing the most general spherically symmetric spacetime:

$$ds^2 = c^2 d\tau^2 = n(r) c^2 dt^2 - d\underline{r} \cdot d\underline{r} \quad -(1)$$

here:

$$d\underline{r} \cdot d\underline{r} = \frac{dr^2}{n(r)} + r^2 d\phi^2 \quad -(2)$$

in plane polar coordinates (r, ϕ). By definition:

$$d\underline{r} = \frac{\partial \underline{r}}{\partial r} dr + \frac{\partial \underline{r}}{\partial \phi} d\phi \quad -(3)$$

so

$$d\underline{r} \cdot d\underline{r} = \left(\frac{\partial \underline{r}}{\partial r} dr + \frac{\partial \underline{r}}{\partial \phi} d\phi \right) \cdot \left(\frac{\partial \underline{r}}{\partial r} dr + \frac{\partial \underline{r}}{\partial \phi} d\phi \right)$$

$$= \frac{dr^2}{n(r)} + r^2 d\phi^2 \quad -(4)$$

A possible solution is:

$$\left(\frac{\partial \underline{r}}{\partial r} \right)^2 dr^2 = \frac{dr^2}{n(r)} \quad -(5)$$

$$\left(\frac{\partial \underline{r}}{\partial \phi} \right)^2 d\phi^2 = r^2 d\phi^2 \quad -(6)$$

$$\left(\frac{\partial \underline{r}}{\partial r} \right) \cdot \left(\frac{\partial \underline{r}}{\partial \phi} \right) dr\, d\phi = 0 \quad -(7)$$

Eqs. (5) to (7) imply:

$$\underline{r} = \frac{r}{n(r)^{1/2}} \underline{e}_r + r\phi\, \underline{e}_\phi \quad -(8)$$

So the radial coordinate of n_1 theory is:

$$r_1 = \frac{r}{n(r)^{1/2}} \quad -(9)$$

So momentum quantization is changed fundamentally in Note 434(3) to:

$$p_1 \phi = -i\hbar \frac{\partial \phi}{\partial r_1} \quad -(10)$$

Energy quantization is changed to:

$$E_1 \phi = i\hbar \frac{\partial \phi}{\partial t_1} \quad -(11)$$

From eq. (1):

$$\boxed{dt_1 = n(r)^{1/2} dt} \quad -(12)$$

So

$$\boxed{E_1 \phi = \frac{i\hbar}{n(r)^{1/2}} \frac{\partial \phi}{\partial t}} \quad -(13)$$

and:

$$\langle E_1 \rangle = i\hbar \int \phi^* \frac{1}{n(r)^{1/2}} \frac{\partial \phi}{\partial t} d\tau \quad -(14)$$

In the limit of:

$$n(r) = 1 \quad -(15)$$

$$\langle E \rangle = i\hbar \int \phi^* \frac{\partial \phi}{\partial t} d\tau \quad -(16)$$

and using the wave function:

$$\phi = e^{-i\omega t} \quad -(17)$$

we obtain:

$$\langle E \rangle = \hbar\omega \quad -(18)$$

This familiar Planck quantization is changed in m space to:

$$\langle E_1 \rangle = i\hbar \int \phi^* \frac{1}{m(r)^{1/2}} \frac{\partial \phi}{\partial t} d\tau \quad -(19)$$

<u>So the m space introduces new energy levels of photons.</u>

<u>Summary of Quantization in m Space</u>

This is:

$$E_1 \phi = i\hbar \frac{\partial \phi}{\partial t_1} \quad -(20)$$

$$= \frac{i\hbar}{m(r)^{1/2}} \frac{\partial \phi}{\partial t}$$

and

$$\underline{p_1} \phi = -i\hbar \underline{\nabla_1} \phi \quad -(22)$$

for example:

$$p_1 \phi = -i\hbar \frac{\partial \phi}{\partial r_1} \quad -(23)$$

$$= -i\hbar \left(\frac{2m(r)^{3/2}}{2m(r) - \frac{dm(r)}{dr}} \right) \frac{d\phi}{dr}$$

So new momentum levels appear in m space.