Possible Definitions of a Torsion in the Riemann-Minkowski Space

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The asymmetry of the electromagnetic field tensor led people to the idea of a torsion. As the Geometric theory of fields demonstrates, the field tensor is first a curve parameter of the world-lines, and is performed from surfaces with minimal and maximal mean RIEMANNian curvature [1,2]. It might be itself a kind of torsion. With it, the definition from another special torsion is actually unnecessary. However, people like the idea of the origin of electromagnetism from an "outer" curvature respectively torsion.

Let us see if definitions are possible that harmonize with mentioned curve parameter.

Outer geometrical quantities can be relevant only then, if they are related to the "inner" coördinates, i.e. those of the basical manifold. The basical manifold is here the space-time. Exclusively the tensor set of the Second fundamental form

$$\psi = \sum_{\sigma} \Omega_{\sigma ij} \Omega_{\sigma kl} \mathsf{d}x^i \mathsf{d}x^j \mathsf{d}x^k \mathsf{d}x^l \tag{1}$$

(in which σ means the dimensions above those of the basical manifold) meets this condition. These basical tensors are symmetrical, like the metric tensor.

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The RICCI tensor follows from

$$R_{ik} = R^l{}_{ikl} = \sum_\sigma (\Omega_\sigma{}^l{}_i \Omega_{\sigma kl} - \Omega_\sigma{}^l{}_l \Omega_{\sigma ik}) \quad + \quad Z_{ik} \quad , \tag{2}$$

in which Z means an additional term with huge lots of degrees of freedom, as to see at EISENHART [3]. One may compare it with the RICCI tensor as follows from electromagnetism [1]

$$R_{ik} = \frac{1}{4}\, g_{ik} F_{ab} F^{ab} - F_{ia} F_k{}^a \qquad . \tag{3}$$

One does not need any try going from a "distance" ψ. A plausible ansatz were for the vector potential

$$A_i = \sum_\sigma \Omega_{\sigma ij} \frac{\mathsf{d}x^j}{\mathsf{d}s} \qquad . \tag{4}$$

The field tensor were then

$$F_{ik} = A_{i,k} - A_{k,i} = \sum_\sigma [(\Omega_{\sigma ij;k} - \Omega_{\sigma kj;i}) \frac{\mathsf{d}x^j}{\mathsf{d}s} + \Omega_{\sigma ij} (\frac{\mathsf{d}x^j}{\mathsf{d}s})_{;k} - \Omega_{\sigma kj} (\frac{\mathsf{d}x^j}{\mathsf{d}s})_{;i}] \quad . \tag{5}$$

This led to complicated connections, that were fulfillable with the degrees of freedom. The ansatz

$$F_{ik} = \text{const} \cdot \sum_\sigma (\Omega_{\sigma ia} \delta^a_k - \Omega_{\sigma ka} \delta^a_i) \tag{6}$$

(δ means the KRONECKER aka mixed metric tensor)
looks as though it could better fit in parts. However, it becomes identically zero.

We see that "outer" definitions are the first that comes along, if they do not vanish anyway. With it, they are simply irrelevant, mathematically and physically.
Such idea may be nice, but it is unnecessary for the lots of degrees of freedom. However, any claim that electromagnetism had nothing to do with geometry (HEHL [4]), is absolutely unfounded. Just the RIEMANNian four-manifold of signature 2 involves electromagnetism.

References

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