CHAPTER EIGHT

ECE COSMOLOGY.

8.1. INTRODUCTION

Astronomy is one of the oldest of the sciences and has become a precise subject area. Cosmology began to develop as a subject when the observations of the orbit of Mars by Tycho Brahe were analyzed by Johannes Kepler to give three planetary laws reduced by Newton to universal gravitation and the equivalence of gravitational and inertial mass. The famous Newtonian dynamics were developed to include rotational motions in non inertial frames by Euler, Bernoulli, Coriolis and others, and Laplace developed his elegant celestial mechanics. Lagrange developed the subject of dynamics from a different perspective, and using more general concepts which were taken up by Hamilton to produce the Hamilton equations and the idea of the Hamiltonian. The latter became the basis of quantum mechanics. Orbital theory can be developed elegantly with the idea of the Lagrangian and the Euler Lagrange equations. For example, conservation of angular momentum and the Euler Lagrange equations can be used to show that if the orbit of a mass m around a mass M is observed to be an ellipse, then the force between m and M is inversely proportional to the square of the distance r between m and M - the famous inverse square law as inferred by Newton. The same method also gives the three Kepler laws of planetary motion. However the Lagrangian method is more general than that of Newton because it can give the force law for any orbit.

In the eighteenth and early nineteenth centuries the orbits of all masses m around a mass M were thought to be ellipses to an excellent approximation, with M at one focus of the ellipse, so the subject was thought to be complete, and m travelled on the ellipse. The orbits of planets could be observed with precision, and objects such as galaxies were unknown. So the famous Newtonian concept of universal gravitation was thought to be as near to perfection as human intellect could devise. Newtonian dynamics worked for astronomy and also back on the ground. The apocryphal apple was governed by the acceleration due to gravity g of the earth. The apple and the moon were governed by the same law, universal gravitation.

The gods however are offended by human pretence to perfection, the orbit of a planet precesses, a point of the ellipse such as its perihelion moves forward a little every orbit. In the Newtonian dynamics the elliptical orbit does not move forward if one considers only m and M and the force between them. From precise astronomical observations of orbits by ancient astronomers the precession of the perihelion had been known well before Newton's time. In Newton's time, the seventeenth century, it was thought to be caused by the gravitational pull of other planets. It is a very tiny effect so was not thought to be due to any flaw in Newton's universal gravitation. When the human intellect contrives something that it thinks to be perfect, no data are allowed to stand in the way, and it is human nature to hang on to a theory even though the data show that the theory is not quite right. Sometimes the theory is totally wrong and always gave an illusion of the truth. The precession of planetary orbits can indeed be explained to a large extent by Newtonian concepts, but there seems to be a tiny part of the precession that cannot be explained.

Following the Michelson Morley experiment the entire subject of dynamics was changed and the concept of special relativity introduced as described in chapter one of this book. The Newtonian and Lagrangian dynamics were recovered as limits of special relativity. However, special relativity is restricted to the Lorentz transform and a constant inter frame velocity. In order to consider acceleration and similar effects a new relativity was needed. Another profound change in thought occurred when Einstein and others decided to base dynamics on geometry. This was also Kepler's idea, and went back to the ancient Greeks, who thought of geometry as beauty itself, or perfect beauty. Effectively this means that the

Lorentz transform becomes the general coordinate transform. It is not in any way clear to the human intuition that space should become part of time, that the familiar three dimensions should be abandoned, and that the familiar concepts of Euclid should be replaced by a different geometry. The very idea of a different geometry had been considered only by a few mathematicians up to about 1905.

Among the first to consider such as geometry was Riemann in the early nineteenth century, followed in the eighteen sixties by Christoffel. These two prominent mathematicians devised the concept of metric and connection. The metric is a symmetric object by definition, but the connection has no particular symmetry in the lower two of its three indices. About forty years later Ricci and Levi Civita devised the concept of curvature of space of any dimension, including four dimensional spacetime, that of special relativity. In physics concomitant progress was being made by Noether, who linked the conservation laws of physics to symmetry laws. The subject of physics introduced the canonical energy momentum tensor, which is also symmetric in its indices. In mathematics, in about 1900, Levi-Civita defined the Christoffel connection as being symmetric. This was an axiom, or hypothesis, not a rigorous proof. In 1900 it was not known that there existed a fundamental property of any mathematical space in any dimension, the torsion.

In 1902 Bianchi inferred an identity in which a well defined cyclic sum of curvature tensors vanishes. This is known as the first Bianchi identity, from which the second Bianchi identity can be inferred. The two Bianchi identities were also inferred in ignorance of the existence of torsion, and using a symmetric connection. The ingredients available to Einstein from 1905 to 1915 were therefore the second Bianchi identity and the Noether Theorem, thought to be fundamental principles of geometry and physics. Proceeding on the ancient basis that geometry gives physics, Einstein attempted for a decade to arrive at a field equation linking the two concepts. This was finally published in 1915 and asserts that the second Bianchi identity is proportional to the covariant derivative of the canonical energy momentum tensor. With the benefit of hindsight this is an over complicated procedure. By Ockham's Razor a simpler theory is preferred, and that theory is ECE theory. In addition the Einstein field equation was arrived at in ignorance of torsion. So it was bound to fail qualitatively, and has indeed done so. The velocity curve of a whirlpool galaxy shows that the Einstein theory is incorrect qualitatively, or completely. The proof of this is given later in this chapter.

At first the field equation of Einstein seemed to be logical, but on closer inspection it contains an assumption made a priori, i.e. guesswork. This is the assumption of the symmetric connection made by Levi-Civita fifteen years before the field equation appeared. The second Bianchi identity used by Einstein relies on a symmetric connection, so is true if and only if the torsion is zero. This was of course unknown to Einstein and also unknown to Levi-Civita and Ricci. The procedure used in deriving the Einstein field equation is to reduce the second Bianchi identity to the covariant derivative of the Einstein tensor, which is symmetric in its lower two indices, and which is made up of a combination of the Ricci tensor and the Ricci scalar. Unknown to Einstein and all his contemporaries this procedure is true if and only if the torsion is zero. If the torsion is finite it fails completely as explained in UFT88 on www.aias.us .

The field equation was criticized immediately and severely by Schwarzschild in a letter to Einstein of December 1915 as explained earlier in this book. Apart from the assumption of a symmetric connection, there are other flaws in the attempted first solution of the field equation by Einstein. Schwarzschild solved the equation using a metric which does not contain a singularity. So it was known as early as 1915 that there are no black holes and big bang, concepts which were ridiculed by Einstein and Hoyle independently. The cold truth is that these concepts are just mathematical flaws. Experimental data have shown many times over that there was no big bang, and black holes have never been discovered. They are simply asserted to exist by dogmatists. The confusion was greatly compounded by the introduction of a metric that was attributed falsely to Schwarzschild. This metric contains singularities or infinities, so by definition should be rejected as a valid solution of the Einstein field equation. The Schwarzschild metrics, true (1915), and false, fail completely in whirlpool galaxies. This fact has been known for sixty years. A plethora of such metrics have been inferred in a century of work on the Einstein field equation but all fail completely in view of the failure of the field equation in whirlpool galaxies and in view of the fact that they all neglect torsion (M. W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" referred to in chapter one).

The existence of torsion is a fundamental building block of ECE theory, which set out in 2003 to rebuild general relativity using a rigorously correct geometry, one which does not contain guesswork. So it is essential to prove that torsion cannot be discarded in any valid geometry. In the Cartan geometry used in ECE theory the torsion is defined by the first Maurer Cartan structure equation, inferred in the twenties. This procedure has been explained earlier in this book and the basis of ECE cosmology and unified field theory is that torsion and curvature are identically non zero in any valid geometry. The reason is that they are both generated by the commutator of covariant derivatives acting on any tensor in any space of any dimension. They are always produced simultaneously, and the commutator always produces the two structure equations of Cartan simultaneously. The commutator always produces the torsion tensor as the difference of two anti symmetric connections, so the anti symmetry of the connection is the anti symmetry of the commutator.

A symmetric connection produces a symmetric commutator which vanishes, and a symmetric connection means that the torsion vanishes. This means that the curvature vanishes if the torsion vanishes because torsion and curvature are always produced simultaneously by the commutator. A null commutator means both a null torsion and null curvature, so a symmetric connection means a null torsion AND a null curvature.

The incorrect procedure used by the Einsteinian general relativity is to omit the torsion tensor, and to assume that the commutator produces only the curvature. This is mathematical nonsense that has become dogma. The fact that the torsion always exists means that the first and second Bianchi identities are changed completely in structure. The first Bianchi identity becomes the Cartan identity and the second Bianchi identity becomes the equation given in chapter one. These mathematical flaws are obvious in retrospect, and were compounded greatly through the illusion of accuracy of the Einstein theory in the solar system. In chapter 8.2 the correct explanation for light deflection by gravitation is given in terms of the spin connection of ECE theory, which is also capable of giving a satisfactory explanation of the velocity curve of a whirlpool galaxy. Currently both the ECE and the Einsteinian theories are influential in science, but obvious and drastic flaws in geometry cannot remain indefinitely without being remedied. The fundamental aim of ECE theory is to improve on the ideas used by Einstein and his contemporaries, ideas which go back to Kepler and to ancient times.

8.2 ECE THEORY OF LIGHT DEFLECTION DUE TO GRAVITATION.

Consider as in UFT 215 the linear orbital velocity in cylindrical polar coordinates (r , θ):

$$\underline{v} = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta \qquad -(1)$$

where $\underline{e}_r$ and $\underline{e}_\theta$ are the unit vectors of the cylindrical polar system. The velocity squared is:

$$v^2 = \dot{r}^2 + r^2\dot{\theta}^2 \qquad -(2)$$

The precession of an elliptical orbit can be described by the equation:

$$r = \frac{\alpha}{1 + \epsilon\cos(x\theta)} \quad -(3)$$

when x is much less than unity. In this equation, α is the half right latitude and ϵ is the eccentricity. When x becomes large, some very interesting mathematical results are obtained, the subject area of precessing conical sections which show fractal behaviour as described and illustrated in the UFT papers on www.aias.us. However in astronomy the factor x is close to unity for all types of precessing orbits, in the solar system and in binary systems which exhibit the largest precessions. When x is exactly one, the subject of conical sections is recovered, for example static ellipse, the static hyperbola and so on.

Elementary kinematics of plane polar coordinates produce the acceleration:

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\,\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\underline{e}_\theta \quad -(4)$$

This is a well known general result described in several UFT papers. From the equation (3) of precessing conical sections

$$\frac{dr}{d\theta} = \frac{x\epsilon}{\alpha}\, r^2 \sin(x\theta). \quad -(5)$$

From lagrangian dynamics the conserved orbital angular momentum is well known to be:

$$L = mr^2\dot{\theta} = mr^2\frac{d\theta}{dt}. \quad -(6)$$

Therefore:

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \frac{xL\epsilon}{m\alpha}\sin(x\theta) \quad -(7)$$

and from Eq. (6):

$$\dot{\theta} = \frac{L}{mr^2} . \quad -(8)$$

The second derivatives are:

$$\ddot{r} = \frac{x^2 L^2 \epsilon}{m^2 \alpha r^2} \cos(x\theta) \quad -(9)$$

and:

$$\ddot{\theta} = -\frac{2L^2 x \epsilon}{m^2 r^3 \alpha} \sin(x\theta) \quad -(10)$$

and the angular dependent part of the acceleration vanishes:

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0. \quad -(11)$$

The radial part is given by:

$$\ddot{r} - r\dot{\theta}^2 = \frac{x^2 L^2 \epsilon}{m^2 \alpha r^2} \cos(x\theta) - \frac{L^2}{m^2 r^3} . \quad -(12)$$

From Eq. (3):

$$\cos(x\theta) = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(13)$$

and the acceleration of an object in orbit is:

$$\underline{a} = \left(\frac{L}{m}\right)^2 \left(\frac{(x^2 - 1)}{r^3} - \frac{x^2}{\alpha r^2}\right) \underline{e}_r . \quad -(14)$$

The force is defined conventionally as:

$$\underline{F} = m\underline{a} \quad -(15)$$

If there is no precession then:

$$x = 1 \quad -(16)$$

and the force law reduces to the inverse square law:

$$\underline{F} = -\frac{L^2}{m\alpha r^2}\,\underline{e}_r . \quad -(17)$$

This is the Newtonian inverse square law if:

$$\alpha = \frac{L^2}{m^2 M G} . \quad -(18)$$

The same force law is obtained elegantly from Lagrangian dynamics, which gives the following equation for any orbit:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{m r^2}{L} F(r) . \quad -(19)$$

From Eqs. (3) and (19):

$$F(r) = \frac{L^2}{m}\left(\frac{(x^2 - 1)}{r^3} - \frac{x^2}{\alpha r^2}\right) \quad -(20)$$

which is the same as Eq. (14).

The square of the orbital velocity can therefore be expressed as:

$$v^2 = \left(\frac{L}{m\alpha}\right)^2 \left[\frac{2x^2\alpha}{r} - x^2(1-\epsilon^2) + \frac{\alpha^2}{r^2}(1-x^2)\right] \quad -(21)$$

and when

$$x = 1 \quad -(22)$$

the Keplerian equation for orbital linear velocity is obtained:

$$v^2 \xrightarrow[x=1]{} \left(\frac{L}{m\alpha}\right)^2 \left[\frac{2\alpha}{r} - (1-\epsilon^2)\right] \quad -(23)$$

thus checking that the theory is correct and self consistent. At the distance R_0 of closest approach of m to M in an orbit:

$$R_0 = \frac{\alpha}{1+\epsilon} \quad -(24)$$

so Eq. (21) becomes:

$$v^2 = \frac{L^2}{m^2 R_0}\left[\frac{x^2}{\alpha}(1+\epsilon) - \frac{(x^2-1)}{R_0}\right] \quad -(25)$$

and solving for the eccentricity ϵ gives:

$$\epsilon = \frac{m^2 \alpha R_0}{x^2 L^2}\left(v^2 - \frac{L^2}{m^2}\left(\frac{x^2-1}{R_0}\right)\right) - 1 \quad -(26)$$

This equation can be used in the problem of determining the angle of deflection of a hyperbolic orbit of m around M.

The total deflection for a hyperbola, as in UFT 216, is 2ϕ :

$$\Delta\phi = 2\phi = 2\sin^{-1}\frac{1}{\epsilon} \quad -(27)$$

where

$$\phi = \tan^{-1}\frac{a}{b} \quad -(28)$$

where a and b are the major and minor semi axes. Therefore:

$$\Delta\phi = 2\sin^{-1}\frac{1}{\epsilon} = 2\tan^{-1}\frac{a}{b} \quad -(29)$$

where the eccentricity is defined by:

$$\epsilon = \left(1+\frac{b^2}{a^2}\right)^{1/2}. \quad -(30)$$

The half right latitude is defined by:

$$\alpha = \frac{b^2}{a}. \quad -(31)$$

At the distance of closest approach of m to M in a hyperbolic orbit:

$$R_0 = \frac{\alpha}{1+\epsilon} \quad -(32)$$

so:

$$\cos(x\theta) = 1 \quad -(33)$$

as in Eq. (24).

For very small angles of deflection such as that observed in the deflection of light from a distant source by the sun:

$$\sin\phi \sim \phi = \frac{1}{\epsilon} = \left[\frac{m^2 \alpha R_0}{x^2 L^2}\left(v^2 - \frac{L^2}{m^2}\left(\frac{x^2-1}{R_0^2}\right)\right) - 1\right]^{-1} \quad -(34)$$

If v could be measured experimentally, m can be found. For light v is very close to c and m is the mass of the photon. Theoretically, photon mass can be obtained in this way. In the Newtonian limit:

$$x = 1 \quad -(35)$$

and

$$\sin\phi \sim \phi = \frac{1}{\epsilon} = \left[\frac{m^2 \alpha R_0 v^2}{L^2} - 1\right]^{-1} \quad -(36)$$

in which the Newtonian half right latitude is:

$$\alpha = \frac{L^2}{m^2 M G}. \quad -(37)$$

So the well known Newtonian theory of the orbital deflection is recovered:

$$\sin\phi \sim \phi = \frac{1}{\epsilon} = \left(\frac{R_0 v^2}{MG} - 1\right)^{-1}. \quad -(38)$$

Note that m cancels out of the calculation in the Newtonian limit, but does not cancel in the rigorous equation (34). If the photon velocity is assumed to be c for all practical purposes, i.e. to be very close to c, then

$$\Delta\phi = 2\phi = \frac{2MG}{R_0 c^2} \quad -(39)$$

to an excellent approximation. This is the famous Newtonian value for light deflection by gravitation.

The experimentally observed value is always:

$$\Delta\phi = 2\phi = \frac{4MG}{R_0 c^2} \quad -(40)$$

to high precision, for electromagnetic radiation grazing any object of mass M. This is twice the Newtonian value.

The reason for this famous result cannot be found in the deeply flawed Einsteinian theory, but a straightforward explanation can be found using the principles of this book.

Consider the vector format of the first Maurer Cartan structure equation given here in the notation of chapter one:

$$\mathbf{T}^a(\mathrm{orb}) = -\nabla q^a_0 - \frac{\partial \mathbf{q}^a}{\partial t} - \omega^a_{\ 0b}\,\mathbf{q}^b + q^b_{\ 0}\,\boldsymbol{\omega}^a_{\ b} \quad -(41)$$

and

$$\mathbf{T}^a(\mathrm{spin}) = \nabla \times \mathbf{q}^a - \boldsymbol{\omega}^a_{\ b} \times \mathbf{q}^b. \quad -(42)$$

The fundamental ECE hypothesis was devised for electromagnetism and defines the electromagnetic potential in terms of the tetrad:

$$A^a_\mu = A^{(0)}\,q^a_\mu. \quad -(43)$$

Now define the linear momentum tetrad:

$$p^a_\mu = p^{(0)}\,q^a_\mu \quad -(44)$$

in an analogous manner, using the minimal prescription:

$$p^a_\mu \rightarrow p^a_\mu + eA^a_\mu. \quad -(45)$$

It follows from Eqs. (41) and (44) that the orbital force of ECE theory is:

$$\mathbf{F}^a(\mathrm{orb}) = -\nabla \phi^a_0 - \frac{\partial \mathbf{p}^a}{\partial t} - \omega^a_{\ 0b}\,\mathbf{p}^b + \phi^b_{\ 0}\,\boldsymbol{\omega}^a_{\ b} \quad -(46)$$

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and that the spin force is:

$$\underline{F}^a(\text{spin}) = \underline{\nabla} \times \underline{p}^a - \underline{\omega}^a{}_b \times \underline{p}^b . \quad -(47)$$

In the simplified single polarization theory:

$$\underline{F}(\text{orb}) = -\underline{\nabla}\phi - \frac{\partial \underline{p}}{\partial t} - \omega_0 \underline{p} + \phi \underline{\omega} \quad -(48)$$

and:

$$\underline{F}(\text{spin}) = \underline{\nabla} \times \underline{p} - \underline{\omega} \times \underline{p} . \quad -(49)$$

In the non relativistic limit the spin connection vanishes and:

$$\underline{F}(\text{orb}) = -\underline{\nabla}\phi - \frac{\partial \underline{p}}{\partial t} . \quad -(50)$$

The famous equivalence of inertial and gravitational mass is recovered from Eq. (50) using the anti symmetry law of ECE theory described earlier in this book. So:

$$-\frac{\partial \underline{p}}{\partial t} = -\underline{\nabla}\phi \quad -(51)$$

and:

$$\phi = -\frac{mMG}{r} \quad -(52)$$

where ϕ is the gravitational potential. This is defined in direct analogy to the electromagnetic scalar potential ϕ_e as follows:

$$p_\mu^a = \left(\frac{\phi^a}{c} , -\underline{p}^a \right) -(53)$$

and

$$A^a_\mu = \left(\frac{\phi^a_e}{c}, -\underline{A}^a \right). \quad -(54)$$

In Newtonian dynamics:

$$\phi = -\frac{mMG}{r} \quad -(55)$$

so the force is:

$$F = -\frac{mMG}{r^2} \quad -(56)$$

and the acceleration due to gravity is:

$$g = \frac{MG}{r^2}. \quad -(57)$$

This powerful and precise result of ECE theory was first inferred in UFT 141. The ECE theory is therefore precise to one part in ten to the power seventeen, the precision of the experimental proof of the equivalence of gravitational and inertial mass. The equivalence is due to Cartan geometry.

The calculation of light deflection due to gravitation proceeds by applying the ECE anti symmetry law to Eq. (48) to find that:

$$-\underline{\nabla}\phi + \underline{\omega}\phi = -\frac{d\underline{p}}{dt} - \omega_0\underline{p} \quad -(58)$$

in which it has been assumed that:

$$\frac{d\underline{p}}{dt} = \frac{\partial\underline{p}}{\partial t} \quad -(59)$$

So the force is:

$$\underline{F} = 2\left(-\frac{d\underline{p}}{dt} - \omega_0 \underline{p}\right) = -2\left(\underline{\nabla}\phi - \underline{\omega}\phi\right) \quad -(60)$$

The factor two in Eq. (60) can be eliminated without affecting the physics by assuming that:

$$p^a_\mu = \frac{p^{(0)}}{2} q^a_\mu \quad -(61)$$

so the orbital force becomes:

$$\underline{F} = -\frac{d\underline{p}}{dt} - \omega_0 \underline{p} = -\underline{\nabla}\phi + \underline{\omega}\phi \quad -(62)$$

an equation which gives the equivalence principle (51) for vanishing spin connection.

Now define:

$$\underline{p} = p_r \underline{e}_r \quad -(63)$$

$$\underline{\omega} = \omega_r \underline{e}_r \quad -(64)$$

and compare Eqs. (20) and (62) to find that:

$$F = -\frac{\partial\phi}{\partial r} + \phi\,\omega_r = -\frac{kx^2}{r^2} - \frac{k(1-x^2)\alpha}{r^3} \quad -(63)$$

For small deviations from a Newtonian orbit as in planetary precession or any observable precession in astronomy:

$$-\frac{\partial\phi}{\partial r} = -\frac{kx^2}{r^2} \quad -(64)$$

i. e. :

$$x \sim 1 \quad -(65)$$

to an excellent approximation. From Eqs. (63) and (64) :

$$\phi\,\omega_r = -\frac{k\alpha}{r^3}(1-x^2) \quad -(66)$$

in an almost Newtonian approximation. In this approximation the gravitational potential is well known to be:

$$\phi = -\frac{k}{r} \quad -(67)$$

so the spin connection can be expressed in terms of x as follows:

$$\omega_r = (1-x^2)\frac{\alpha}{r^2} = (1-x^2)\frac{b^2}{ar^2}. \quad -(68)$$

Using Eq. (68), the correction needed to produce Eq. (40) form Eq. (39) is:

$$\frac{R_0c^2}{MG} \rightarrow \frac{R_0c^2}{MG} + \frac{\alpha}{R_0}\left(\frac{1-x^2}{x^2}\right). \quad -(69)$$

Using Eq. (32) it is found that:

$$2\alpha\phi = 2\frac{R_0c^2}{MG} + 2(1+\epsilon)\left(\frac{1-x^2}{x^2}\right). \quad -(70)$$

Experimentally:

$$(1+\epsilon)\left(\frac{1-x^2}{x^2}\right) = \frac{R_0c^2}{MG} \quad -(71)$$

and using Eq. (27):

$$\frac{1}{\epsilon} = \sin\left(\frac{\Delta\phi}{2}\right). \quad -(72)$$

For small deflections:

$$\frac{1}{\epsilon} \sim \frac{\Delta\phi}{2} \quad -(73)$$

so to an excellent approximation:

$$\left(1+\frac{2}{\Delta\phi}\right)\left(\frac{1-x^2}{x^2}\right)^{-1}=\frac{R_0c^2}{MG}. \quad -(74)$$

However by experiment:

$$\Delta\phi=\frac{4R_0c^2}{MG}, \quad -(75)$$

$$x\sim 1,$$

so using Eq. (68):

$$\omega_r=\frac{\Delta\phi}{4}\left(1+\frac{2}{\Delta\phi}\right)^{-1}\frac{\alpha}{r^2}. \quad -(76)$$

From Eq. (32):

$$\alpha=R_0(1+\epsilon)=R_0\left(1+\frac{2}{\Delta\phi}\right) \quad -(77)$$

and from Eqs. (76) and (77)

$$\omega_r=\frac{\Delta\phi}{4}\frac{R_0}{r^2}. \quad -(78)$$

This is a universal spin connection that describes all electromagnetic deflections from any relevant object M in the universe. This spin connection also describes planetary precession through its relation to x, Eq. (68). The procedure used to derive this result also gives the equivalence principle. Finally at distance of closest approach:

$$\omega_r=\frac{\Delta\phi}{4R_0} \quad -(79)$$

a very simple result that can be tabulated in astronomy for any relevant object of mass M.

8.3 THE VELOCITY CURVE OF A WHIRLPOOL GALAXY

Whirlpool galaxies are familiar objects in cosmology and are very complex in structure. However there is one feature that makes them useful for the study of the fundamental theories of cosmology such as those of Newton and Einstein, and ECE, and that is the velocity curve, the plot of the velocity of a star orbiting the centre of a galaxy versus the distance between the star and the centre. It was discovered experimentally in the late fifties that the velocity becomes constant as r goes to infinity. The first part of this section will give the basic kinematics of the orbit and will show that both the Newton and Einstein theories fail completely to describe the velocity curve. The second part will describe how ECE theory gives a plausible explanation of the velocity curve without the use of random empiricism such as dark matter. It appears that the theory of dark matter has been refuted experimentally, leaving ECE cosmology as the only explanation.

Consider the radial vector in the plane of any orbit:

$$\underline{r} = r\,\underline{e}_r \qquad -(80)$$

where $\underline{e}_r$ is the radial unit vector. The velocity of an object of mass m in orbit is defined as:

$$\underline{v} = \frac{d\underline{r}}{dt} = \frac{dr}{dt}\,\underline{e}_r + r\,\frac{d\underline{e}_r}{dt} \qquad -(81)$$

because in plane polar coordinates the unit vector $\underline{e}_r$ is a function of time so the Leibnitz theorem applies. In the Cartesian system the unit vectors $\underline{i}$ and $\underline{j}$ are not functions of time. The unit vectors of the plane polar system are defined by:

$$\underline{e}_r = \cos\theta\,\underline{i} + \sin\theta\,\underline{j} \qquad -(82)$$

$$\underline{e}_\theta = -\sin\theta\,\underline{i} + \cos\theta\,\underline{j} \qquad -(83)$$

and it follows that:

$$\frac{d\underline{e}_r}{dt} = \frac{d\theta}{dt}\,\underline{e}_\theta = \omega\,\underline{e}_\theta \qquad -(84)$$

as described in UFT 236. The velocity in a plane is therefore:

$$\underline{v} = \frac{dr}{dt}\,\underline{e}_r + \omega r\,\underline{e}_\theta = \frac{dr}{dt}\,\underline{e}_r + \underline{\omega} \times \underline{r} \qquad -(85)$$

in which the angular velocity vector:

$$\underline{\omega} = \frac{d\theta}{dt}\,\underline{k} \qquad -(86)$$

is the Cartan spin connection as proven in UFT 235 on www.aias.us. Therefore this spin connection is related to the universal spin connection inferred in Section 8.2 giving a coherent cosmology for the solar system and whirlpool galaxies. As we shall prove, the Newton and Einstein theories fail completely to do so.

Using the chain rule:

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} \qquad -(87)$$

it is found that the velocity is defined for any orbit by:

$$v^2 = \omega^2\left(\left(\frac{dr}{d\theta}\right)^2 + r^2\right) \qquad -(88)$$

and is therefore defined by the angular velocity or spin connection magnitude:

$$\omega = \frac{d\theta}{dt}. \qquad -(89)$$

The orbit itself is defined by $dr/d\theta$, because any planar orbit is defined by r as a function of θ. The angular momentum of any planar orbit is defined by:

$$\underline{L} = \underline{r} \times \underline{p} = m\,\underline{r} \times \underline{v} \qquad -(90)$$

and its magnitude is:

$$L = mr^2 \omega. \quad -(91)$$

Therefore for any planar orbit:

$$v^2 = \left(\frac{L}{mr}\right)^2 + \left(\frac{L}{mr^2}\left(\frac{dr}{d\theta}\right)\right)^2 \quad -(92)$$

and as r becomes infinite:

$$r \rightarrow \infty \quad -(93)$$

the velocity reaches the limit:

$$\frac{dr}{d\theta} = \left(\frac{mv_\infty}{L}\right) r^2 \quad -(94)$$

where v_∞ is the velocity for infinite r. In whirlpool galaxies v_∞ is a constant by experimental observation. Therefore:

$$\frac{d\theta}{dr} = \left(\frac{L}{mv_\infty}\right)\frac{1}{r^2} \quad -(95)$$

and

$$\theta = \frac{L}{mv_\infty}\int \frac{dr}{r^2} = -\left(\frac{L}{mv_\infty}\right)\frac{1}{r} \quad -(96)$$

which is the equation of a hyperbolic spiral orbit. In UFT 76 on www.aias.us this hyperbolic spiral orbit was compared with the observed M101 whirlpool galaxy. So the essentials of galactic dynamics can be understood from the simple first principles of kinematics, defining the angular velocity as the spin connection of ECE theory.

Newtonian dynamics fails completely to describe this result because it produces a static conical section:

$$r = \frac{\alpha}{1 + \epsilon \cos\theta} \quad -(97)$$

with an inverse square law of attraction. From Eq. (97):

$$\frac{dr}{d\theta} = \frac{\epsilon r^2}{\alpha} \sin\theta \quad - (98)$$

and using this result in Eq. (88):

$$v^2 = \omega^2 r^2 \left(1 + \left(\frac{\epsilon r}{\alpha} \right)^2 \sin^2\theta \right) \quad - (99)$$

where:

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{1}{\epsilon^2} \left(\frac{\alpha}{r} - 1 \right)^2 . \quad - (100)$$

So the Newtonian velocity is:

$$v^2 = \omega^2 r^2 \left(\frac{2\alpha}{r} - \left(\frac{r}{\alpha} \right)^2 \left(1 - \epsilon^2 \right) \right) . \quad - (101)$$

The semi major axis of an elliptical orbit is defined by:

$$a = \frac{\alpha}{1 - \epsilon^2} \quad - (102)$$

so Newtonian dynamics produces:

$$v^2 = \frac{1}{\alpha} \left(\frac{L}{m} \right)^2 \left(\frac{2}{r} - \frac{1}{a} \right) . \quad - (103)$$

Using the Newtonian half right latitude:

$$\alpha = \frac{L^2}{m^2 M G} \quad - (104)$$

gives:

$$v^2 = MG \left(\frac{2}{r} - \frac{1}{a} \right) \quad - (105)$$

Note that:

$$\frac{1}{a} = \frac{1-\epsilon^2}{\alpha} = \frac{1}{r}\left(1+\epsilon\cos\theta\right)\left(1-\epsilon^2\right) \quad -(106)$$

so the Newtonian velocity is:

$$v^2(\text{Newton}) = \frac{MG}{r}\left(2-\left(1-\epsilon^2\right)\left(1+\epsilon\cos\theta\right)\right) \quad -(107)$$

It follows that:

$$v(\text{Newton}) \xrightarrow[r\to\infty]{} 0 \quad -(108)$$

so the theory fails completely to describe the velocity curve of a whirlpool galaxy.

The Einstein theory does no better because it produces a precessing ellipse , Eq. (3), from which:

$$\frac{dr}{d\theta} = \frac{x\epsilon r^2}{\alpha}\sin(x\theta). \quad -(109)$$

Using Eq. (109) in Eq. (88) gives:

$$v^2 = \left(\frac{L}{mr}\right)^2\left(1+\left(\frac{x\epsilon\sin(x\theta)}{1+\epsilon\cos(x\theta)}\right)^2\right) \quad -(110)$$

and again it is found that:

$$v(\text{Einstein}) \xrightarrow[r\to\infty]{} 0 \quad -(111)$$

and the Einstein theory fails completely to describe the dynamics of a whirlpool galaxy. This leaves ECE theory as the only correct and general theory of cosmology. The latter can be developed by considering again the acceleration in plane polar coordinates

$$\underline{a} = \frac{d\underline{v}}{dt} = \left(\ddot{r} - r\dot{\theta}^2\right)\underline{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\underline{e}_\theta . \quad -(112)$$

As shown in UFT 235 this can be expressed as:

$$\left(\ddot{r} - r\dot{\theta}^2\right)\underline{e}_r = \frac{d^2 r}{dt^2}\underline{e}_r + \underline{\omega} \times \left(\underline{\omega} \times \underline{r}\right) \quad -(113)$$

and

$$\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\underline{e}_\theta = \frac{d\underline{\omega}}{dt} \times \underline{r} + 2\underline{\omega} \times \underline{\dot{r}} \quad -(114)$$

Eq. (114) is the Coriolis acceleration and $\underline{\omega} \times \left(\underline{\omega} \times \underline{r}\right)$ is the centrifugal acceleration. In the UFT papers it is shown that the Coriolis acceleration vanishes for all planar orbits (see Eq. (11)). Using the chain rule it can be shown as in the UFT papers that:

$$\frac{d^2 r}{dt^2} = \left(\frac{L}{mr}\right)^2 \left(\frac{dr}{d\theta}\right) \frac{d}{dr}\left(\frac{1}{r^2}\frac{dr}{d\theta}\right). \quad -(115)$$

The centrifugal acceleration is defined by:

$$\underline{\omega} \times \left(\underline{\omega} \times \underline{r}\right) = -\omega^2 r\,\underline{e}_r = -\frac{L^2}{m^2 r^3}\underline{e}_r \quad -(116)$$

so the total acceleration is defined by:

$$\underline{a} = \left(\frac{L}{mr}\right)^2 \left[\left(\frac{dr}{d\theta}\right)\frac{d}{dr}\left(\frac{1}{r^2}\frac{dr}{d\theta}\right) - \frac{1}{r}\right]\underline{e}_r \quad -(117)$$

for all planar orbits.

In this equation:

$$\frac{d}{dr}\left(\frac{1}{r^2}\frac{dr}{d\theta}\right) = \frac{d\theta}{dr}\frac{d}{d\theta}\left(\frac{1}{r^2}\frac{dr}{d\theta}\right) \quad -(118)$$

so:

$$\underline{a} = \left(\frac{L}{mr}\right)^2 \left[\frac{d}{d\theta}\left(\frac{1}{r^2}\frac{dr}{d\theta}\right) - \frac{1}{r}\right] \underline{e}_r \qquad -(119)$$

Now note that:

$$\frac{d}{d\theta}\left(\frac{1}{r}\right) = \frac{d}{dr}\left(\frac{1}{r}\right)\frac{dr}{d\theta} \qquad -(120)$$

so:

$$\frac{d}{d\theta}\left(\frac{1}{r^2}\frac{dr}{d\theta}\right) = \frac{1}{r^2}\frac{d}{d\theta}\left(\frac{dr}{d\theta}\right) = -\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right). \qquad -(121)$$

Therefore the acceleration is:

$$\underline{a} = -\left(\frac{L}{mr}\right)^2\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right)\underline{e}_r \qquad -(122)$$

and using the definition of force:

$$\underline{F} = m\,\underline{a} \qquad -(123)$$

which is Eq (19) derived from lagrangian dynamics. This analysis of any planar orbit is therefore rigorously self consistent.

The Lagrangian method of deriving Eq. (123) sets up the Lagrangian:

$$\mathcal{L} = \frac{1}{2}mv^2 - U \qquad -(124)$$

in which the velocity is defined by:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2. \qquad -(125)$$

The force is derived from the potential energy as follows:

$$F = -\frac{\partial U}{\partial r} \qquad -(126)$$

The two Euler Lagrange equations are:

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right), \quad \frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{r}}\right) \quad -(127)$$

and the angular momentum is defined by the lagrangian to be a constant of motion:

$$L = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = mr^2 \frac{d\theta}{dt} = \text{constant}. \quad -(128)$$

Eq. (122) is the result of pure kinematics in a plane, and is also an equation of Cartan geometry. It is the result of the fundamental expression for acceleration in a plane. Eq. (122) is also an equation of Cartan geometry because the spin connection is the angular velocity.

The covariant derivative of Cartan may be defined for use in classical kinematics in three dimensional space. For any vector $\underline{V}$ the covariant derivative is:

$$\frac{D\underline{V}}{dt} = \left(\frac{d\underline{V}}{dt}\right)_{\text{axes fixed}} + \underline{\omega} \times \underline{V} \quad -(129)$$

where the spin connection vector is the angular velocity ω . In plane polar coordinates define:

$$\underline{V} = V\underline{e}_r \quad -(130)$$

for simplicity of development. The velocity is then defined by:

$$\underline{v} = \frac{D\underline{r}}{dt} = \frac{d\underline{r}}{dt} + \underline{\omega} \times \underline{r} \quad -(131)$$

where:

$$\frac{d\underline{r}}{dt} = \left(\frac{d\underline{r}}{dt}\right)_{\text{axes fixed}} \quad -(132)$$

By definition:

$$\frac{D\underline{r}}{dt} = \frac{D}{dt}\left(r\underline{e}_r\right) = \frac{dr}{dt}\underline{e}_r + r\frac{d\underline{e}_r}{dt} \quad -(133)$$

so:

$$\left(\frac{d\underline{r}}{dt}\right)_{\text{axes fixed}} = \left(\frac{dr}{dt}\right)\underline{e}_r \quad -(134)$$

and

$$\underline{\omega} \times \underline{r} = r\frac{d\underline{e}_r}{dt} \quad -(135)$$

The acceleration is defined by:

$$\underline{a} = \frac{D\underline{v}}{dt} = \frac{d\underline{v}}{dt} + \underline{\omega} \times \underline{v} \quad -(136)$$

where:

$$\frac{d\underline{v}}{dt} = \left(\frac{d\underline{v}}{dt}\right)_{\text{axes fixed}} \quad -(137)$$

From fundamental kinematics as described above:

$$\underline{a} = \frac{d\underline{v}}{dt} + \underline{\omega} \times \underline{v} = \left(\ddot{r} - \omega^2 r\right)\underline{e}_r + \left(r\frac{d\omega}{dt} + 2\frac{dr}{dt}\omega\right)\underline{e}_\theta \quad -(138)$$

where the unit vectors of the plane polar coordinates system are defined by:

$$\underline{e}_r \times \underline{e}_\theta = \underline{k} \quad -(139)$$

$$\underline{k} \times \underline{e}_r = \underline{e}_\theta \quad -(140)$$

$$\underline{e}_\theta \times \underline{k} = \underline{e}_r \quad -(141)$$

Therefore:

$$\frac{d\underline{v}}{dt} + \underline{\omega} \times \underline{v} = \frac{d^2 r}{dt^2}\,\underline{e}_r + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \frac{d\underline{\omega}}{dr} \times \underline{r} + 2\underline{\omega} \times \left(\frac{dr}{dt}\,\underline{e}_r\right). \quad -(142)$$

From Eq. (131)

$$\underline{v} = \frac{dr}{dt} + \underline{\omega} \times \underline{r} \quad -(143)$$

so in Eq. (136):

$$\underline{a} = \frac{d^2 r}{dt^2}\,\underline{e}_r + \frac{d\underline{\omega}}{dt} \times \underline{r} + \underline{\omega} \times \left(\frac{dr}{dt}\right)_{\text{axes fixed}} + \underline{\omega} \times \frac{dr}{dt}\,\underline{e}_r + \underline{\omega} \times (\underline{\omega} \times \underline{r}) \quad -(144)$$

In this equation:

$$\underline{\omega} \times \left(\frac{dr}{dt}\right)_{\text{axes fixed}} = \underline{\omega} \times \frac{dr}{dt}\,\underline{e}_r \quad -(145)$$

so:

$$\underline{a} = \frac{d^2 r}{dt^2}\,\underline{e}_r + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \frac{d\underline{\omega}}{dt} \times \underline{r} + 2\underline{\omega} \times \left(\frac{dr}{dt}\,\underline{e}_r\right) \quad -(146)$$

which is Eq. (142), QED.

The covariant derivatives used in these calculations are examples of the Cartan covariant derivative:

$$D_\mu V^a = \partial_\mu V^a + \omega^a_{\ \mu b} V^b. \quad -(147)$$

The well known centripetal acceleration:

$$\underline{a} = \underline{\omega} \times (\underline{\omega} \times \underline{r}) \quad -(148)$$

and the Coriolis acceleration:

$$\underline{a} = \frac{d\underline{\omega}}{dt} \times \underline{r} + 2\underline{\omega} \times \left(\frac{dr}{dt} \underline{e}_r \right) \quad -(149)$$

are produced by the plane polar system of coordinates. These accelerations do not exist in the Cartesian system and depend entirely on the existence of the spin connection of Cartan.

As shown already the Coriolis acceleration vanishes for all closed planar orbits and the acceleration simplifies to:

$$\underline{a} = \left(\ddot{r} - \omega^2 r \right) \underline{e}_r = \frac{d^2 r}{dt^2} \underline{e}_r + \underline{\omega} \times \left(\underline{\omega} \times \underline{r} \right) . \quad -(150)$$

For example the acceleration due to gravity is:

$$\underline{g} = \frac{d^2 r}{dt^2} \underline{e}_r + \underline{\omega} \times \left(\underline{\omega} \times \underline{r} \right) \quad -(151)$$

and includes the centripetal acceleration:

$$\underline{\omega} \times \left(\underline{\omega} \times \underline{r} \right) = - \omega^2 r \underline{e}_r . \quad -(152)$$

The acceleration due to gravity in the plane polar system is the sum of $\underline{g}$ in the Cartesian system:

$$\underline{g}\,(\text{Cartesian}) = \frac{d^2 r}{dt^2} \underline{e}_r \quad -(153)$$

and the centripetal acceleration. To make this point clearer consider the acceleration of an elliptical orbit or closed elliptical trajectory in the plane polar system. It is:

$$\underline{a} = - \frac{L^2}{m^2 r^2 \alpha} \underline{e}_r \quad -(154)$$

where the angular momentum is a constant of motion and defined by:

$$L = |\underline{L}| = |\underline{r} \times \underline{p}| = m r^2 \omega . \quad -(155)$$

The acceleration due to gravity of the elliptical motion of a mass m is:

$$\underline{g} = -\frac{L^2}{m^2 r^2 \alpha} \underline{e}_r \quad -(156)$$

in plane polar coordinates. The Newtonian result is recovered using the half right latitude:

$$\alpha = \frac{L^2}{m^2 \underline{M} G} \quad -(157)$$

so:

$$\underline{g} = -\frac{MG}{r^2} \underline{e}_r . \quad -(158)$$

The only force present in the plane polar system of coordinates is:

$$\underline{F} = m \underline{g} = -\frac{m M G}{r^2} \underline{e}_r \quad -(159)$$

which is the equivalence principle, Q. E. D.

The acceleration in the Cartesian system of coordinates from Eq. (151) is:

$$\underline{a}\,(\text{Cartesian}) = \underline{g} - \underline{\omega} \times (\underline{\omega} \times \underline{r}) \quad -(160)$$

in which the centrifugal acceleration is:

$$-\underline{\omega} \times (\underline{\omega} \times \underline{r}) = \omega^2 r \, \underline{e}_r . \quad -(161)$$

Therefore in the Cartesian system the acceleration produced by the same elliptical trajectory

is:

$$\left(\frac{d^2 r}{dt^2}\right)_{\text{Cartesian}} \underline{e}_r = \left(-\frac{L^2}{m^2 r^2 \alpha} + \omega^2 r\right) \underline{e}_r \quad -(162)$$

It generalizes the Newtonian theory to give:

$$\left(\frac{d^2 r}{dt^2}\right)_{\text{Cartesian}} \underline{e}_r = \left(-\frac{MG}{r^2} + \frac{L^2}{m^2 r^3}\right) \underline{e}_r \quad -(163)$$

and the familiar force:

$$\underline{F} = m \left(\frac{d^2 r}{dt^2}\right)_{\text{Cartesian}} \underline{e}_r = \left(-\frac{mMG}{r^2} + \frac{L^2}{m r^3}\right) \underline{e}_r \quad -(164)$$

of the textbooks. From a comparison of Eqs. (159) and (164) the forces in the plane polar and Cartesian systems are different. If the frame of reference is static with respect to the observer the force is Eq. (164). If the frame of reference is rotating with respect to the observer the force is defined by Eq. (159).

The easiest way to approach this analysis is always to calculate the acceleration firstly in plane polar coordinates and to realize that one term of the resultant expression is the acceleration in the Cartesian system. For an observer on earth orbiting the sun, the relevant expression is that in the Cartesian frame, because the latter is also fixed on the earth and does not move with respect to the observer. In other words the observer is in his own frame of reference. For an observer on the sun the relevant expression is that in the plane polar system of coordinates, because the earth rotates with respect to the observer fixed on the sun.

The observer on the earth experiences the centrifugal acceleration:

$$-\underline{\omega} \times (\underline{\omega} \times \underline{r}) = \omega^2 r \, \underline{e}_r \qquad -(165)$$

directed outwards from the earth. This is the origin of the everyday centrifugal force. The observer on the sun experiences the centripetal acceleration:

$$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -\omega^2 r \, \underline{e}_r \qquad -(166)$$

directed towards the sun and towards the observer. The entire analysis rests on the spin connection and on the fact that in the plane polar system the frame itself is rotating and thus generates the spin connection by definition.

8.4 DESCRIPTION OF ORBITS WITH THE MINKOWSKI FORCE EQUATION.

In UFT 238 on www.aias.us an entirely new approach to orbital theory was taken using the Minkowski force equation. This is a course that relativity theory could have taken, but cosmology followed the use of Einstein's flawed geometry, a subject that became known as general relativity. The Minkowski force equation is the Newton force equation with proper time τ replacing time t. This equation was inferred by Minkowski shortly after Einstein's introduction of the idea of relativistic momentum. A completely general kinematic theory of orbits can be developed in this way. It reduces to the Newtonian theory but never to the Einsteinian theory. Newtonian dynamics does not give any of the forces that are generated as discussed in Section 8.3 using plane polar coordinates and a system of rotating coordinates. It turns out that the space part of the Minkowski four force produces new and unexpected orbital properties that can be tested experimentally.

The relativistic force law and relativistic orbits of the Minkowski equation can be derived by considering the relativistic velocity in plane polar coordinates:

$$\mathbf{v} = \frac{d\mathbf{r}}{d\tau} = \gamma \frac{d\mathbf{r}}{dt} \quad -(167)$$

where τ is the proper time and γ the Lorentz factor:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(168)$$

$$-(169)$$

The relativistic acceleration is:

$$\mathbf{a} = \frac{d}{d\tau}\left(\frac{d\mathbf{r}}{d\tau}\right) = \frac{d}{d\tau}\left(\gamma \frac{d\mathbf{r}}{dt}\right) = \gamma \frac{d}{dt}\left(\gamma \frac{d\mathbf{r}}{dt}\right).$$

Using the Leibnitz Thoerem:

$$\mathbf{a} = \gamma\left(\frac{d\gamma}{dt}\frac{d\mathbf{r}}{dt} + \gamma \frac{d}{dt}\left(\frac{d\mathbf{r}}{dt}\right)\right). \quad -(170)$$

The velocity v appearing in the Lorentz factor is defined by the infinitesimal line element:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - d\mathbf{r} \cdot d\mathbf{r} \quad -(171)$$

where:

$$d\mathbf{r} \cdot d\mathbf{r} = v^2 dt^2 \quad -(172)$$

Therefore

$$c^2 d\tau^2 = \left(c^2 - v^2\right) dt^2 \quad -(173)$$

and the Lorentz factor is:

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}. \quad -(174)$$

In plane polar coordinates:

$$d\underline{r} \cdot d\underline{r} = dr^2 + r^2 d\theta^2. \quad -(175)$$

Therefore:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2. \quad -(176)$$

The radial vector in plane polar coordinates is:

$$\underset{\sim}{r} = r\,\underline{e}_r \quad -(177)$$

therefore the non relativistic velocity is:

$$\underline{v} = \frac{d}{dt}\left(r\,\underline{e}_r\right) = \frac{dr}{dt}\underline{e}_r + r\frac{d\underline{e}_r}{dt} = \frac{dr}{dt}\underline{e}_r + \omega r\,\underline{e}_\theta$$

$$= \frac{dr}{dt}\underline{e}_r + \underline{\omega} \times \underline{r} = \left(\frac{L_0}{m}\right)\left(\frac{1}{r}\underline{e}_\theta - \frac{d}{d\theta}\left(\frac{1}{r}\right)\underline{e}_r\right). \quad -(178)$$

For a particle of mass m in an orbit, its relativistic momentum is:

$$\underline{p} = \gamma m \frac{d\underline{r}}{dt} = m\frac{d\underline{r}}{d\tau} \quad -(179)$$

an equation which can be rearranged as follows:

$$p^2c^2 = \gamma^2 m^2 c^4 \left(\frac{v}{c}\right)^2 = \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2}\right) = \gamma^2 m^2 c^4 - m^2 c^4 \quad -(180)$$

giving the Einstein energy equation:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(181)$$

in which

$$E = \gamma m c^2 \quad -(182)$$

is the total energy and

$$E_0 = m c^2 \quad -(183)$$

is the rest energy. The relativistic total angular momentum is:

$$L = m r^2 \frac{d\theta}{d\tau} = \gamma L_0 . \quad -(184)$$

The concept of Minkowski force equation uses acceleration, so this is a plausible new approach to all orbits. The Einstein energy equation can be derived from the infinitesimal line element (171) and developed as:

$$m c^2 = m c^2 \left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dr}{d\tau}\right)^2 - r^2 \left(\frac{d\theta}{d\tau}\right)^2$$
$$= \gamma^2 m c^2 - \left(\left(\frac{dr}{d\tau}\right)^2 + r^2 \left(\frac{d\theta}{d\tau}\right)^2\right) = \frac{E^2}{m c^2} - \frac{p^2}{c^2} \quad -(185)$$

So

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(186)$$

Q. E. D. The relativistic linear momentum in Eq. (185) is:

$$p^2 = m^2 \left(\left(\frac{dr}{d\tau}\right)^2 + r^2 \left(\frac{d\theta}{d\tau}\right)^2\right) \quad -(187)$$

which is Eq. (179), Q. E. D. The definition of relativistic acceleration is

$$\underline{a} = \frac{d}{d\tau}\left(\frac{d\underline{r}}{d\tau}\right) = \gamma\left(\frac{d\gamma}{dt}\frac{d\underline{r}}{dt} + \gamma\frac{d}{dt}\left(\frac{d\underline{r}}{dt}\right)\right) \quad -(188)$$

in which:

$$\frac{d\underline{r}}{dt} = \frac{dr}{dt}\underline{e}_r + \underline{\omega}\times\underline{r} \quad -(189)$$

and

$$\frac{d}{dt}\left(\frac{d\underline{r}}{dt}\right) = \frac{d^2r}{dt^2}\underline{e}_r + \frac{d\underline{\omega}}{dt}\times\underline{r} + 2\underline{\omega}\times\frac{dr}{dt}\underline{e}_r + \underline{\omega}\times(\underline{\omega}\times\underline{r}) \quad -(190)$$

Using the chain rule:

$$\frac{d\gamma}{dt} = \frac{d\gamma}{dv}\frac{dv}{dt} \quad -(191)$$

where v is the velocity of the Lorentz factor defined in Eq. (174). Therefore:

$$\frac{d\gamma}{dv} = \frac{d}{dv}\left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \gamma^3\frac{v}{c^2} \quad -(192)$$

and in plane polar coordinates:

$$\underline{a} = \gamma^4\frac{v}{c^2}\frac{dv}{dt}\frac{d\underline{r}}{dt} + \gamma^2\frac{d}{dt}\left(\frac{d\underline{r}}{dt}\right) \quad -(193)$$

$$= \left(\frac{d\gamma}{d\tau}\frac{dr}{dt} + \gamma^2\frac{d^2r}{dt^2}\right)\underline{e}_r + \gamma^2\underline{\omega}\times(\underline{\omega}\times\underline{r}) + \frac{d\gamma}{d\tau}\underline{\omega}\times\underline{r} + \gamma^2\left(\frac{d\underline{\omega}}{dt}\times\underline{r} + 2\underline{\omega}\times\frac{dr}{dt}\underline{e}_r\right)$$

In static Cartesian coordinates on the other hand;

$$\underline{a} = \frac{d}{d\tau}\left(\gamma\frac{d\underline{r}}{dt}\right) = \gamma\frac{d\gamma}{dt}\frac{d\underline{r}}{dt} + \gamma^2\frac{d}{dt}\left(\frac{d\underline{r}}{dt}\right) \quad -(194)$$

so:

$$\underline{a}\left(\text{Cartesian}\right) = \left(\gamma \frac{d\gamma}{dt}\frac{dr}{dt} + \gamma^2 \frac{d^2 r}{dt^2}\right)\underline{e}_r \quad -(195)$$

in which:

$$v = \frac{dr}{dt}, \quad \frac{d^2 r}{dt^2} = \frac{dv}{dt}, \quad \frac{d\gamma}{dv} = \gamma^3 \frac{v}{c^2} \quad -(196)$$

and

$$\frac{d\gamma}{dt} = \frac{d\gamma}{dv}\frac{dv}{dt} = \frac{\gamma^3 v}{c^2}\frac{dv}{dt}. \quad -(197)$$

Therefore:

$$\underline{a}\left(\text{Cartesian}\right) = \left(\gamma^4 \frac{v^2}{c^2} + \gamma^2\right)\frac{dv}{dt}\underline{e}_r \quad -(198)$$

in which:

$$\frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2}. \quad -(199)$$

Therefore the Cartesian acceleration is:

$$\underline{a}\left(\text{Cartesian}\right) = \gamma^4 \frac{d^2 r}{dt^2}\underline{e}_r. \quad -(200)$$

Using Eq. (200) in Eq. (193):

$$\underline{a}\left(\text{plane polar}\right) = \gamma^4 \frac{d^2 r}{dt^2}\underline{e}_r + \gamma^2 \underline{\omega}\times(\underline{\omega}\times\underline{r}) + \frac{d\gamma}{d\tau}\underline{\omega}\times\underline{r} + \gamma^2\left(\frac{d\underline{\omega}}{dt}\times\underline{r} + 2\underline{\omega}\times\frac{dr}{dt}\underline{e}_r\right) \quad -(201)$$

which is the expression for relativistic acceleration in plane polar coordinates.

It can be proven as follows that the relativistic Coriolis acceleration vanishes for all planar orbits. The general expression for relativistic Coriolis acceleration is:

$$\underline{a}\,(\text{Coriolis}) = \gamma^2\left(r\frac{d}{dt}\frac{d\theta}{dt} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right)\underline{e}_\theta \quad -(202)$$

in which the total ~~relativistic~~ non relativistic angular momentum is:

$$L_0 = mr^2\frac{d\theta}{dt}. \quad -(203)$$

It follows that:

$$\frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d}{dt}\left(\frac{L_0}{mr^2}\right) = \frac{d}{dr}\left(\frac{L_0}{mr^2}\right)\frac{dr}{dt} = -\frac{2L_0}{mr^3}\frac{dr}{dt} \quad -(204).$$

so:

$$\underline{a}\,(\text{Coriolis}) = \left(-\frac{2L_0}{mr^2}\frac{dr}{dt} + \frac{2L_0}{mr^2}\frac{dr}{dt}\right)\underline{e}_\theta = \underline{0} \quad -(205)$$

Q. E. D.

Therefore the relativistic acceleration for all planar orbits is:

$$\underline{a} = \gamma^4\frac{d^2r}{dt^2}\underline{e}_r + \gamma^2\underline{\omega}\times(\underline{\omega}\times\underline{r}) + \frac{d\gamma}{d\tau}\underline{\omega}\times\underline{r}. \quad -(206)$$

The relativistic centripetal component of this orbit is:

$$\underline{a}\,(\text{centripetal}) = \gamma^2\underline{\omega}\times(\underline{\omega}\times\underline{r}) = -\frac{L^2}{m^2r^3}\underline{e}_r. \quad -(207)$$

In Eq. (206):

$$\frac{d\gamma}{d\tau} = \gamma\frac{d\gamma}{dv}\frac{dv}{dt} = \frac{\gamma^4}{c^2}v\frac{dv}{dt} = \frac{\gamma^4}{c^2}\frac{dr}{dt}\frac{d^2r}{dt^2} \quad -(208)$$

and therefore the acceleration becomes:

$$\underline{a} = \gamma^4 \frac{d^2 r}{dt^2}\underline{e}_r - \frac{L^2}{m^2 r^3}\underline{e}_r + \frac{\gamma^4}{c^2}\frac{dr}{dt}\frac{d^2 r}{dt^2}\omega r\,\underline{e}_\theta \quad -(209)$$

in which the relativistic total angular momentum is

$$L = \gamma L_0 = m r^2 \frac{d\theta}{d\tau} = \gamma m r^2 \omega . \quad -(210)$$

The relativistic force law is therefore the mass m multiplied by the relativistic acceleration:

$$\underline{a} = \left(\gamma^4 \frac{d^2 r}{dt^2} - \frac{L^2}{m^2 r^3}\right)\underline{e}_r + \frac{\gamma^4}{c^2}\frac{dr}{dt}\frac{d^2 r}{dt^2}\underline{\omega}\times\underline{r} \quad -(211)$$

in which:

$$\underline{\omega}\times\underline{r} = \omega r\,\underline{e}_\theta . \quad -(212)$$

This equation can be transformed into a format where the relativistic force can be calculated from the observation of any planar orbit. The result is the relativistic generalization of Eq, (122).

Consider the relativistic acceleration:

$$\underline{a} = \gamma^4 \frac{d^2 r}{dt^2}\underline{e}_r + \gamma^2\underline{\omega}\times(\underline{\omega}\times\underline{r}) + \frac{d\gamma}{d\tau}\underline{\omega}\times\underline{r} \quad -(213)$$

in which the relativistic momentum is:

$$\underline{p} = m\frac{d\underline{r}}{d\tau} . \quad -(214)$$

It follows that:

$$\frac{d^2 r}{dt^2} = -\left(\frac{L}{\gamma m r}\right)^2 \frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) \quad -(215)$$

and that:

$$\underline{a} = -\left(\left(\frac{\gamma L}{mr}\right)^2 \frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{L^2}{m^2 r^3}\right)\underline{e}_r + \frac{\gamma^4}{c^2}\frac{dr}{dt}\frac{d^2 r}{dt^2}\,\omega r\,\underline{e}_\theta \quad -(216)$$

It also follows as in UFT 238 that:

$$\frac{dr}{dt} = -\frac{L}{m\gamma}\frac{d}{d\theta}\left(\frac{1}{r}\right) \quad -(217)$$

so the required relativistic generalization of Eq. (122) is:

$$\underline{a} = -\left(\frac{L}{mr}\right)^2\left(\gamma^2 \frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right)\underline{e}_r + \frac{L^4}{m^4 c^2 r^3}\frac{d}{d\theta}\left(\frac{1}{r}\right)\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)\underline{e}_\theta \quad -(218)$$

For the purposes of graphics and animation it is convenient to express the Lorentz factor in terms of the angle θ. The result as derived in UFT 238 is:

$$v^2 = \left(\frac{L_0}{md}\right)^2\left(1 + \epsilon^2 + 2\epsilon\cos\theta\right) \quad -(219)$$

In summary, the relativistic force for any planar orbit is defined by:

$$\underline{F} = -\frac{L^2}{mr^2}\left(\gamma^2 \frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right)\underline{e}_r + \frac{L^4}{m^4 c^2 r^3}\frac{d}{d\theta}\left(\frac{1}{r}\right)\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)\underline{e}_\theta \quad -(220)$$

in which the Lorentz factor is:

$$\gamma = \left(1 - \left(\frac{L_0}{mc}\right)^2\left(\frac{1}{r^2} + \left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2\right)\right)^{-1/2} \quad -(221)$$

and in which the relativistic total angular momentum is:

$$L = \gamma L_0 = \gamma m r^2 \frac{d\theta}{d\tau} = \gamma m r^2 \omega \quad -(222)$$