435(1): Properties of the Time Dependent Schroedinger Equation in n Space

Consider: $$E = i\hbar \frac{d\psi}{dt_1} = H_1 \psi \quad -(1)$$

It follows that: $$\hbar\omega \int \psi^* \frac{1}{n(r)^{1/2}} \psi \, d\tau = \langle H_1 \rangle \quad -(2)$$

Denoting: $$E_1 = \langle H_1 \rangle \quad -(3)$$

it follows that: $$E_1 = \hbar\omega \int \psi^* \frac{1}{n(r)^{1/2}} \psi \, d\tau \quad -(4)$$

For example, the Lamb shift in atomic H increases the energy of $2^2S_{1/2}$, and changes its frequency from ω to ω_1. The new frequency is:

$$\omega_1 = \omega \int \psi_H^* \frac{1}{n(r)^{1/2}} \psi_H \, d\tau \quad -(5)$$

so $$\boxed{\frac{\omega_1}{\omega} = \int \psi_H^* \frac{1}{n(r)^{1/2}} \psi_H \, d\tau} \quad -(6)$$

Both ω_1 and ω are known with great precision experimentally, so $n(r)^{1/2}$ can be found for the energy change in $^2S_{1/2}$. There is no energy change in $^2P_{1/2}$ so in this case:

$$n(r)(^2P_{1/2}) = 1 \quad -(7)$$

2) So in this interpretation, each energy level is associated with its own $n(r)^{-1+1/2}$ function. The Lamb shifted energy levels each have $n(r)$ different from one. The unshifted levels all have $n(r)$ of unity.

This method removes the necessity of having to model $n(r)$, and each Lamb shift is caused by its characteristic $n(r)$ function.

The energy levels of the Dirac eqn are:

$$E_{nJ} = E_n \left(1 + \left(\frac{\alpha}{n}\right)^2 \left(\frac{n}{J+1/2} - \frac{3}{4}\right)\right) \quad -(7)$$

where E_n are the energy levels of the Schroedinger eqn. Here J is the total angular momentum quantum number:

$$J = L+S, \cdots, L-S \quad -(8)$$

and

$$L = 0, \cdots, n-1 \quad -(9)$$

Here α is the fine structure constant. In the Dirac eqn there is no Lamb shift because J is the same for $^2P_{1/2}$ and $^2S_{1/2}$:

$$^2S_{1/2} : n=1,\ L=0,\ S=1/2,\ J=1/2$$
$$^2P_{1/2} : n=1,\ L=1,\ S=-1/2,\ J=1/2$$

However, in generally covariant H eqn of Heavy, the Lamb shift is given by eqn (6). The Dirac eqn is accurately solved, so the ψ_H wavefunctions are used in wavefunction of the complicated, so the first approximation.
