The m theory of the rest energies of any particle

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3 Numerical analysis and graphics

3.1 Some examples with Bessel functions

In this section we further inspect some details of m theory applied to elementary particles. In UFT 431 we had identified Bessel functions as possible solutions to the wave equation. Before discussing the wave equation of m theory in more detail in the next section, we consider the suitability of Bessel functions in the wave function context.

In Fig. 1 the Bessel function $j_1(x)$ is graphed as an example, together with its derivative $dj_1(x)/dx$ and its integral $\int j_1(x)dx$. Differentiation gives a sum of other Bessel functions, integration leads to an expression with a hypergeometric series. It is seen that all three expressions give similarly oscillating functions with a certain phase shift.

Alternatively, we can consider the first parameter a of the Bessel function as a variable, evaluating $j_a(x_0)$, $dj_a(x_0)/da$ and $\int j_a(x_0)da$ for a fixed $x_0 = 1$. The corresponding results are graphed in Fig. 2, indicating that increasing a leads to functions falling asymptotically to zero.

A wave function must be normalizable:

$$\int \psi^*(r)\,\psi(r)\,r^2 dr = N \tag{12}$$

for the radial coordinate r with $N < \infty$. This is not the case for Bessel functions and squared Bessel functions. Therefore we have to augment them by a function dropping fast enough to zero. We define

$$\psi(r) := j_{r_0^2}(r)\exp(-\frac{r}{2r_0}) \tag{13}$$

which gives $N = 0.930$ for $r_0 = 2$. The wave function has to be normalized with this factor:

$$\psi(r) \rightarrow \frac{1}{\sqrt{N}}\psi(r). \tag{14}$$

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With this normalized wave function we can compute the expectation value of the m function. We define m(r) as in earlier papers by

$$\mathrm{m}(r) = 2 - \exp\left(\log(2)\exp(-\frac{r}{r_0})\right). \tag{15}$$

Then the expectation value is

$$\int \psi^*(r)\,\mathrm{m}(r)\,\psi(r) r^2 dr = 0.945. \tag{16}$$

For demonstration we have graphed in Fig. 3 the original Bessel function for $r_0 = 2$, the modified wave function (13) and the integrand of the expectation integral integral (16). It is clearly seen that the modified functions drop zo zero. The calculation of the expectation vaule can be formulated scale invariantly, i.e. using the true particle radius in fm does not change the result. The masses of elementary particles will be computed in a later paper.

3.2 Some details on the wave equation

The wave equation was derived from fundamentals of ECE theory in UFT 51. The ECE Lemma, Eq. (7.24) of UFT 51, reads:

$$\Box\, q^a{}_\mu = R\, q^a{}_\mu \tag{17}$$

with tetrad $q^a{}_\mu$ and scalar curvature R. The Einstein Ansatz (7.38/39) is

$$R = -kT \tag{18}$$

where k is the Einstein constant and T is the energy-momentum scalar. In quantum physicls we have to replace this by

$$kT \to \left(\frac{mc}{\hbar}\right)^2, \tag{19}$$

which leads to the Proca equation (7.18) for photon mass m_p:

$$\left(\Box + \left(\frac{m_p c}{\hbar}\right)^2\right) A^\nu = 0 \tag{20}$$

where A^ν are the components of the electromagnetic potential. Alternatively to the Proca equation follows the Dirac equation (7.48) with spinors ϕ:

$$\left(\Box + \left(\frac{m_e c}{\hbar}\right)^2\right) \phi = 0 \tag{21}$$

for electron mass m_e. Using only the space part of the d'Alembert operator

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \tag{22}$$

we obtain

$$\left(-\nabla^2 + \left(\frac{mc}{\hbar}\right)^2\right) \psi = 0 \tag{23}$$

for the wave function ψ of a particle with mass m.

The signs in the wave equation – although seemingly a minor difference – are very important. The solutions of the differential equation (in one dimension)

$$\frac{d^2\psi(x)}{dx^2} + \kappa^2\psi(x) = 0 \tag{24}$$

are oscillatory:

$$\psi(x) = k_1 \sin(\kappa x) + k_2 \cos(\kappa x), \tag{25}$$

while the solutions of

$$-\frac{d^2\psi(x)}{dx^2} + \kappa^2\psi(x) = 0 \tag{26}$$

are exponential:

$$\psi(x) = k_1 \exp(-\kappa x) + k_2 \exp(\kappa x). \tag{27}$$

Obviously Eq. (23) is of type (26) and has exponential solutions. Setting the constant $k_2 = 0$ gives an exponentially decreasing wave function and charge density, which is physically meaningful. For spherical problems, the corresponding radial differential equation (with spherical ∇^2) is not analytically solvable. The solutions are exponential as above in the far field limit. When the differential equation contains a radius-dependent κ as is the case of m theory, see Eq. (6):

$$\left(-\nabla^2 + \mathrm{m}(r)\left(\frac{mc}{\hbar}\right)^2\right)\psi = 0, \tag{28}$$

then the exponential solution is augmented by oscillations similarly as in Fig. 3.

3.3 Towards a radial function for elementary particles

Eq. (28) is similar to the radial Schrödinger equation with angular momentum zero. It is an eigenvalue equation for the mass m with eigenfunctions ψ. The same solution method as for the radial Schrödinger equation should be applicable. We solved a similar problem in UFT 260 for the so-called Partons.

In the Schrödinger equation the spherical operator ∇^2 is simplified by the function substitution

$$\psi(r) = \frac{\phi(r)}{r}. \tag{29}$$

The Schrödinger equation then reads

$$\left(\frac{d^2}{dr^2} + k^2(r)\right)\phi = 0 \tag{30}$$

with the non-differential factor

$$k^2(r) = \frac{2m}{\hbar^2}\left(E - \frac{l(l+1)\hbar^2}{2mr^2} - V(r)\right). \tag{31}$$

We can use the same substitution (29) for Eq. (28). Then we have

$$\left(\frac{d^2}{dr^2}+k^2(r)\right)\phi=0 \tag{32}$$

with

$$k^2(r)=-\mathrm{m}(r)\left(\frac{mc}{\hbar}\right)^2. \tag{33}$$

Notice that $k^2(r)$ is negative. Solving the radial Schrödinger equation is tricky because the boundary conditions cannot be given by defining ϕ and $d\phi/dr$ at one point. Instead two function values of ϕ have to be given at two points so that the solution does not diverge for large r. Non-divergence appears only for discrete values of E, the eigenvalues. A special numerical scheme is commonly used for the solution procedure, called Fox-Goodwin or Numerov method. This method has been applied in UFT 260 for solving the radial equation for Partons. The method has still to be worked out for Eqs. (32/33). We present only an example where ϕ and $d\phi/dr$ have been given at $r=0$ so that the standard Runge-Kutta solver of Maxima can be applied. In Fig. 4 the functions $\phi(r)$ and $\psi(r)$ are graphed for certain parameters. It is seen that the solution ψ resembles a hyperbola while ϕ is nearly linear. It is not clear if the application of the Numerov method will give physical solutions because the factor $k^2(r)$ is purely negative. These complicated numerical problems have to be solved in future.

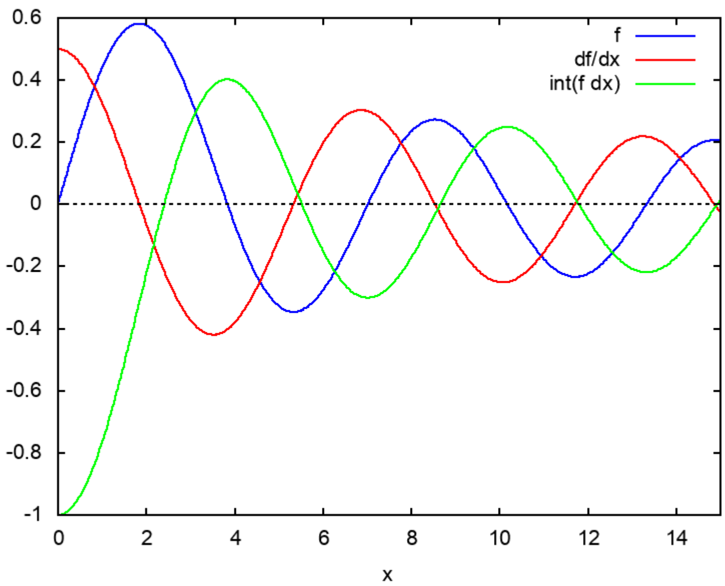

Figure 1: Example for Bessel function $j_1(x)$, its derivative and integral.

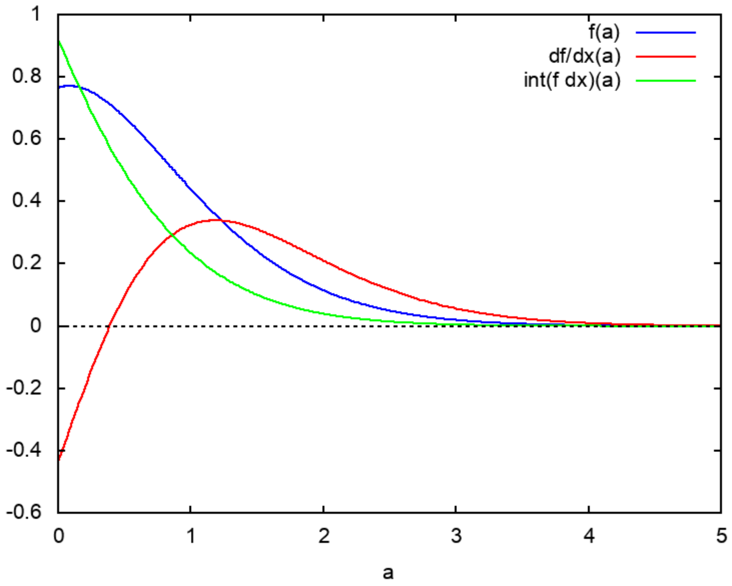

Figure 2: Example for Bessel function $j_a(x_0)$ for fixed x_0, its derivative and integral.

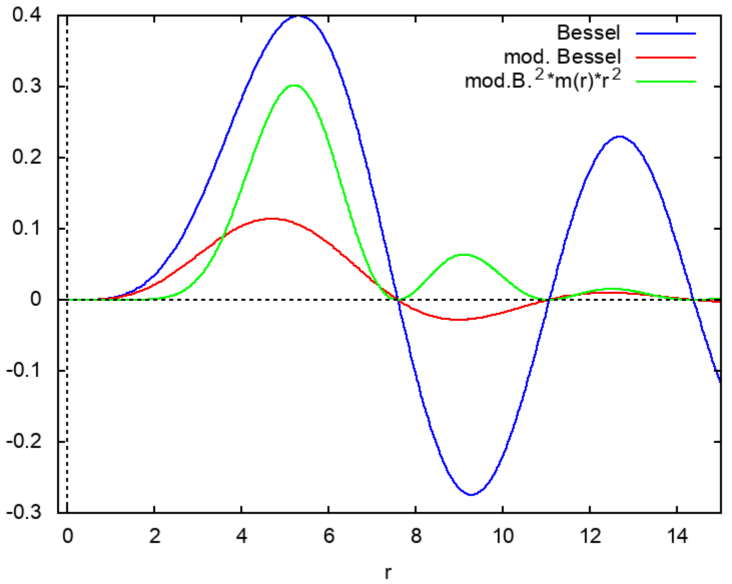

Figure 3: Bessel function, modified Bessel function and spherical integrand of Eq. (16).

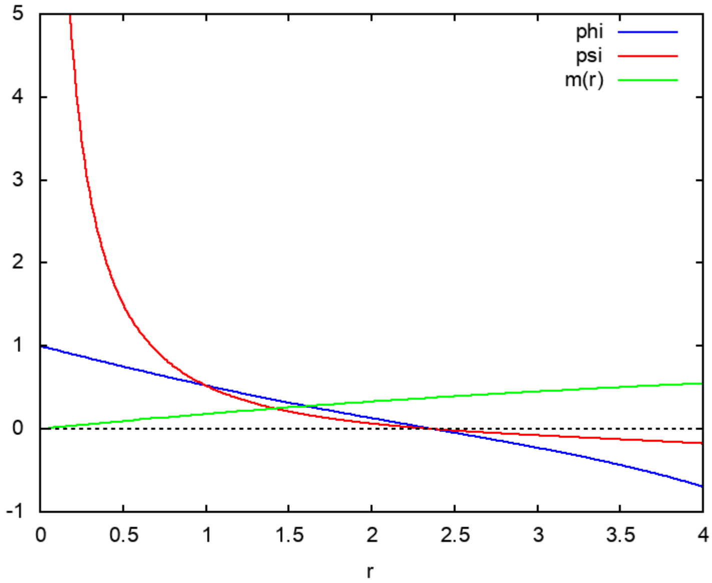

Figure 4: Preliminary solution of Eqs. (32/33), and function m(r).