The full path of calculation through Cartan geometry

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Abstract

During the development of ECE theory, several aspects of Cartan geometry were touched. In this article we present the big picture, how physics evolves over the entire range of Cartan geometry. The tetrad corresponds to a given potential, and over several stages all types of connections are computed up to the torsion forms, which correspond to physical force fields. We put together all relevant equations of Cartan geometry. The potential is simplified by using a novel restriction to polarization. This simplification is translated to the tangent space of Cartan geometry by choosing the unit vectors of this space to be parallel to those in the base manifold. This leads to a diagonal tetrad matrix. Examples are given for some physical systems. In particular, a new justification for the Evans B(3) field is found.

Keywords: Cartan geometry, general relativity, ECE theory, electrodynamics.

1 Introduction

The development of ECE theory is based on Cartan geometry. Elements of geometry are interpreted as physical quantities by applying the ECE axioms to the geometry. This is basically multiplying the geometrical quantities by constants with physical dimensions so that equations of physics are obtained. This has been worked out in great detail in the course of ECE development [1]- [6]. All papers concentrate on special aspects. In this article we cover a span over the whole range of Cartan geometry. Starting with the tetrad, which corresponds to the physical potentials, we end up with electromagnetic or gravitational field quantities. The great progress is that all kinds of geometrical tensors and connections are computable in this path. The earlier difficulty was to find the Christoffel (Gamma) connections for a given problem. This is barely

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possibly without mathematical tools. In this paper we use computer algebra to solve the equations for the Gamma and spin connections.

The equations are of algebraic type. For the Gamma connections, a linear equation system has to be solved. Thus, the complexity is brought into the development not by complicated mathematics, but by the large number of equations and variables to be handled. This is the reason, why in Einsteinian theory only very simple systems (mostly with spherical symmetry) could be handled in the past.

A second difficulty arises from the fact that Cartan geometry introduces so-called polarization indices. These indices arise from the usage of tangent space of the Cartan base manifold. So an electric field vector $\mathbf{E}$ becomes an indexed vector $\mathbf{E}^a$ in ECE theory. Several methods have been developed to get rid of this index in case of physical situations, for example using only one index or averaging over all index values [7]. In this paper we introduce another method: assuming that the basis vectors of base manifold and tangent space are parallel. Then the tetrad is reduced to a diagonal matrix and the polarization index is identical to the coordinate index.

In section 2 we summarize the equations of ECE theory used, and in section 3 we give some examples of the overall method. Further, more mathematical, examples can be found in section 3 of the ECE textbook [8].

2 The full path through Cartan geometry

2.1 Listing the equations

We remember that the ECE axioms connect the electromagnetic potential $A^a{}_\mu$ and electromagnetid field tensor $F^a{}_{\mu\nu}$ with the tetrad $q^a{}_\mu$ and torsion tensor $T^a{}_{\mu\nu}$:

$$A^a{}_\mu := A^{(0)} q^a{}_\mu, \tag{1}$$

$$F^a{}_{\mu\nu} := A^{(0)} T^a{}_{\mu\nu}, \tag{2}$$

or in form notation:

$$A^a := A^{(0)} q^a, \tag{3}$$

$$F^a := A^{(0)} T^a, \tag{4}$$

where $A^{(0)}$ is a constant introducing physical units. Since $A^a{}_\mu$ is a vector potential, $A^{(0)}$ has the units of Vs/m. Torsion is defined in units of $1/m$, and, consequently, F has units of a magnetic field (Tesla or Vs/m^2). q^a is a 1-form and T^a is a 2-form. Correspondingly, the potential A^a is a 1-form and the field tensor F^a is a 2-form of Cartan geometry, consisting of the field components $\mathbf{E}^a$ and $\mathbf{B}^a$.

We start the computational part with a given potential, so the tetrad elements are known.

From the tetrad, we can compute the metric tensor $g_{\mu\nu}$ and its contravariant counter-part $g^{\mu\nu}$, which, in algebraic terminology, is the inverse matrix of $g_{\mu\nu}$:

$$g_{\mu\nu} = n\ q^a{}_\mu q^b{}_\nu\, \eta_{ab}, \tag{5}$$

$$g^{\mu\nu} = \frac{1}{n}\ q^\mu{}_a q^\nu{}_b\, \eta^{ab}. \tag{6}$$

η^{ab} is the metric of Minkowski space and n the space dimension, normally 4.

Metric compatibility enables us to determine the Γ connection from the linear equation system

$$D_\sigma g_{\mu\nu} = \partial_\sigma g_{\mu\nu} - \Gamma^\lambda{}_{\sigma\mu} g_{\lambda\nu} - \Gamma^\lambda{}_{\sigma\nu} g_{\mu\lambda} = 0. \tag{7}$$

As discussed in great detail in the ECE papers, the Γ connection is asymmetric in its lower indices, and the relevant parts are the antisymmetric parts on the non-diagonal elements. Therefore we require explicitly

$$\Gamma^\rho{}_{\mu\nu} = -\Gamma^\rho{}_{\nu\mu} \tag{8}$$

for all $\mu \neq \nu$. Then the solution is unique up to four undetermined constants, for which suitable choices have to be made in specific applications.

The spin connection is computable from the tetrad elements and the Γ connection:

$$\omega^a{}_{\mu b} = q^a{}_\nu q^\lambda{}_b \Gamma^\nu{}_{\mu\lambda} - q^\lambda{}_b \partial_\mu q^a{}_\lambda. \tag{9}$$

With these prerequisites, we can compute the curvature and torsion tensors:

$$R^\lambda{}_{\mu\nu\rho} = \partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\nu \Gamma^\lambda_{\mu\rho} + \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\nu\rho} - \Gamma^\lambda_{\nu\sigma}\Gamma^\sigma_{\mu\rho}, \tag{10}$$

$$T^\lambda{}_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}. \tag{11}$$

For ECE theory, we need the curvature and torsion forms, which are obtained from the above tensors by

$$R^a{}_{b\mu\nu} = q^a{}_\rho q^\sigma{}_b R^\rho{}_{\sigma\mu\nu}, \tag{12}$$

$$T^a{}_{\mu\nu} = q^a{}_\lambda T^\lambda{}_{\mu\nu}\,. \tag{13}$$

Since the vector components refer to the contravariant elements in the force field tensor F, we have to raise the indices in T:

$$T^{a\mu\nu} = \eta^{\mu\rho}\eta^{\nu\sigma} T^a{}_{\rho\sigma}\,, \tag{14}$$

with the inverse Minkowski metric, which is identical to the the covariant form:

$$\eta^{ab} = \eta_{ab}. \tag{15}$$

From ECE theory, the electromagnetic field form F is

$$F^{a\mu\nu} = \begin{bmatrix} F^{a00} & F^{a01} & F^{a02} & F^{a03} \\ F^{a10} & F^{a11} & F^{a12} & F^{a13} \\ F^{a20} & F^{a21} & F^{a22} & F^{a23} \\ F^{a30} & F^{a31} & F^{a32} & F^{a33} \end{bmatrix} \tag{16}$$

$$= \begin{bmatrix} 0 & -E^{a1}/c & -E^{a2}/c & -E^{a3}/c \\ E^{a1}/c & 0 & -B^{a3} & B^{a2} \\ E^{a2}/c & B^{a3} & 0 & -B^{a1} \\ E^{a3}/c & -B^{a2} & B^{a1} & 0 \end{bmatrix}.$$

With Eq. (2) we can identify

$$A^{(0)}\begin{bmatrix} T^{a00} & T^{a01} & T^{a02} & T^{a03} \\ T^{a10} & T^{a11} & T^{a12} & T^{a13} \\ T^{a20} & T^{a21} & T^{a22} & T^{a23} \\ T^{a30} & T^{a31} & T^{a32} & T^{a33} \end{bmatrix} = \frac{1}{c}\begin{bmatrix} 0 & -E^{a1} & -E^{a2} & -E^{a3} \\ E^{a1} & 0 & -cB^{a3} & cB^{a2} \\ E^{a2} & cB^{a3} & 0 & -cB^{a1} \\ E^{a3} & -cB^{a2} & cB^{a1} & 0 \end{bmatrix}, \tag{17}$$

which relates the components of $\mathbf{E}^a$ and $\mathbf{B}^a$ directly to certain torsion elements:

$$\begin{bmatrix} E^{a1} \\ E^{a2} \\ E^{a3} \end{bmatrix} = c\,A^{(0)}\begin{bmatrix} T^{a10} \\ T^{a20} \\ T^{a30} \end{bmatrix}, \tag{18}$$

$$\begin{bmatrix} B^{a1} \\ B^{a2} \\ B^{a3} \end{bmatrix} = A^{(0)}\begin{bmatrix} -T^{a23} \\ T^{a13} \\ -T^{a12} \end{bmatrix}. \tag{19}$$

In addition to the above derivation, there are the Hodge dual quantities to be computed. Only in four dimensions, the Hodge duals of F have the same dimensions as F. Therefore, we restrict consideration to a four-dimensional space. As derived in [7], The dual of the Γ connection, called Λ, and the resulting spin connection $\omega_{(\Lambda)}$ are

$$\Lambda^{\lambda}{}_{\mu\nu} = \frac{1}{2}|g|^{-1/2} g^{\rho\alpha} g^{\sigma\beta} \epsilon_{\rho\sigma\mu\nu} \Gamma^{\lambda}{}_{\alpha\beta}, \tag{20}$$

$$\omega_{(\Lambda)}{}^{a}{}_{\mu b} = q^{a}{}_{\nu} q^{\lambda}{}_{b} \Lambda^{\nu}{}_{\mu\lambda} - q^{\lambda}{}_{b} \partial_\mu q^{a}{}_{\lambda}, \tag{21}$$

with g being the determinant of the metric tensor. The Hodge dual curvature and torsion are, in analogy to their definitions with the Γ connection:

$$\widetilde{R}^{\lambda}{}_{\mu\nu\rho} = \partial_\mu \Lambda^{\lambda}_{\nu\rho} - \partial_\nu \Lambda^{\lambda}_{\mu\rho} + \Lambda^{\lambda}_{\mu\sigma}\Lambda^{\sigma}_{\nu\rho} - \Lambda^{\lambda}_{\nu\sigma}\Lambda^{\sigma}_{\mu\rho}, \tag{22}$$

$$\widetilde{T}^{\lambda}{}_{\mu\nu} = \Lambda^{\lambda}_{\mu\nu} - \Lambda^{\lambda}_{\nu\mu}. \tag{23}$$

Thus, all relevant variables of Cartan geometry can be computed by Eqs. (5-14), and the resulting force fields by Eqs. (18-19). The tetrad, respectively the potential, has to be given for input.

2.2 Simplification of the 4-potential

In ECE theory, the 4-vector of the potential is given by

$$A^{a\mu} = \begin{bmatrix} \frac{\phi^a}{c} \\ A^{a1} \\ A^{a2} \\ A^{a3} \end{bmatrix} \tag{24}$$

where ϕ^a is the scalar potential and A^{a1}, A^{a2}, A^{a3} are the components of the vector potential, which in vector notation can be written as

$$\mathbf{A}^a = \begin{bmatrix} A^{a1} \\ A^{a2} \\ A^{a3} \end{bmatrix}. \tag{25}$$

This corresponds to a vector potential in relativity theory with an additional polarization index a. The 0-component is the scalar potential

$$A^{a0} = \frac{\phi^a}{c} \tag{26}$$

where c is the vacuum velocity of light and a is the polarization index as well. $A^{a\mu}$ is a 2-component quantity and can be written in matrix form as

$$(A^{a\mu}) = \begin{bmatrix} \frac{\phi^{(0)}}{c} & \frac{\phi^{(1)}}{c} & \frac{\phi^{(2)}}{c} & \frac{\phi^{(3)}}{c} \\ 0 & A^{(1)1} & A^{(2)1} & A^{(3)1} \\ 0 & A^{(1)2} & A^{(2)2} & A^{(3)2} \\ 0 & A^{(1)3} & A^{(2)3} & A^{(3)3} \end{bmatrix}. \tag{27}$$

The indices in parentheses are the Latin (tangent space) indices. The first column does not contain elements of the vector potential because the latter is a pure space-like quantity. In the following, we assume that the basis vectors of the base manifold are parallel to the base vectors of the tangent space. Then the indices of both spaces have a one-to-one correspondence $a \leftrightarrow \mu$ and we have only terms with $a = \mu$, no mixed index terms. Eq. (27) then takes the form

$$(A^{a\mu}) = \begin{bmatrix} \frac{\phi^{(0)}}{c} & 0 & 0 & 0 \\ 0 & A^{(1)1} & 0 & 0 \\ 0 & 0 & A^{(2)2} & 0 \\ 0 & 0 & 0 & A^{(3)3} \end{bmatrix}. \tag{28}$$

Omitting the polarization index, we can write $(A^{a\mu})$ with the "conventional" scalar potenital ϕ and vector potential $\mathbf{A}$:

$$(A^{a\mu}) = \begin{bmatrix} \frac{\phi}{c} & 0 & 0 & 0 \\ 0 & A^1 & 0 & 0 \\ 0 & 0 & A^2 & 0 \\ 0 & 0 & 0 & A^3 \end{bmatrix}. \tag{29}$$

Then the tetrad is defined by Eq. (1), however $A^a{}_\mu$ is to be given in its covariant form. Therefore we have first to translate the vector potential A^μ to its covariant form

$$A_\mu = \eta_{\mu\nu} A^\nu \tag{30}$$

which gives a sign change in the components of the $\mathbf{A}$ vector. Finally we have

$$(q^a{}_\mu) = \frac{(A^a{}_\mu)}{A^{(0)}} = \frac{1}{A^{(0)}} \begin{bmatrix} \frac{\phi}{c} & 0 & 0 & 0 \\ 0 & -A^1 & 0 & 0 \\ 0 & 0 & -A^2 & 0 \\ 0 & 0 & 0 & -A^3 \end{bmatrix}. \tag{31}$$

3 Examples

We present some examples of static and dynamic potentials and compute all quantities of Cartan geometry, up to the electric and magnetic fields.

3.1 Coulomb potential

One of the simplest and most important cases of electrodynamics is the Coulomb potential. In 4-vector notation, the potential is the 0-component

$$A^0 = \frac{\phi(r)}{c} = \frac{1}{c}\frac{q_e}{4\pi\epsilon_0 r} \tag{32}$$

where q_e is the central point charge and r is the radial coordinate of a spherical coordinate system

$$(X^\mu) = \begin{bmatrix} t \\ r \\ \theta \\ \phi \end{bmatrix}. \tag{33}$$

According to Eq. (31), the potential corresponds to the first diagonal element of the tetrad:

$$\phi(r) = c\ A^{(0)} q^{(0)}{}_0. \tag{34}$$

Inserting the potential into the q matrix gives

$$(q^a{}_\mu) = \frac{1}{2}\frac{(A^a{}_\mu)}{A^{(0)}} = \frac{1}{A^{(0)}}\begin{bmatrix} \frac{\phi(r)}{c} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{35}$$

which is a singular matrix. Cartan Geometry, however, is only defined with non-singular tetrads. Therefore, a vector potential is necessarily required in addition to a scalar potential. We choose the simplest form, a constant vector potential, which gives no magnetostatic field. The final form of the tetrad then is

$$(q^a{}_\mu) = \frac{1}{2}\begin{bmatrix} \frac{C_0}{r} & 0 & 0 & 0 \\ 0 & -C_1 & 0 & 0 \\ 0 & 0 & -C_2 & 0 \\ 0 & 0 & 0 & -C_3 \end{bmatrix}, \tag{36}$$

where we have replaced

$$C_0 = \frac{q_e}{A^{(0)}\, c\, 4\pi\epsilon_0} \tag{37}$$

and the C_i are arbitrary constants for $i = 1, 2, 3$. For simplicity of results, we assume $C_i > 0$ and omit the factors $A^{(0)}$ and c. Then, the vector potential is

$$\mathbf{A} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}. \tag{38}$$

Applying the Cartan geometry, Eqs. (5-8) gives Γ connections with four unspecified parameters D_1 to D_4:

$$\Gamma^0{}_{01} = \frac{1}{r} \tag{39}$$
$$\Gamma^0{}_{10} = -\frac{1}{r}$$
$$\Gamma^0{}_{12} = \frac{D_4\, {C_2}^2 r^2}{{C_0}^2}$$
$$\Gamma^0{}_{13} = -\frac{D_3\, {C_1}^2 r^2}{{C_0}^2}$$
$$\ldots$$

It is possible to set the D_i to zero:

$$D_1 = D_2 = D_3 = D_4 = 0. \tag{40}$$

Then, only three non-vanishing connections remain:

$$\Gamma^0{}_{01} = \frac{1}{r}, \tag{41}$$
$$\Gamma^0{}_{10} = -\frac{1}{r}, \tag{42}$$
$$\Gamma^1{}_{00} = \frac{C_0^2}{C_1^2\, r^3}. \tag{43}$$

The first pair is antisymmetric, while the third connection is a diagonal element which does not contribute to torsion.

Applying Eqs. (9-14), the non-vanishing spin connections are

$$\omega^{(0)}{}_{0(1)} = -\frac{C_0}{C_1\, r^2}, \tag{44}$$
$$\omega^{(1)}{}_{0(0)} = -\frac{C_0}{C_1\, r^2}, \tag{45}$$

which are antisymmetric indices in a and b. (Observe that the upper index a has to be lowered for comparison, which gives a sign change for the second connection element.) The Hodge duals of the Γ connection are

$$\Lambda^0{}_{23} = -\frac{C_2 C_3}{|C_0| C_1}, \tag{46}$$
$$\Lambda^0{}_{32} = \frac{C_2 C_3}{|C_0| C_1}, \tag{47}$$

being only constants, and the non-zero Λ spin connections are

$$\omega_{(\Lambda)}{}^{(0)}{}_{1(0)} = \frac{1}{r}, \tag{48}$$
$$\omega_{(\Lambda)}{}^{(0)}{}_{2(3)} = \frac{C_0 C_2}{|C_0| C_1 r}, \tag{49}$$
$$\omega_{(\Lambda)}{}^{(0)}{}_{3(2)} = -\frac{C_0 C_3}{|C_0| C_1 r}. \tag{50}$$

It is important to note that the connection $\omega_{(\Lambda)}{}^{(0)}{}_{1(0)}$ has the form that has already been derived in early UFT papers. In those papers, the spin connections Γ and Λ have not been discerned. Which one was meant, depended on the field equations used. In the inhomogeneous equations (Coulomb and Ampere-Maxwell law), the Λ spin connections appear.

The non-vanishing torsion and curvature tensor elements are

$$T^0{}_{01} = -T^0{}_{10} = \frac{2}{r}, \tag{51}$$

$$R^0{}_{101} = -R^0{}_{110} = \frac{2}{r^2}, \tag{52}$$

$$R^1{}_{001} = -R^1{}_{010} = \frac{2C_0^2}{C_1^2 r^4}, \tag{53}$$

which are all antisymmetric in the last two indices. The same holds for the torsion and curvature forms:

$$T^{(0)}{}_{01} = -T^{(0)}{}_{10} = \frac{C_0}{r^2}, \tag{54}$$

$$R^{(0)}{}_{(1)01} = -R^{(0)}{}_{(1)10} = -\frac{2C_0}{C_1 r^3}, \tag{55}$$

$$R^{(1)}{}_{(0)01} = -R^{(1)}{}_{(0)10} = -\frac{2C_0}{C_1 r^3}. \tag{56}$$

The final result according to Eqs. (18-19) are the electric fields

$$\mathbf{E}^{(0)} = c\,A^{(0)} \begin{bmatrix} \frac{C_0}{r^2} \\ 0 \\ 0 \end{bmatrix}, \tag{57}$$

$$\mathbf{E}^{(1)} = \mathbf{E}^{(2)} = \mathbf{E}^{(3)} = \mathbf{0}, \tag{58}$$

and the magnetic fields

$$\mathbf{B}^{(0)} = \mathbf{B}^{(1)} = \mathbf{B}^{(2)} = \mathbf{B}^{(3)} = \mathbf{0}. \tag{59}$$

Only the electric 0-component of polarization is not a zero vector, and all polarizations of the magnetic field vanish. This is exactly the classical result

$$\mathbf{E}^{(0)} = \mathbf{E} = \frac{q_e}{4\pi\epsilon_0\, r^2}. \tag{60}$$

3.2 Vector potential of a straight current wire

The next example is the vector potential of an infinite straight current wire [9]. In cylindrical coordinates, the vector potential is given by

$$\mathbf{A} = \begin{bmatrix} A_r \\ A_\theta \\ A_Z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ C_3 \log(r) \end{bmatrix}. \tag{61}$$

The wire is placed in the Z direction. The constant C_3 is defined by

$$C_3 = -\frac{\pi a^2 j_Z}{2\pi\epsilon_0 c^2} \tag{62}$$

where a is the thickness of the wire, j_Z the current, and ϵ_0 the vacuum permittivity.

Applying the same mechanism as in the preceding example leads to the tetrad matrix

$$(q^a{}_\mu) = \frac{1}{2}\frac{1}{A^{(0)}}\begin{bmatrix} C_0 & 0 & 0 & 0 \\ 0 & -C_1 & 0 & 0 \\ 0 & 0 & -C_2 & 0 \\ 0 & 0 & 0 & -C_3\log(r) \end{bmatrix}. \tag{63}$$

We assume $C_0 > 0, C_1 > 0, C_2 > 0, C_3 < 0$ similar as in the former example. The equations of Cartan geometry are applied as described before. We present only the spin connection results:

$$\omega^{(1)}{}_{3(3)} = -\omega^{(3)}{}_{3(1)} = \frac{C_1}{C_3\, r}, \tag{64}$$

$$\tag{65}$$

and

$$\omega_{(\Lambda)}{}^{(3)}{}_{0(2)} = \frac{C_0}{C_1 r|\log(r)|}, \tag{66}$$

$$\omega_{(\Lambda)}{}^{(3)}{}_{1(3)} = -\frac{1}{r\log(r)}, \tag{67}$$

$$\omega_{(\Lambda)}{}^{(3)}{}_{2(0)} = \frac{C_2}{C_1 r|\log(r)|}. \tag{68}$$

We have no antisymmetry for the spin connection $\omega_{(\Lambda)}$ again. The resulting electric fields are

$$\mathbf{E}^{(a)} = \mathbf{0}, \tag{69}$$

and the magnetic fields are

$$\mathbf{B}^{(0)} = \mathbf{B}^{(1)} = \mathbf{B}^{(2)} = \mathbf{0}, \tag{70}$$

with the third polarization field being

$$\mathbf{B}^{(3)} = \begin{bmatrix} 0 \\ -\frac{C_3}{r} \\ 0 \end{bmatrix}. \tag{71}$$

This is exactly the field obtained from the classical calculation

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}. \tag{72}$$

3.3 Circularly polarized electromagnetic wave

The third example is a rotating electromagnetic field in form of a circularly polarized wave. The basis vectors of such a wave are rotating in the XY plane of cartesian coordinates. In classical electromagetism there is no Z component, but in the Evans B$^{(3)}$ field theory there is one. The vector potential is also a rotating field, phase shifted by 90 degrees in the XY plane. We define the tetrad matrix as before by

$$(q^a{}_\mu) = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\cos(\omega t - \mathbf{kR}) & -\sin(\omega t - \mathbf{kR}) & 0 \\ 0 & \sin(\omega t - \mathbf{kR}) & -\cos(\omega t - \mathbf{kR}) & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{73}$$

with time angular velocity ω, wave vector $\mathbf{k} = [k^1, k^2, k^3]$ and coordinate vector $\mathbf{R} = [X, Y, Z]$. When setting the constants appearing in the Γ connection to zero as before, no electric and magnetic fields come out. However, there are spin connections which are identical for the Γ and Λ connection:

$$\omega^{(1)}{}_{0(2)} = -\omega^{(2)}{}_{0(1)} = -\frac{\omega}{c}, \tag{74}$$

$$\omega^{(1)}{}_{1(2)} = -\omega^{(2)}{}_{1(1)} = k^1, \tag{75}$$

$$\omega^{(1)}{}_{2(2)} = -\omega^{(2)}{}_{2(1)} = k^2, \tag{76}$$

$$\omega^{(1)}{}_{3(2)} = -\omega^{(2)}{}_{3(1)} = k^3. \tag{77}$$

The metric g is identical to the Minkowski metric. This shows that a field defined in "flat" space can have a spin connection of general relativity.

The situation changes when setting the constants inferred by the Γ connection not all to zero. We define

$$D_1 := \kappa \neq 0 \tag{78}$$

where κ is a wave number, for exampe $\kappa = 1/\mathrm{m}$. The other constants remain zero. While the metric is the Minkowski metric again, because it does not depend on the connections, more terms in the spin connections appear. Most interesting is the final result for the electric and magnetic fields:

$$\mathbf{E}^{(1)} = A^{(0)} c\, \kappa \begin{bmatrix} -\sin(\omega t - \mathbf{kR}) \\ \cos(\omega t - \mathbf{kR}) \\ 0 \end{bmatrix}, \tag{79}$$

$$\mathbf{E}^{(2)} = A^{(0)} c\, \kappa \begin{bmatrix} -\cos(\omega t - \mathbf{kR}) \\ -\sin(\omega t - \mathbf{kR}) \\ 0 \end{bmatrix}, \tag{80}$$

$$\tag{81}$$

and

$$\mathbf{B}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ A^{(0)}\kappa \end{bmatrix}. \tag{82}$$

All other fields are zero. The fields have the right dimensions of V/m and Tesla. The electric fields are rotated against the potential vectors as expected. Highly interesting is that a constant magnetic field in Z direction appears, which in principle is the $\mathrm{B}^{(3)}$ field of Evans. This is a consequence of the fact that the tetrad always has to be non-singular. Therefore at least one component A^{a3} has to be present which is the case in Eq. (73).

3.4 Discussion and conclusions

t.b.d.

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