436(3): General Law for Spectral Effect of the Vacuum

As in previous notes the energy levels of a spectrum in the presence of the vacuum are given by:

$$E = \hbar\omega \int \psi^* n(r)^{1/2} \psi \, d\tau, \quad -(1)$$

in which the volume element is:

$$d\tau = r^2 \sin\theta \, dr \, d\phi \, d\theta \quad -(2)$$

in spherical polar coordinates. In these coordinates the surface of a sphere is:

$$S = \int_0^{2\pi} d\phi \int_0^{\pi} r^2 \sin\theta \, d\theta = 4\pi r^2 \quad -(3)$$

and the volume of a sphere is:

$$V = \int_0^r 4\pi r^2 \, dr = \frac{4}{3}\pi r^3 \quad -(4)$$

The Born normalization condition is:

$$\int \psi^* \psi \, d\tau = 1 \quad -(5)$$

If ψ^* is the simple complex conjugate of ψ, then

$$\psi^* \psi = 1 \quad -(6)$$

and

$$\int d\tau = \frac{4}{3}\pi r^3 = 1 \quad -(7)$$

This corresponds with a volume V_0 of one cubic metre - the unit volume. To ensure correct units, eq. (1) must be written in terms of energy density:

$$E = \hbar\omega \frac{\int \phi^* m(r)^{1/2} \phi \, d\tau}{\int \phi^* \phi \, d\tau} \quad -(8)$$

$$= \frac{\hbar\omega}{V_0} \int \phi^* m(r)^{1/2} \phi \, d\tau$$

If ϕ^* is the simple conjugate of ϕ then:

$$E = \frac{\hbar\omega}{V_0} \int m(r)^{1/2} d\tau$$

$$= \frac{\hbar\omega}{V_0} \int_0^r 4\pi m(r)^{1/2} r^2 dr \quad -(9)$$

Define the volume:

$$V_1 = \int_0^r 4\pi m(r)^{1/2} r^2 dr \quad -(10)$$

then

$$\boxed{E = \hbar\omega \left(\frac{V_1}{V_0} \right)} \quad -(11)$$

In general:

$$\frac{V_1}{V_0} = \frac{\int_0^r 4\pi m(r)^{1/2} r^2 dr}{\int_0^r 4\pi r^2 dr} \quad -(12)$$