+38(2): Constant m Limit of the Orbit.

The orbit equations for constant m reduce to:

$$\ddot{r} - r\dot{\phi}^2 = -mG\left(\frac{m^{1/2}}{\gamma^3 r^2} + \frac{\dot{\phi}}{\gamma c^2 m^{1/2}}\right) \quad -(1)$$

and

$$r\ddot{\phi} + 2\dot{\phi}\dot{r} = \frac{mG}{r^2}\left(\frac{r\dot{\phi}\dot{r}}{\gamma c^2 m^{1/2}}\right) \quad -(2)$$

where:

$$\gamma = \left(m - \frac{\dot{r}^2 + r^2\dot{\phi}^2}{c^2 m}\right)^{-1/2} \quad -(3)$$

Eqs. (1) and (2) give a precessing orbit which shrinks to a point as:

$$\underline{m} \rightarrow \infty \quad -(4)$$

In the particular case:

$$m = 1 \quad -(5)$$

these equations give the orbit of special relativity. For other values of m the orbit departs more and more from that of special relativity.

The Newtonian equations of the orbit are:

$$\ddot{r} - r\dot{\phi}^2 = -\frac{mG}{r^2} \quad -(6)$$

and

$$r\ddot{\phi} + 2\dot{\phi}\dot{r} = 0 \quad -(7)$$

The Newtonian limit is defined by:

$$v^2 \ll c^2 \quad -(8)$$

$$m = 1 \quad -(9)$$

so:

$$\frac{\dot{\phi}}{2} \ll \frac{1}{r^2} \quad -(10)$$

in eq. (1), and

$$\frac{r\ddot{\phi}r}{c^2} \rightarrow 0 \quad -(11)$$

Eq. (2).

It would be very interesting to investigate the behaviour of the orbital precession as

$$M \rightarrow \infty \quad -(12)$$

i.e. as the orbit collapses to a point. This point can be given the appellation "dark star" after Michel in 1738.

The complete orbit equations are eqns (7) and (8) of Note 438(1), in which appear $n(r)$ and $dn(r)/dr$. They should exhibit a variety of very interesting orbits as M goes to infinity and the orbit collapses to a dark star. All the astronomical data on "black holes" can be reinterpreted as data on the dark star.