336(1): The Harmonic Oscillator in m Space

The usual time independent wave function of the harmonic oscillator in non relativistic quantum mechanics is:

$$\phi(r) = N H(y) \exp\left(-\frac{y^2}{2}\right) \quad -(1)$$

where N is a normalization factor and $H(y)$ are the Hermite polynomials. Here:

$$y = \left(\frac{m\omega}{\hbar}\right)^{1/2} r \quad -(2)$$

where:

$$\omega = \left(\frac{k}{m}\right)^{1/2} \quad -(3)$$

Its non relativistic energy levels are:

$$E = \left(n + \frac{1}{2}\right)\hbar\omega, \quad -(4)$$

and when

$$n = 0 \quad -(5)$$

the zero point energy is:

$$E_0 = \frac{1}{2}\hbar\omega \quad -(6)$$

and has no classical counterpart. The usual potential energy of the harmonic oscillator is:

$$U(r) = \frac{1}{2} k r^2 \quad -(7)$$

and the usual non relativistic kinetic energy is:

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$$T = \frac{p^2}{2m} \quad -(8)$$

Therefore its Hamiltonian in classical physics is

$$H_0 = \frac{p^2}{2m} + \frac{1}{2} k r^2 \quad -(9)$$

In m space its Hamiltonian is:

$$H = \frac{p^2}{2m\,m(r)} + \frac{1}{2}\frac{k r^2}{m(r)}$$

$$= \frac{H_0}{m(r)} \quad -(10)$$

In m space the Schroedinger equation of the harmonic oscillator is:

$$H\phi = E\phi \quad -(11)$$

i.e.:

$$-\frac{\hbar^2}{2m}\nabla^2\left(\frac{\phi}{m(r)}\right) + \frac{1}{2}\frac{k r^2}{m(r)}\phi = E\phi \quad -(12)$$

here $\phi(r)$ is given by Eq. (4).

In the limit of:

$$m(r) \rightarrow 1 \quad -(13)$$

$$E = \left(n + \frac{1}{2}\right)\hbar\omega \quad -(14)$$

where:

$$n = 0, 1, 2, \ldots$$

and the wave function is given by Eq (4) under condition (13).

3) The normalization constant in the limit (13) is:

$$N = \frac{1}{\left(2^n n! \pi^{1/2}\right)^{1/2}} \quad -(15)$$

and $H_n(y)$ in the limit (13) are the Hermite polynomials.

Therefore the usual wave function is:

$$\phi(r) = N H(y) \exp\left(-\frac{y^2}{2}\right) \quad -(16)$$

where

$$y = \left(\frac{m\omega}{\hbar}\right)^{1/2} r \quad -(17)$$

and the usual Schroedinger equation:

$$H\phi = E\phi \quad -(18)$$

gives the energy levels:

$$E = \left(n + \frac{1}{2}\right)\hbar\omega \quad -(19)$$

In order to obtain this well known result from the time dependent Schroedinger equation:

$$i\hbar \frac{\partial \phi}{\partial t} = H\phi = E\phi \quad -(20)$$

the total wave function of the harmonic oscillator must be:

$$\boxed{\phi(r, t) = \exp\left(-i\left(n + \frac{1}{2}\right)\omega t\right)\phi(r)} \quad -(21)$$

It follows from eq. (21) that:

$$\frac{\partial \phi}{\partial t} = -i\omega t\left(n + \frac{1}{2}\right)\phi \quad -(22)$$

) and

$$E\psi = i\hbar \frac{d\psi}{dt} = \left(n + \frac{1}{2}\right)\hbar\omega\psi \quad -(23)$$

$$\langle E \rangle = \int \psi^* E \psi \, d\tau = \left(n + \frac{1}{2}\right)\hbar\omega \quad -(24)$$

Q.E.D.

However, it is known that the most general form of the wave function is that of ECE space. This is a wave function of general relativity. Eq. (1) on the other hand, is not relativistic, and the time dependent wave function, eq. (21), is not relativistic.

The generally covariant wave function of general relativity is that of ECE theory:

$$\psi(r,t) = \exp\left(-i\left(n + \frac{1}{2}\right)m^{1/2}(r)\,\omega t\right)\psi(r) \quad -(25)$$

where:

$$\psi(r) = NH(y_1)\exp\left(-\frac{y_1^2}{2}\right) \quad -(26)$$

where

$$y_1 = \left(\frac{m\omega}{\hbar}\right)^{1/2}\frac{r}{m(r)^{1/2}} \quad -(27)$$

Using the wave function (25) in the time dependent Schroedinger equation (20)

5) gives the generally covariant energy levels:

$$\boxed{E = \left(n + \frac{1}{2}\right)\hbar\omega \int \psi^* n(r)^{1/2} \psi \, d\tau} \quad -(28)$$

of the harmonic oscillator.

Eq. (28) describes the effect of the vacuum on the harmonic oscillator. "The vacuum" is defined as n space.

The wave function to be used in eq. (28) is given by eq. (26). In the limit:

$$n(r) \rightarrow 1 \quad -(29)$$

Eq. (28) reduces to:

$$E = \left(n + \frac{1}{2}\right)\hbar\omega \int \psi^* \psi \, d\tau \quad -(30)$$

$$= \left(n + \frac{1}{2}\right)\hbar\omega$$

which is eq. (4), Q.E.D.

Depending on the choice of $n(r)$, there are many more energy levels of the harmonic oscillator in general relativity than in non-relativistic quantum mechanics (the Schrodinger equation). The effect of the vacuum is to produce these new energy levels. They could be observed for example in vibrational spectroscopy.

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