ESSAY 116: NEW HYPERFINE STRUCTURE FROM THE DIRAC EQUATION

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The famous Dirac equation can be classified as a structure of ECE2 theory because the former equation is based on special relativity. The equations of ECE2 are those of a theory that is Lorentz covariant in a space with finite torsion and curvature.

Sommerfeld had lain the groundwork for relativistic quantum mechanics in 1913, as described in Essay 115, he had discovered the magnetic quantum number in 1916, and the inner quantum number in 1921. This is now known as the J quantum number. It is related to the orbital angular momentum quantum number L by the Clebsch Gordan series. The latter defines the way in which the spin quantum number S can be added to L to give J. Sommerfeld predicted the spin of the electron in this way. This became known as the Bohr Sommerfeld hypothesis and was confirmed in the Stern Gerlach experiment of 1922. At about the same time, one of Sommerfeld's graduate students, Pauli, defined the spin quantum number in 1924 by making the spin operator directly proportional to the Pauli matrix operator.

Pauli was born in Vienna and became an assistant of Born and a professor at ETH Zuerich. He rarely published papers, preferring to develop his ideas in correspondence. At this time it was known that Sommerfeld quantization, and later Schroedinger quantization, produced the main features of atomic spectra, notably those of the hydrogen atom, but the fine structure of the spectra still posed challenges. Thomas had inferred his Thomas precession factor of two at the Niels Bohr Institute, but in a very complicated way that bore no discernible resemblance to the then known quantum theory. In 1924 it was known only that the Thomas factor is a large correction, not a tiny relativistic effect.

Paul Dirac put all the pieces together by developing the hamiltonian of special relativity in a basis made up of Pauli matrices, known as the SU(2) basis. The usual basis is made up of unit vectors such as the Cartesian unit vectors, and is called O(3). The lagrangian of special relativity could also be developed in the SU(2) basis. By expressing the hamiltonian in an SU(2) basis, Dirac found that it is possible to explain the Thomas factor, the half integral spin of the electron, and the fine structure of atomic and molecular spectra. This work also led to the prediction of electron spin resonance and nuclear magnetic resonance spectroscopy, confirmed almost twenty years later in about 1945 by van Vleck and others. The Dirac equation also produces what is known as the spin orbit fine structure, and the Dirac equation also predicts the existence of anti particles, discovered experimentally in the thirties. During the development of the UFT papers the Dirac equation has been inferred from Cartan geometry and has evolved into the fermion equation. The latter keeps all the advantages of the Dirac equation and disposes of unobservables such as the Dirac sea and negative energy.

In ECE2 theory, the four potential used in the Dirac equation has been derived from curvature, notably the vector potential has been derived from spin curvature in geometry. During the course of this development it was noticed that the conventional solution of the Dirac equation erroneously replaces the relativistic momentum by the non relativistic momentum, and in so doing misses several terms which lead to new types of observable spectra, a new type of hyperfine structure. The root cause of this error is obscure, probably it was made early on about ninety years ago. The Dirac equation is based on the hamiltonian of special relativity, defined in terms of the Einstein energy equation which is a re expression of the relativistic momentum. The quantization proposed by Dirac is based on the assumption that the four derivative is proportional to the relativistic four momentum. In producing fine structure the quantized four momentum acts on a term that includes the unquantized but relativistic four momentum, denoted p.

When the Dirac equation is correctly worked out, as in work in progress, using the relativistic momentum, hitherto unknown hyperfine structure appears with its own spectral fingerprint, and is present in all atoms, molecules and materials. This is a discovery that opens up a vast new area of spectroscopy and computational quantum chemistry and also gives a new and rigorous test of the Dirac equation.