```
(%i1)  kill(all);
(%o0)  done

(%i1)  assume(b>a,a>0,b>0);
(%o1)  [b>a, a>0, redundant]

(%i2)  integrate(1/x,x,a,b);
(%o2)  log(b)−log(a)
```

## 1 Eq.(11)

```
(%i3)  drdr_vac: 1/(2*epsilon_0*%pi^2)*e^2/(h_bar*c)*(h_bar/(m*c))^2
       *integrate(1/kappa,kappa,%pi/a_0,m*c/h_bar);
```

Is $a_0\ h_{bar}\ (a_0\ c\ m-\pi\ h_{bar})$ positive, negative or zero?p;
Is $c\ h_{bar}\ m$ positive, negative or zero?p;
Is $a_0$ positive or negative?p;

$$(\%o3)\quad \frac{e^2\ h_{bar}\left(\log\left(\frac{c\ m}{h_{bar}}\right)-\log\left(\frac{\pi}{a_0}\right)\right)}{2\ \pi^2\ c^3\ \epsilon_0\ m^2}$$

```
(%i4)  drdr_vac: 1/(2*epsilon_0*%pi^2)*e^2/(h_bar*c)*(h_bar/(m*c))^2
       *integrate(1/kappa,kappa,%pi/a_0,m*c/h_bar);
```

Is $a_0\ h_{bar}\ (a_0\ c\ m-\pi\ h_{bar})$ positive, negative or zero?n;
Is $a_0$ positive or negative?p;
Is $c\ h_{bar}\ m$ positive, negative or zero?p;

$$(\%o4)\quad \frac{e^2\ h_{bar}\left(\log\left(\frac{c\ m}{h_{bar}}\right)-\log\left(\frac{\pi}{a_0}\right)\right)}{2\ \pi^2\ c^3\ \epsilon_0\ m^2}$$

```
(%i5)  alpha_1: e^2/(4*%pi*epsilon_0*h_bar*c);
```

$$(\%o5)\quad \frac{e^2}{4\ \pi\ c\ \epsilon_0\ h_{bar}}$$

```
(%i6)  a_0: 4*%pi*epsilon_0*h_bar^2/(m*e^2);
```

$$(\%o6)\quad \frac{4\ \pi\ \epsilon_0\ {h_{bar}}^2}{e^2\ m}$$

```
(%i7)  drdr_vac_1: logcontract(ev(drdr_vac));
```

$$(\%o7)\quad -\frac{e^2\ \log\left(\frac{e^2}{4\ c\ \epsilon_0\ h_{bar}}\right)\ h_{bar}}{2\ \pi^2\ c^3\ \epsilon_0\ m^2}$$

```
(%i8)  drdr_vac_2: factor(ratsubst(alpha, alpha_1, %));
```

$$(\%o8)\quad -\frac{2\ \alpha\ \log(\pi\ \alpha)\ {h_{bar}}^2}{\pi\ c^2\ m^2}$$

## 2 Eq.(15) (generally)

```
(%i9)  E15: e^2/epsilon_0*rho(0);
```

$$(\%o9)\quad \frac{\rho(0)\ e^2}{\epsilon_0}$$

## 3 Eq.(16) (generally)

```
(%i10)  E16: 1/6*drdr_vac_2*E15;
```

$$(\%o10)\quad -\frac{\rho(0)\ \alpha\ \log(\pi\ \alpha)\ e^2\ {h_{bar}}^2}{3\ \pi\ c^2\ \epsilon_0\ m^2}$$

(%i11) `ratsubst(alpha,alpha_1,E16);`

(%o11) $$-\frac{4\,\rho(0)\,\alpha^2\log(\pi\,\alpha)\,h_{bar}{}^3}{3\,c\,m^2}$$

## 4 Eq.(15) (psi_2s)

(%i12) `R: (Z^(3/2)*(2-(Z*r)/a[0])*%e^(-(Z*r)/(2*a[0])))/(2^(3/2)*a[0]^(3/2));`

(R) $$\frac{Z^{3/2}\left(2-\frac{Z\,r}{a_0}\right)\%e^{-\frac{Z\,r}{2\,a_0}}}{2^{3/2}\,a_0^{3/2}}$$

(%i25) `R2s0: ev(R, [Z=1,r=0]);`

(R2s0) $R$

(%i14) `E15: e^2/epsilon_0*R2s0^2;`

(E15) $$\frac{e^2}{2\,a_0^3\,\epsilon_0}$$

## 5 Eq.(16) (psi_2s)

(%i15) `E16: 1/6*drdr_vac_2*E15;`

(E16) $$-\frac{\alpha\log(\pi\,\alpha)\,e^2\,h_{bar}{}^2}{6\,\pi\,a_0^3\,c^2\,\epsilon_0\,m^2}$$

(%i16) `E16a: ratsubst(a_0,a[0],E16);`

(E16a) $$-\frac{\alpha\log(\pi\,\alpha)\,e^8\,m}{384\,\pi^4\,c^2\,\epsilon_0{}^4\,h_{bar}{}^4}$$

(%i17) `E16b: ratsubst(alpha,alpha_1,E16a);`

(E16b) $$-\frac{2\,\alpha^5\log(\pi\,\alpha)\,c^2\,m}{3}$$

## 6 Eq.(29)

(%i18) `a_0: 5.2917721092e-11;`

(a_0) $5.2917721092\,10^{-11}$

(%i19) `2.623e-27/(36*a_0^2);`

(%o19) $2.601917510622348\,10^{-8}$

## 7 Comparison

(%i20) `kill(R);`

(%o20) *done*

(%i21) `f: 1/sqrt(1-exp(-r/R))-1;`

(f) $$\frac{1}{\sqrt{1-\%e^{-\frac{r}{R}}}}-1$$

(%i22) `E29: integrate(f,r,0,r_1);`

*Is* $r_1$ *positive, negative or zero?* `p;`

*Is* $\%e^{\frac{r_1}{R}}-1$ *positive, negative or zero?* `p;`

(E29) $$\frac{R\log\left(\%e^{-\frac{2\,r_1}{R}}\left(2\,\%e^{\frac{2\,r_1}{R}}+2\sqrt{\%e^{\frac{r_1}{R}}-1}\,\%e^{\frac{3\,r_1}{2\,R}}-\%e^{\frac{r_1}{R}}\right)\right)-2\,R\log\left(-\%e^{-\frac{r_1}{2\,R}}\left(\sqrt{\%e^{\frac{r_1}{R}}-1}-\%e^{\frac{r_1}{2\,R}}\right)\right)-2\,r_1}{2}$$

(%i23)
```
E29a: ev(E29, [R=1.9e-8]);
```

(E29a)
$$\left(1.9\ 10^{-8}\log\left(\%e^{-1.052631578947368\ 10^{8}\ r_1}\left(2\ \%e^{1.052631578947368\ 10^{8}\ r_1}+2\sqrt{\%e^{5.263157894736842\ 10^{7}\ r_1}-1}\ \%e^{7.894736842105263\ 10^{7}\ r_1}-\%e^{5.263157894736842\ 10^{7}\ r_1}\right)\right)-3.8\ 10^{-8}\log\left(-\%e^{-2.631578947368421\ 10^{7}\ r_1}\left(\sqrt{\%e^{5.263157894736842\ 10^{7}\ r_1}-1}-\%e^{2.631578947368421\ 10^{7}\ r_1}\right)\right)-2\ r_1\right)/2$$

(%i24)
```
wxplot2d([E29a], [r_1,0e-5,1.5e-7])$
```

(%t24)