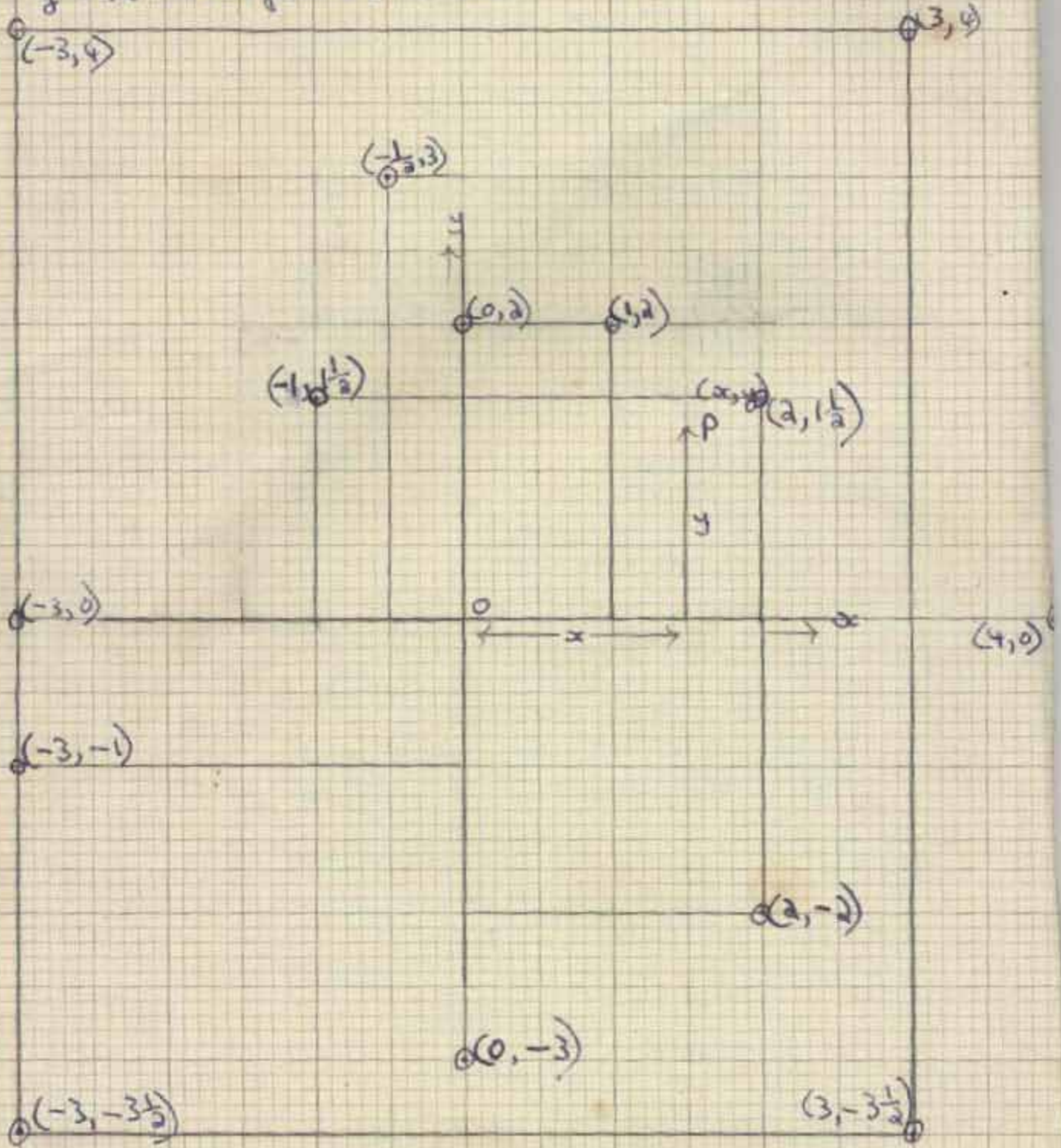
Name MYRON WYN EVANS

Form IV L Date 1964-65

Subject ALGEBRA GRAPHS

NEATNESS IS ESSENTIAL
ALL GRAPHS TO BE DRAWN IN PENCIL
ALL WRITING IS TO BE DONE IN INK

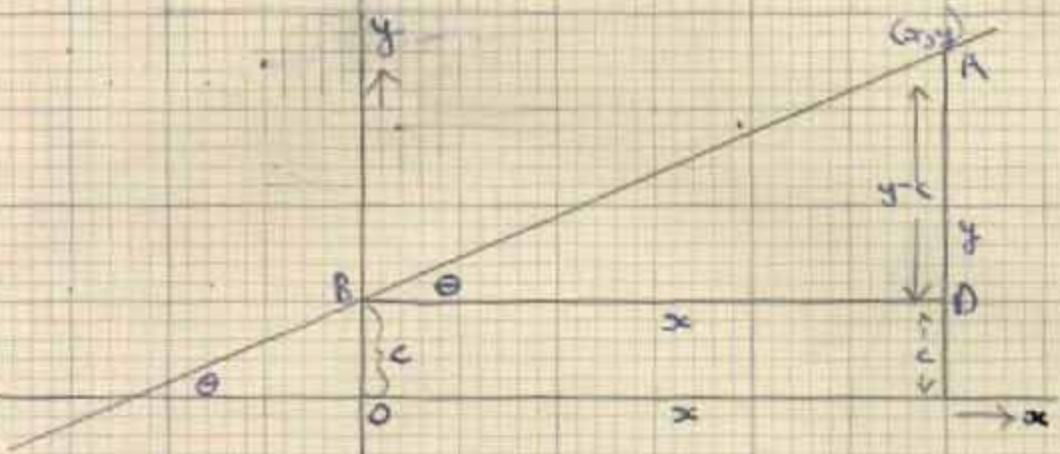
Ox and Oy are the axes
(x, y) are the co-ordinates of the point P
x is the distance from the y axis
y is the distance from the x axis



THE STRAIGHT LINE

A straight line has a constant gradient.

The gradient of the line is the Tangent of the angle which the line makes with the x axis



Let the line have gradient m and make an intercept c on the y axis.

Let A be point (x, y) on the line. Let the line meet the y axis in B through B

Let the line parallel to the x axis meet the line through A parallel to the y axis in D

In $\triangle ABD$

$AD = y - c$

$BD = x$

$\tan\theta = \frac{AD}{BD}$

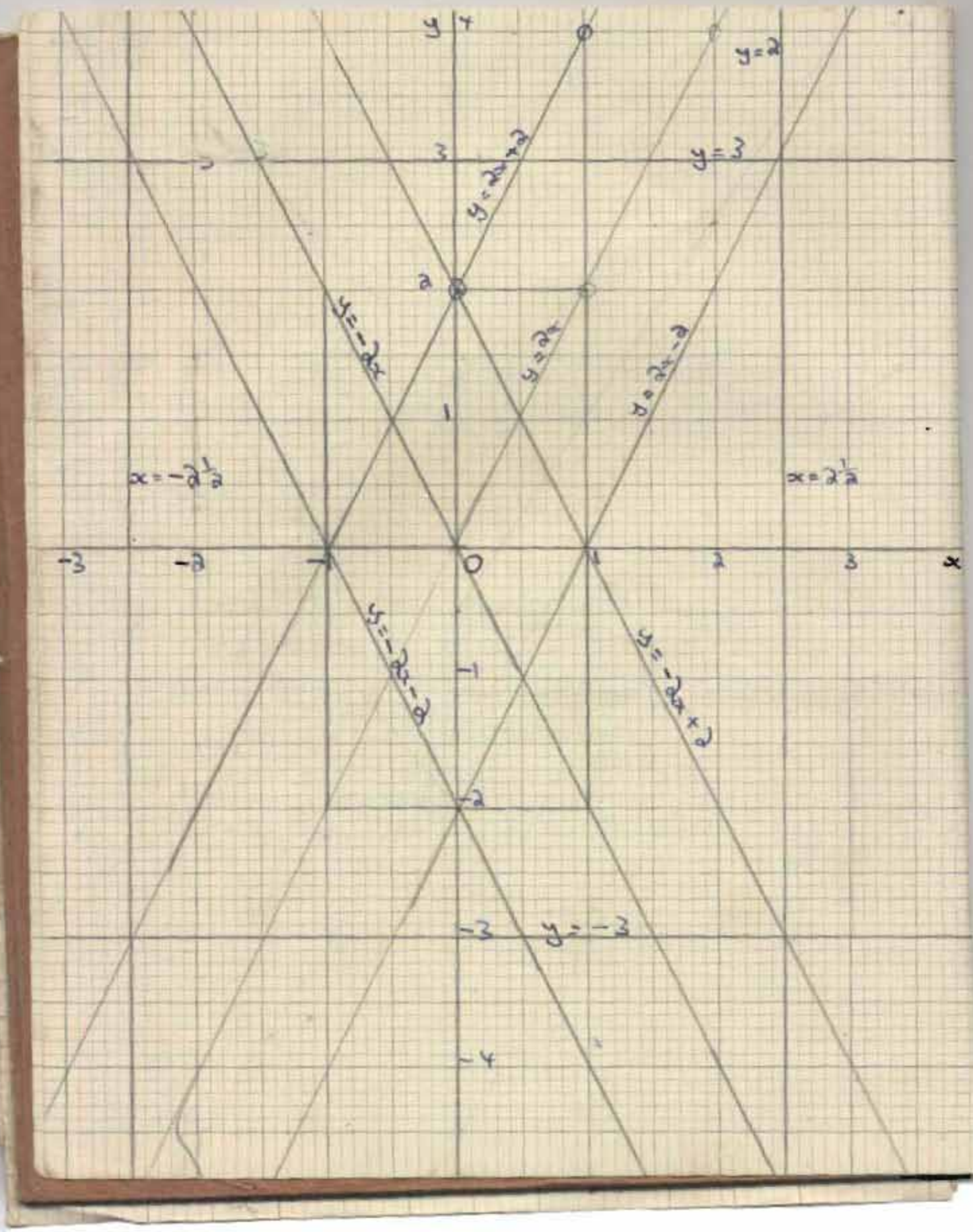
$\therefore m = \frac{y-c}{x}$

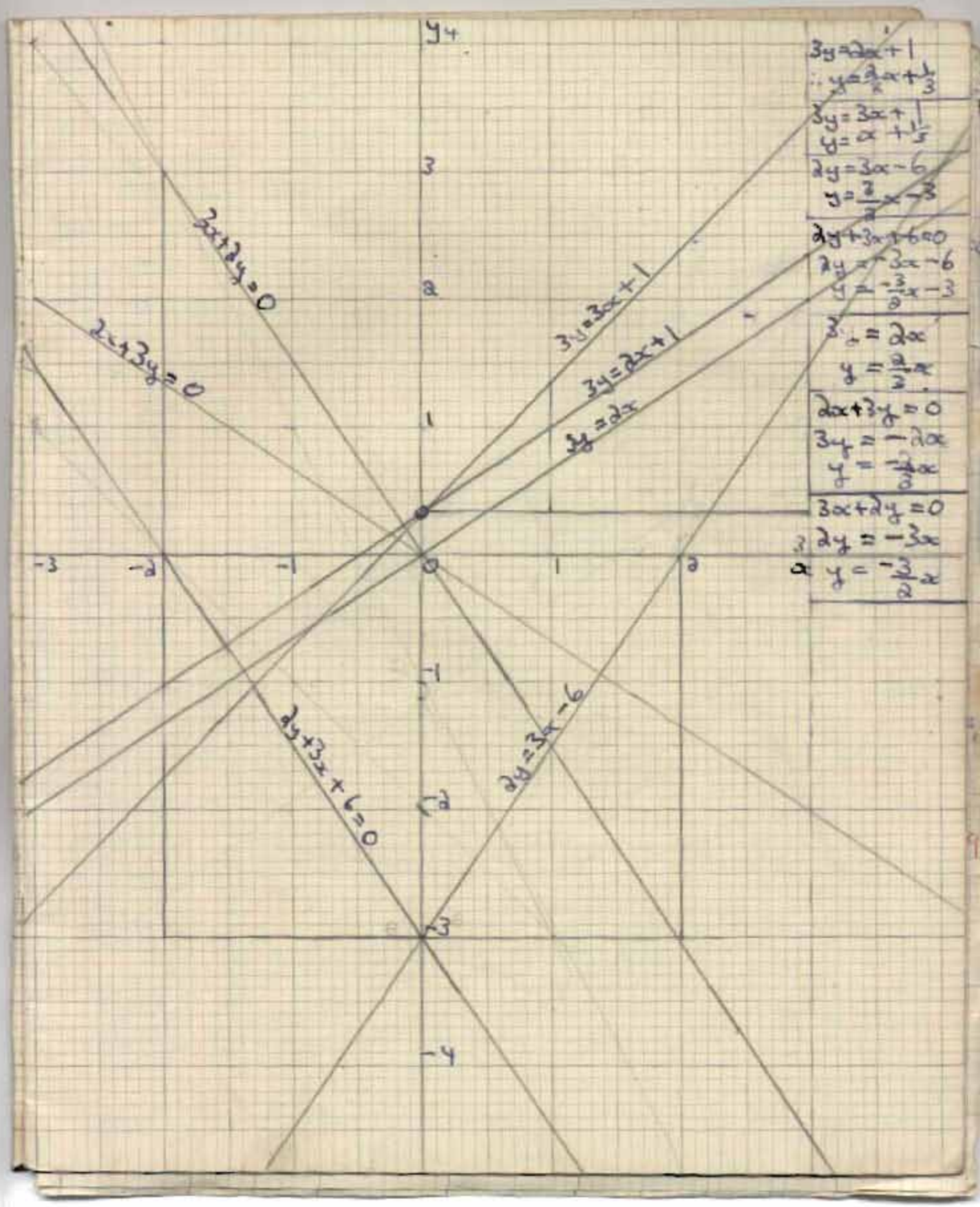
$\therefore mx = y - c$

$\therefore y = mx + c$

$y = mx + c$ is the equation of a straight line with gradient m making an intercept c on the y axis.

$y = mx + c$ is the law connecting x and y so that the point (x, y) lies on a straight line.


y
x
y = 2
y = 3
y = -3
y = 2x + 2
y = -2x
y = 2x
y = -2x - 2
y = 2x - 2
y = -2x + 2
x = -2½
x = 2½



$3y = 3x + 1$

$\therefore y = \frac{3}{3}x + \frac{1}{3}$

$3y = 3x + 1$

$y = x + \frac{1}{3}$

$2y = 3x - 6$

$y = \frac{3}{2}x - 3$

$2y + 3x + 6 = 0$

$2y = -3x - 6$

$y = -\frac{3}{2}x - 3$

$3y = 2x$

$y = \frac{2}{3}x$

$2x + 3y = 0$

$3y = -2x$

$y = -\frac{2}{3}x$

$3x + 2y = 0$

$2y = -3x$

$y = -\frac{3}{2}x$

The following are observed values. Two quantities, x and y, are connected by the law $y = mx + c$. Show that this is an approximately linear law connecting them and find m and c.

x	15.0	22.6	30.0	37.0	45.0	50.0	60.0
y	3.76	5.60	8.60	11.2	14.0	16.2	20.0

$y = mx + c$

If $x = 20$, then $y = 4.7$

If $x = 60$ then $y = 20$

$\therefore 4.7 = m20 + c$

(1) $20 = m60 + c$

$\therefore 15.3 = 40m$

$\therefore m = \frac{15.3}{40}$

$m = 0.3825$

$\therefore 20 = (60 \times 0.3825) + c$

$\therefore 20 = 22.95 + c$

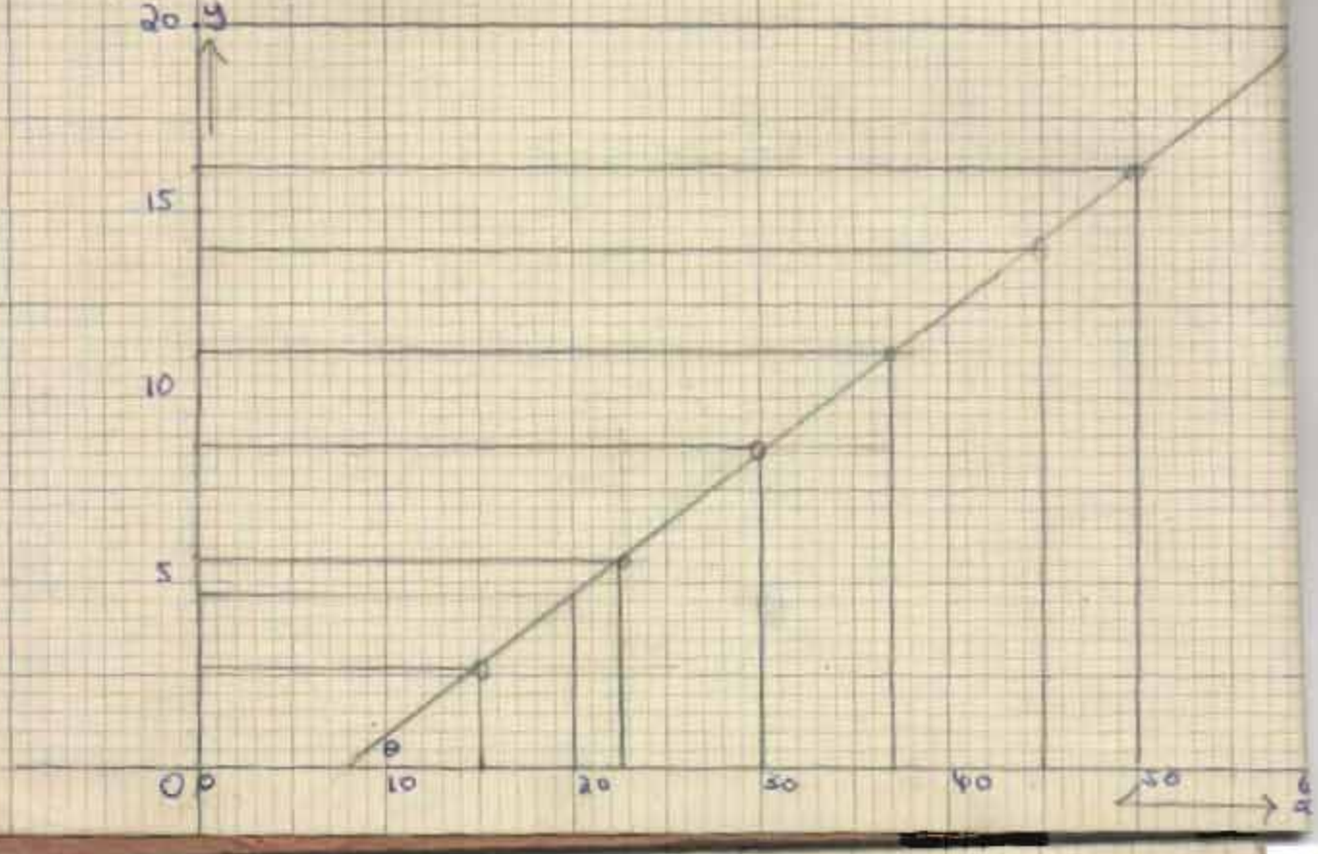
$\therefore -c = 2.95$

$c = -2.95$

ans, $m = 0.3825$

$c = -2.95$

$\therefore y = 0.3825x - 2.95$.



The straight line whose equation is $y = mx + c$ passes through the points $(2, 1)$ $(-3, -2)$ find the value of m and deduce the angle the line makes with the positive direction of the axis of x

$y = mx + c$

The point $(2,1)$ lies on the line

$\therefore 1 = 2m + c$ ——— ①

The point $(-3, -2)$ lies on the line

$\therefore -2 = -3m + c$ ——— ②

② − ① $3 = 5m$

$\therefore m = \frac{3}{5}$

Let θ be the angle the line makes with the positive direction of the x axis.

$\therefore m = \tan\theta$

$\therefore \tan\theta = 0.6$

$\therefore \theta = 30° 58'$

The line makes an angle of $30° 58'$ with the positive direction of the x axis

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A straight line with gradient 3 passes through the point $(1, -1)$. Find the equation of the straight line and find where it cuts i) the y axis ii) the x axis

Let the line be $y = mx + c$

$m = 3$ (given)

$\therefore$ the line is $y = 3x + c$

The point $(1, -1)$ lies on the line

$\therefore -1 = 3.1 + c$

$\therefore$ the line is $y = 3x - 4$

If $x = 0$, $y = -4$

$\therefore$ the line cuts the y axis at the point $(0, -4)$

when $y = 0$

$\therefore 0 = 3x - 4$

$3x = 4$

$x = \frac{4}{3}$

$\therefore$ the line cuts the x axis at the point $(1\frac{1}{3}, 0)$

A steam engine works at a load of P horse power uses W lbs of coal per hour. The following measurements were made.

P	100	80	60	50	35	20
W	2240	1840	1440	1240	940	640

Show that there is a law of the form $W = a + bP$ and find the best values of a and b.

$W = a + bP$

If $P = 100$, $W = 2240$

If $P = 20$, then $W = 640$

$\therefore 1240 = a + 100b$

$640 = a + 20b$

(1) − (2) $600 = 80b$

$\therefore b = 20$

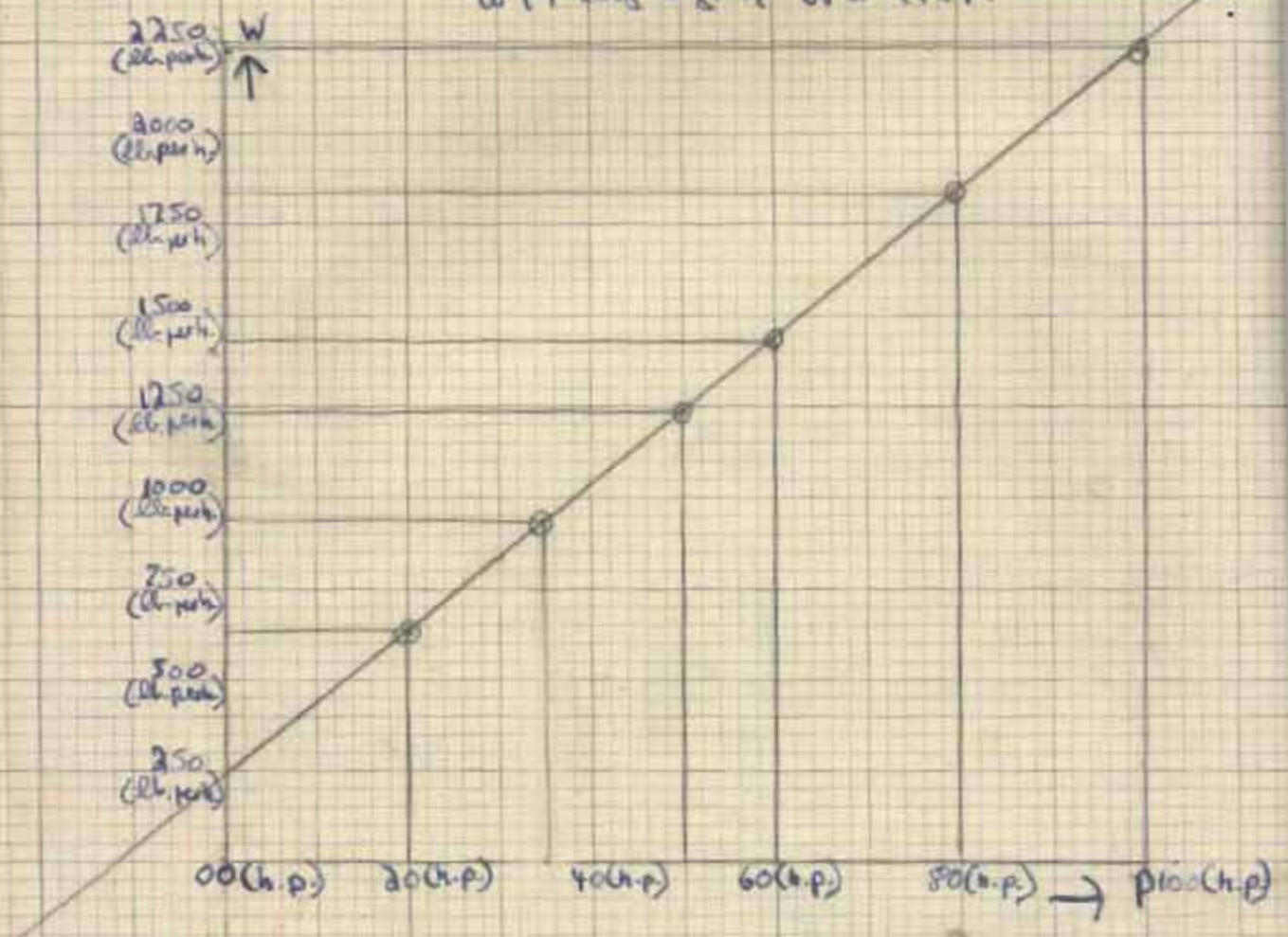
$\therefore 2240 = a + 2000$

$\therefore -a = -240$

$\therefore a = 240$

and $W = 240 + 20P$.

Because the graph is a straight line connecting W to P it is of the form $W = a + bP$, where a and b are constants.



1 Find the line passing through $(-3, 1)$ $(1, -3)$

2 If $m = -\frac{4}{3}$ and passes through the point $(1, -1)$

---

1 $y = mx + c$

the point $(-3, 1)$ lies on the line

$\therefore 1 = -3m + c$ ——— ①

the point $(1, -3)$ lies on the line

$\therefore -3 = m + c$ ——— ②

① − ② $4 = -4m$

$\therefore m = -1$

In ① If $m = -1$

$\therefore 1 = 3 + c$

$\therefore c = -2$

$\therefore$ the line passing through $(-3, 1)$ $(1, -3)$ is $y = -x + -2$

$\therefore y = -x - 2$

$\therefore y = -x - 2$

---

2) Let the line be $y = mx + c$

$m = -\frac{4}{3}$ (given)

$\therefore$ the line is $y = -\frac{4}{3}x + c$

The point $(1, -1)$ lies on the line

$\therefore -1 = -\frac{4}{3} \cdot 1 + c$

$\therefore -c = -\frac{4}{3} + 1$

$\therefore -c = -\frac{1}{3}$

$\therefore c = \frac{1}{3}$

$\therefore$ the line is $y = -\frac{4}{3}x + \frac{1}{3}$ $\therefore 4x + 3y = 3$.

If $x = 0$ then $y = \frac{1}{3}$

$\therefore$ the line meets the y axis at the point $(0, \frac{1}{3})$

When $y = 0$

$0 = -\frac{4}{3}x + \frac{1}{3}$

$\therefore -\frac{4}{3}x = -\frac{1}{3}$

$x = \frac{1}{4}$

$\therefore$ the line cuts the x axis at the point $(\frac{1}{4}, 0)$

1 Find the equation of the line passing through (3,4) (−1,−2)
2 of gradient $\frac{3}{4}$ and passes through the point (−1, −2)

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1. Let the line be $y = mx + c$ where a and b are constants
the point (3,4) lies on the line
$\therefore 4 = 3m + c$ ——— ①
the point (−1, −2) lies on the line
$\therefore -2 = -m + c$ ——— ②
subtract ② from ① $6 = 4m$
$\therefore m = 1\frac{1}{2}$
in ①: if $m = \frac{3}{2}$

$4 = \frac{9}{2} + c$

$\therefore \frac{9}{2} + c = 4$ $\therefore$ the line passing through the points (3,4)(−1,−2)
$c = 4 - \frac{9}{2}$ $= y = \frac{3}{2}x - \frac{1}{2}$ $2y - 3x = -1$
$= -\frac{1}{2}$ $= 2y = 3x - 1$

Let the line be $y = mx + c$ where x and y are constants
$m = \frac{3}{4}$ (given)
$\therefore$ the line is $y = \frac{3}{4}x + c$
the point (−1, −2) lies on the line
$\therefore$ the line is $-2 = -\frac{3}{4} + c$

$c = -1\frac{1}{4}$
$\therefore$ the line is ~~$y = \frac{3}{4} - 1\frac{1}{4} = -8 = 3 - 5$~~
$y = +\frac{3}{4}x - \frac{5}{4} \therefore 4y = 3x - 5$ $\therefore 4y - 3x = -5$
If $x = 0$ then $y = -\frac{5}{4}$
The line meets the y axis at the point $(0, -1\frac{1}{4})$
when $y = 0$
$0 = 3x - 5$
$-3x = -5$
$x = \frac{5}{3}$ $\therefore$ the line meets the x axis at the point $(1\frac{2}{3}, 0)$

Find graphically the solution of the simultaneous equations $y = 2x + 3$, $2y + x = 1$

x	0	1	2
$2x$	0	2	4
$+3$	3	3	3
y	3	5	7

$y = 2x + 3$

x	0	1	2
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$-x$	0	-1	-2
$2y$	1	0	-1
y	$\frac{1}{2}$	0	$-\frac{1}{2}$

$2y + x = 1$

$\therefore x = -1, y = 1$

$2y + x = 1$ —— ①

$y - 2x = 3$ —— ②

$2y - 4x = 6$ —— ③

Subtract $5x = -5$

② from ① $x = -1$

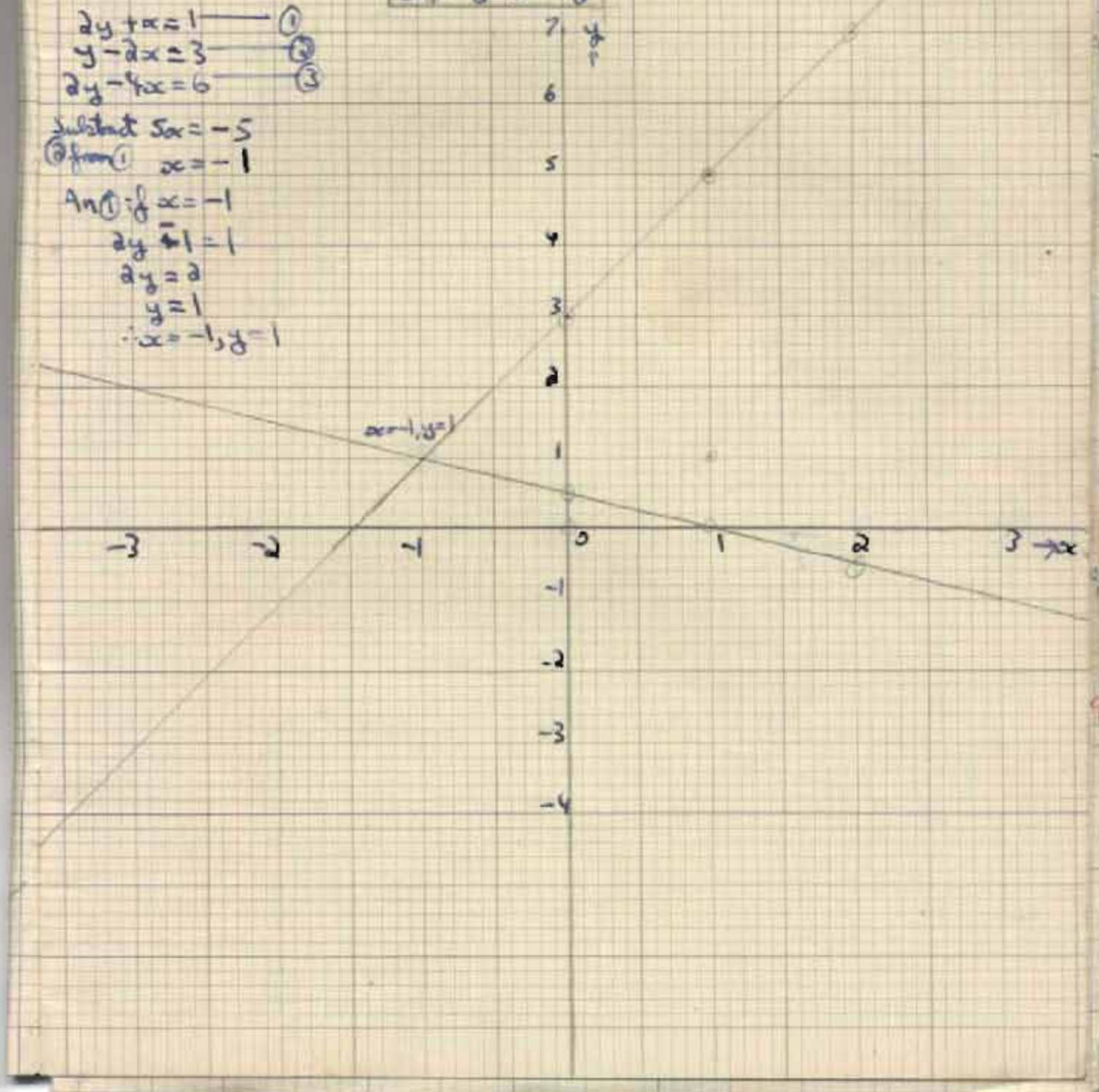
Ans ①: if $x = -1$

$2y - 1 = 1$

$2y = 2$

$y = 1$

$\therefore x = -1, y = 1$



Solve the following simultaneous equations, $3x+4y=2$, $2x-3y=7$
(i) graphically. (ii) by calculation.

x	0	1	2
2	2	2	2
$-3x$	0	-3	-6
$4y$	2	-1	-4
y	$\frac{1}{2}$	$-\frac{1}{4}$	-1

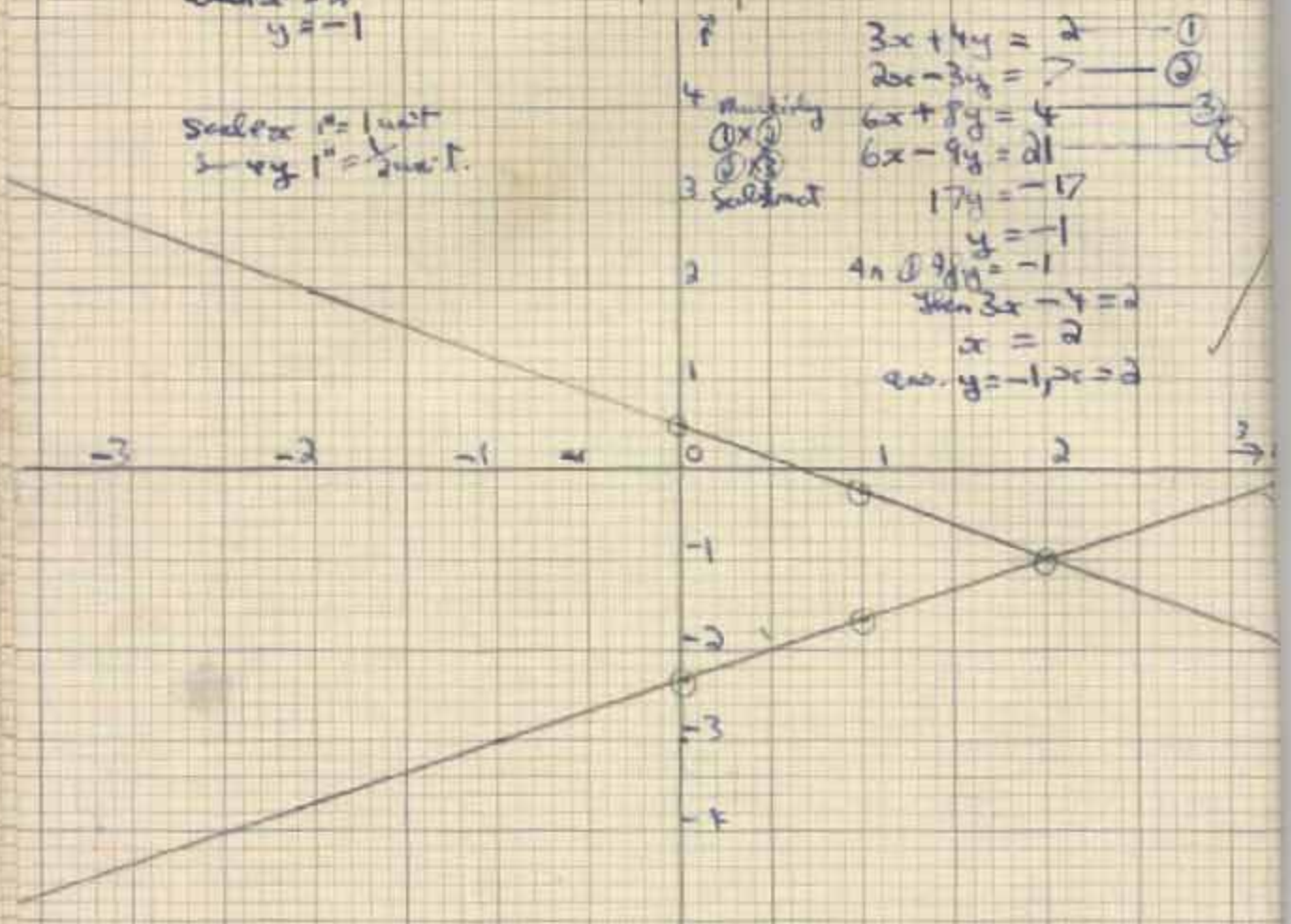
when $x=2$
$y=-1$

scale x: 1" = 1 unit
y: 1" = $\frac{1}{2}$ unit.

$4y = -3x+2$
$-3y = -2x+7$

x	-1	0	1
-7	-7	-7	-7
$+2x$	-2	0	$+2$
$+3y$	-4	-7	-5
y	$=-3$	$-2\frac{1}{3}$	$-1\frac{2}{3}$

$3x+4y=2$ —— ①
$2x-3y=7$ —— ②
multiplying ① × 3: $6x+8y=4$ —— ③
② × 4: $6x-9y=21$ —— ④
subtract ③ − ④: $17y=-17$
$y=-1$
sub $y=-1$ in ①: $3x-4=2$
$3x=6$
$x=2$
ans. $y=-1, x=2$



Draw the lines $y = 3x + 4$, $2y + x = 8$, $x + 2y = 2$

x	0	1	2
$3x$	0	3	
$+4$	4	4	
y	4	7	10

Draw the lines $y = 3x + 4$, $2y + x = 8$, $x + 2y = 2$

x	0	1	2
$3x$	0	3	6
$+4$	4	4	4
y	4	7	10

$y = 3x + 4$

x	0	1	2
8	8	8	8
$-x$	0	-1	-2
$2y$	8	7	6
y	4	$3\frac{1}{2}$	3

$2y = 8 - x$

x	0	1	2
2	2	2	2
$-x$	0	-1	-2
$2y$	2	1	0
y	1	$\frac{1}{2}$	0

$2y = 2 - x$

Write down the solution of the simultaneous equations $y = 3x + 4$ $y = 3x + 4$
$2y + x = 8$ $x + 2y = 2$

The graphical solution to the first simultaneous equation is $x = 0$, $y = 4$.

The graphical solution to the second simultaneous equation is $x = -\frac{6}{7}$ $y = 1\frac{3}{7}$

1. $y - 3x = 4$ —— ①
$2y + x = 8$ —— ②
Multiply ② by 2 ③
① − ③ $7x = 0$
$x = 0$
Sub $x = 0$ in ① $y - 0 = 4$
$y = 4$
$\therefore x = 0, y = 4$

2. $y - 3x = 4$ —— ①
$2y + x = 2$ —— ②
Multiply ① by 2 ③
$2y - 6x = 8$ —— ③
② − ③ $7x = -6$
$x = -\frac{6}{7}$
Sub $x = -\frac{6}{7}$ in ② $2y - \frac{6}{7} = 2$
$2y = 2 + \frac{6}{7}$
$2y = 2\frac{6}{7}$
$y = 1\frac{3}{7}$
$\therefore x = -\frac{6}{7}, y = 1\frac{3}{7}$

Myron Wan Tsaw, Form WL, Graphs (algebra) Part I 16-3-65

Draw the graph of $y = x^2 + 4x$ for values of x between -5 and 1. 1" is the unit on both axes. Use your graph to write down the range of values of x for which $x^2 + 4x$ is less than -3.

(ii) By drawing a suitable straight line graph, solve the equation $x^2 + 3x - 3 = 0$

x	-5	-4	-3	-2	-1	0	1	-4.5	-3.5	-0.5
x^2	25	16	9	4	1	0	1	20.25	12.25	0.25
$4x$	-20	-16	-12	-8	-4	0	4	-18	-14	-2.0
y	5	0	-3	-4	-3	0	5	2.25	-1.25	-1.75

(i) The range of values of x for which $x^2 + 4x$ is less than -3 are

x	-1	-2	-3
x	-1	-2	-3
3	3	3	3
y	2	1	0

?? $x = -3.41$ to $x = -4$ and $x = -4$ to $x = -0.59$

-3.41 & -0.59

(ii) $x^2 + 3x = 3$

$x^2 + 3x - 3 = 0$

$x^2 + 4x = x + 3$

$\therefore y = x + 3$

From graph $y = x + 3$, $\therefore x = 0.8$ and $x = -3.79$.

✓

✓

Good

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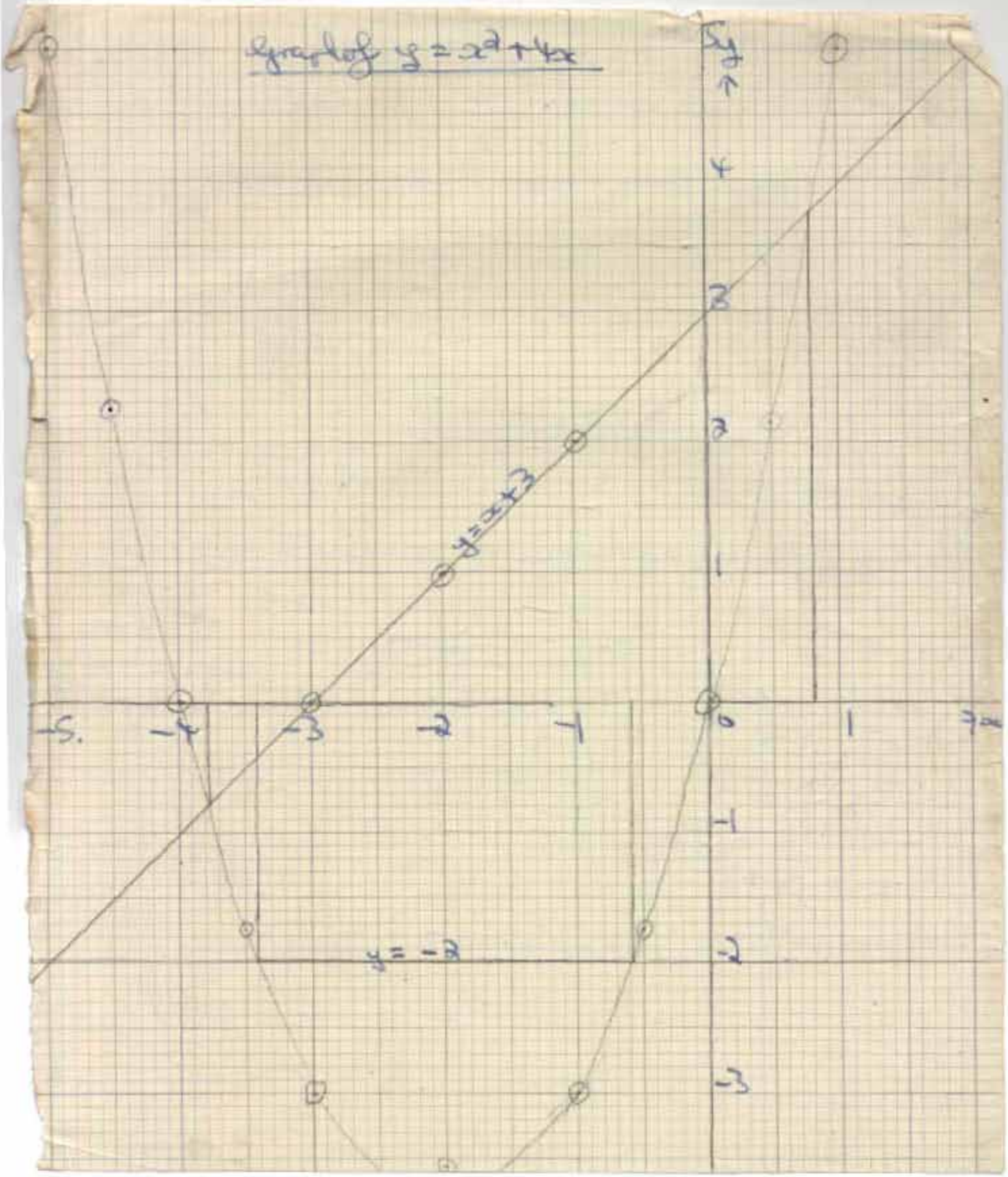
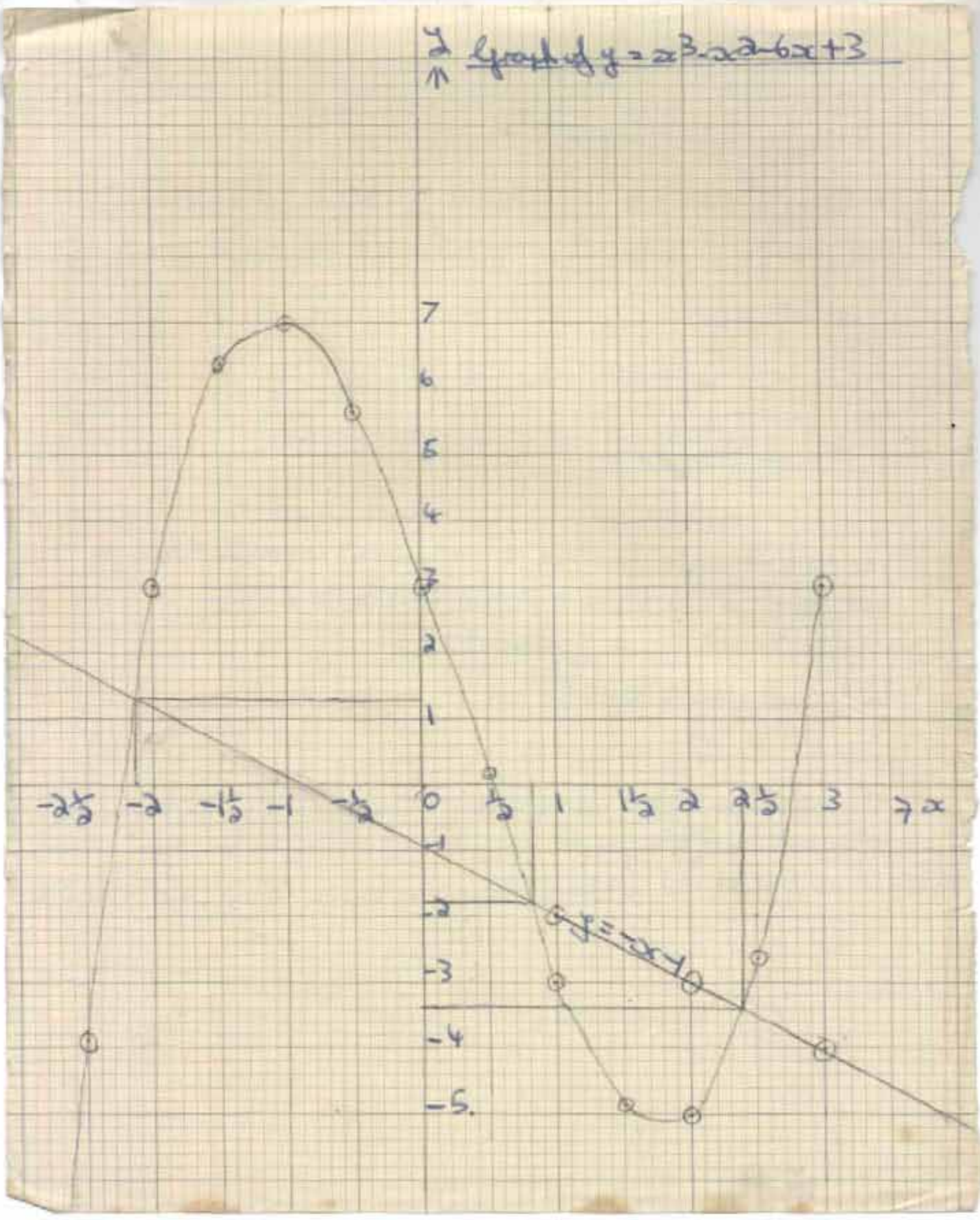

graph of $y = x^2 + 4x$
y
x
y = x + 3
y = -3
-5
-4
-3
-2
-1
0
1
4
3
2
1
-1
-2
-3


Graph of $y = x^3 - x^2 - 6x + 3$
y
x
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
$-2\frac{1}{2}$
-2
$-1\frac{1}{2}$
-1
$-\frac{1}{2}$
$\frac{1}{2}$
1
$1\frac{1}{2}$
2
$2\frac{1}{2}$
3
$y = -2x - 1$

Myron Wyn Wood, Form IVL algebra graphs 9-3-65

Copy and complete the following table, giving values of x^3-x^2-6x+3 for values of x from -2.5 to 3

x	$-2\frac{1}{2}$	$-1\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	-2
x^3-x^2-6x+3	$-3\frac{3}{8}$	$6\frac{3}{8}$	7	$5\frac{5}{8}$	3	$-\frac{1}{8}$	-3	$-4\frac{3}{8}$	-5	$-2\frac{5}{8}$	3	3

x	1	2	3
$-x$	-1	-2	-3
-1	-1	-1	-1
y	-2	-3	-4

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1. The range of values of x for which x^3-x^2-6x+3 is greater than $-x-1$ are $x=-2.12$ to $x=0.82$ and $x>2.4$

2. When x^3-x^2-6x+3 intersects $-x-1$

$$x^3-x^2-6x+3=-x-1$$

$$\therefore x^3-x^2-5x+4=0$$

$\therefore$ the equation which is satisfied by the values of x at the points of intersection of the graph is $x^3-x^2-5x+4=0$

The values of x at the points of intersection are $x=-2.12, 0.82, 2.4$

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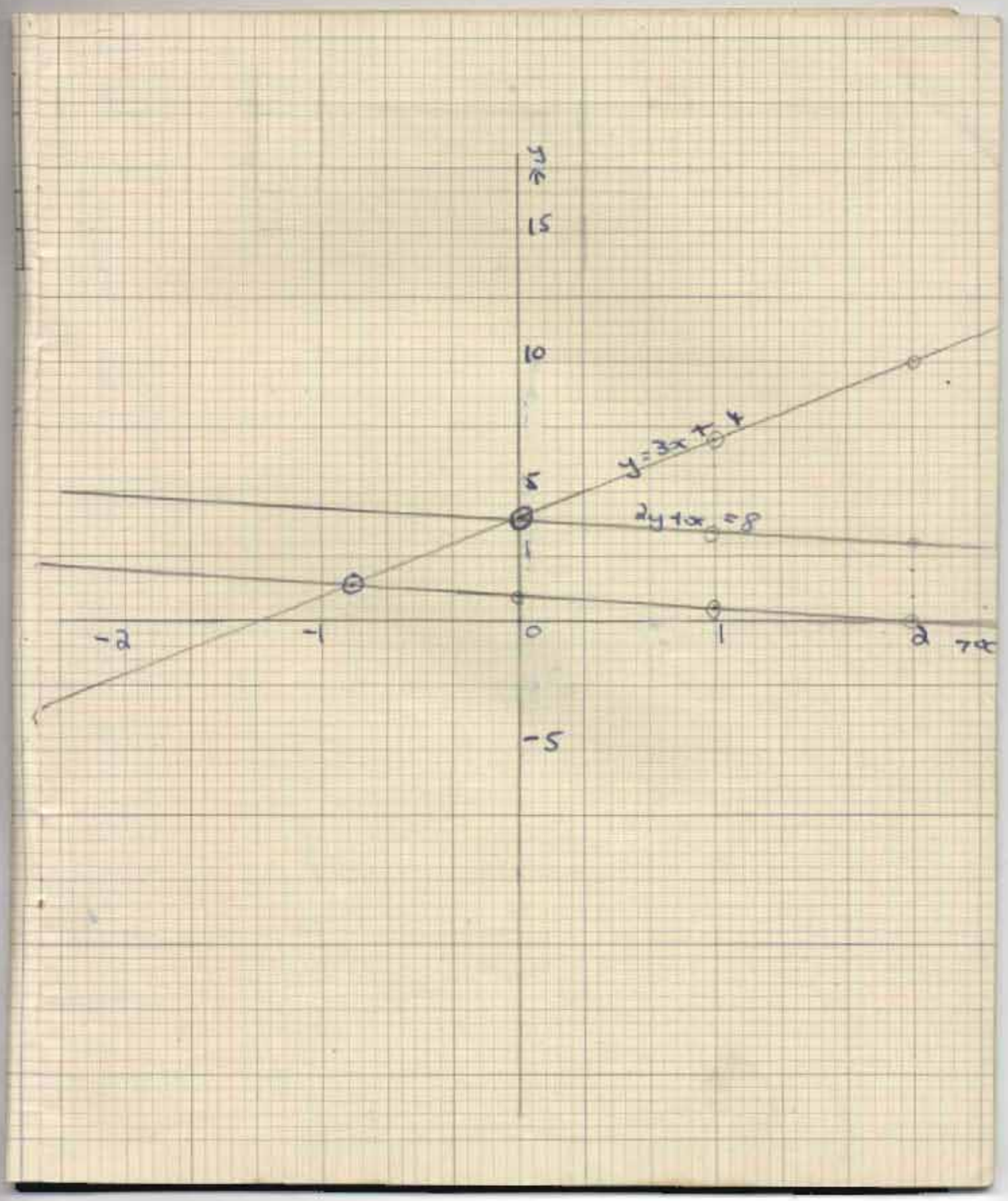
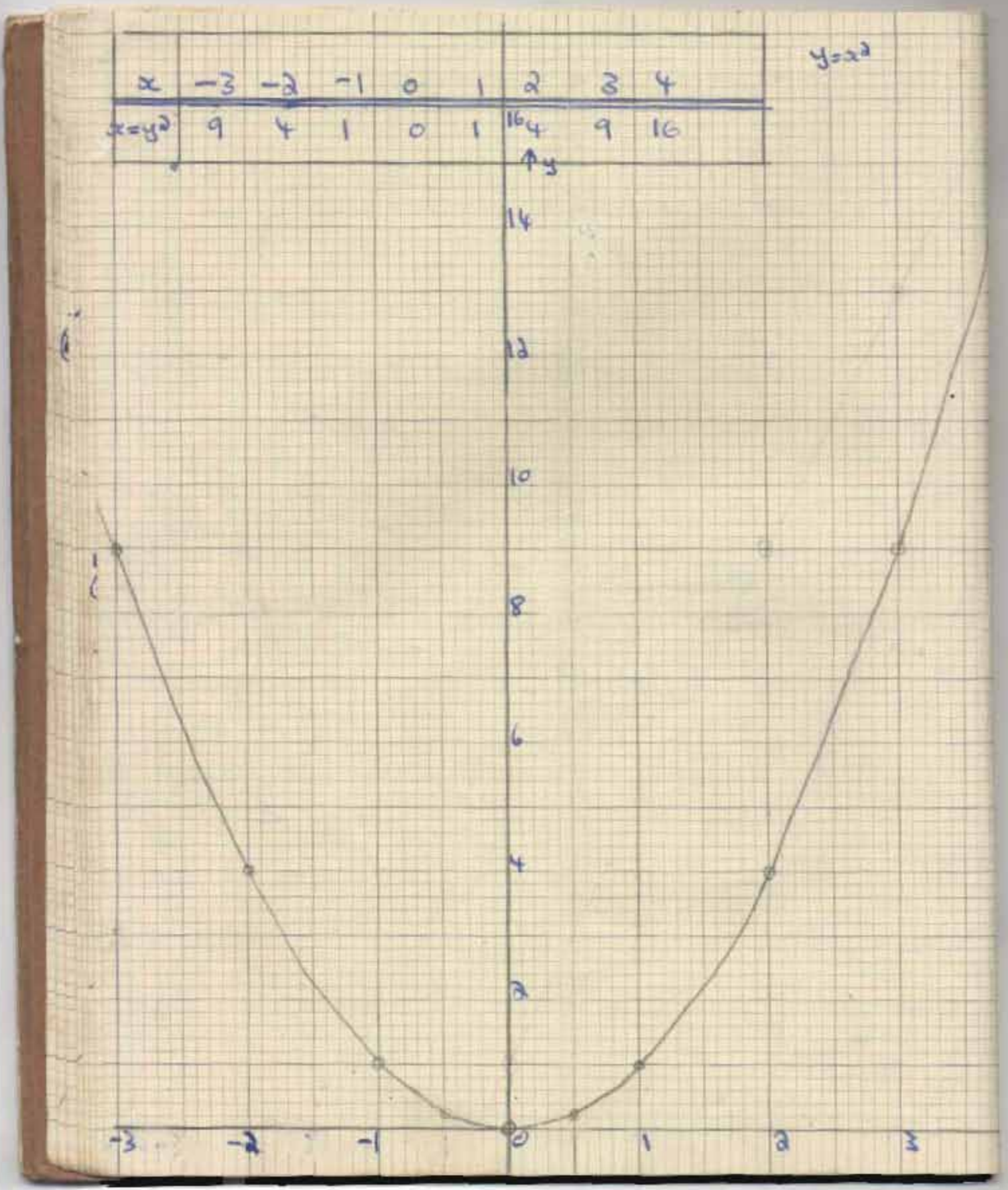

y
15
10
5
1
-5
-2
-1
0
1
2
x
y = 3x + 4
2y + x = 8

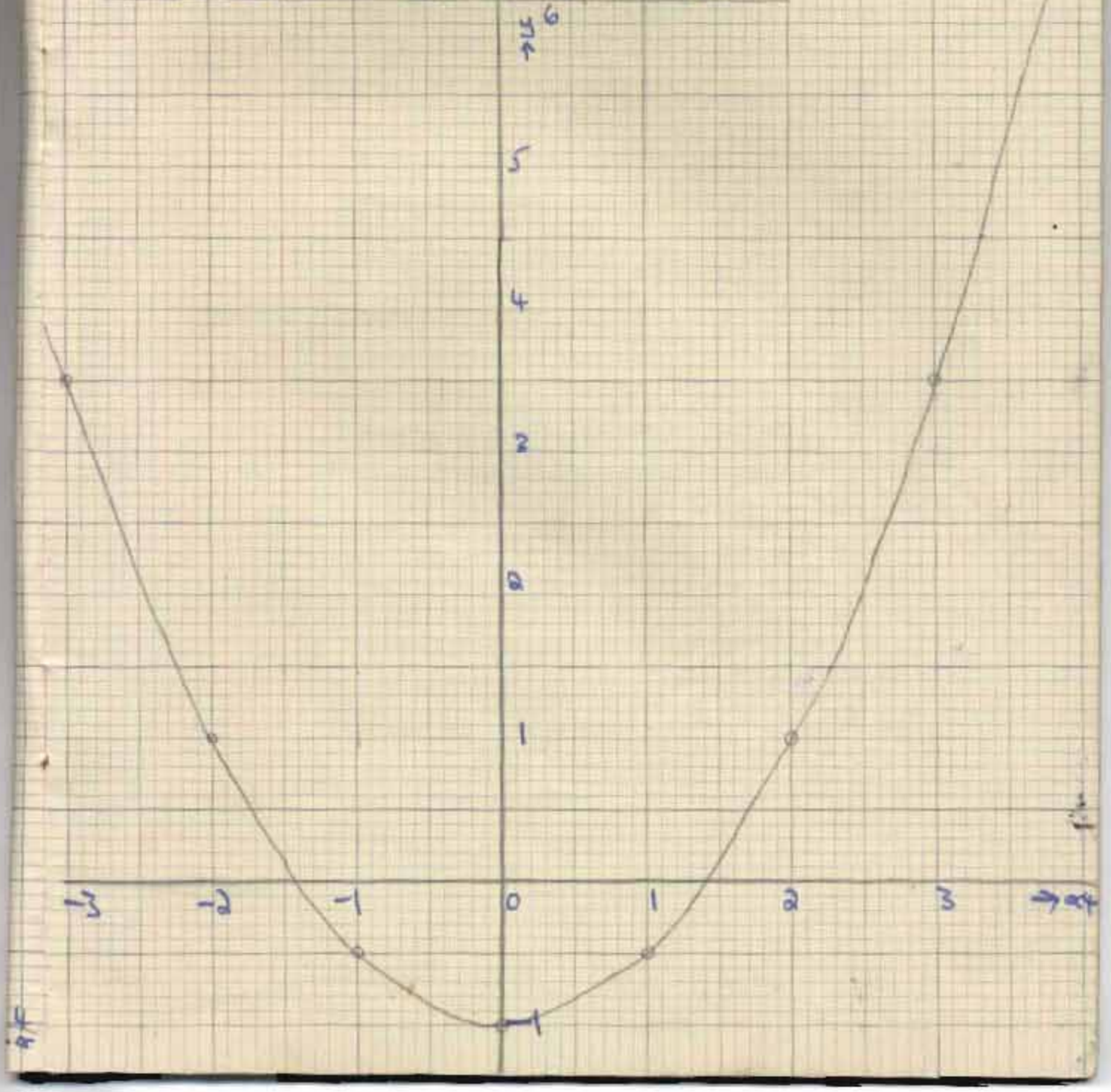
$y=x^2$

x	-3	-2	-1	0	1	2	3	4
$x=y^2$	9	4	1	0	1	4	9	16



$y = \frac{1}{2}x^2 - 1$

x	-3	-2	-1	0	1	2	3	4
x^2	9	4	1	0	1	4	9	16
$\frac{1}{2}x^2$	$4\frac{1}{2}$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2	$4\frac{1}{2}$	8
-1	-1	-1	-1	-1	-1	-1	-1	-1
y	$3\frac{1}{2}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	1	$3\frac{1}{2}$	7



$y=x^2$
$y=-x^2$
$y=2x^2$

$y=\frac{1}{2}x^2$
$y=-\frac{1}{2}x^2$

-4 to $+4$
Homework.

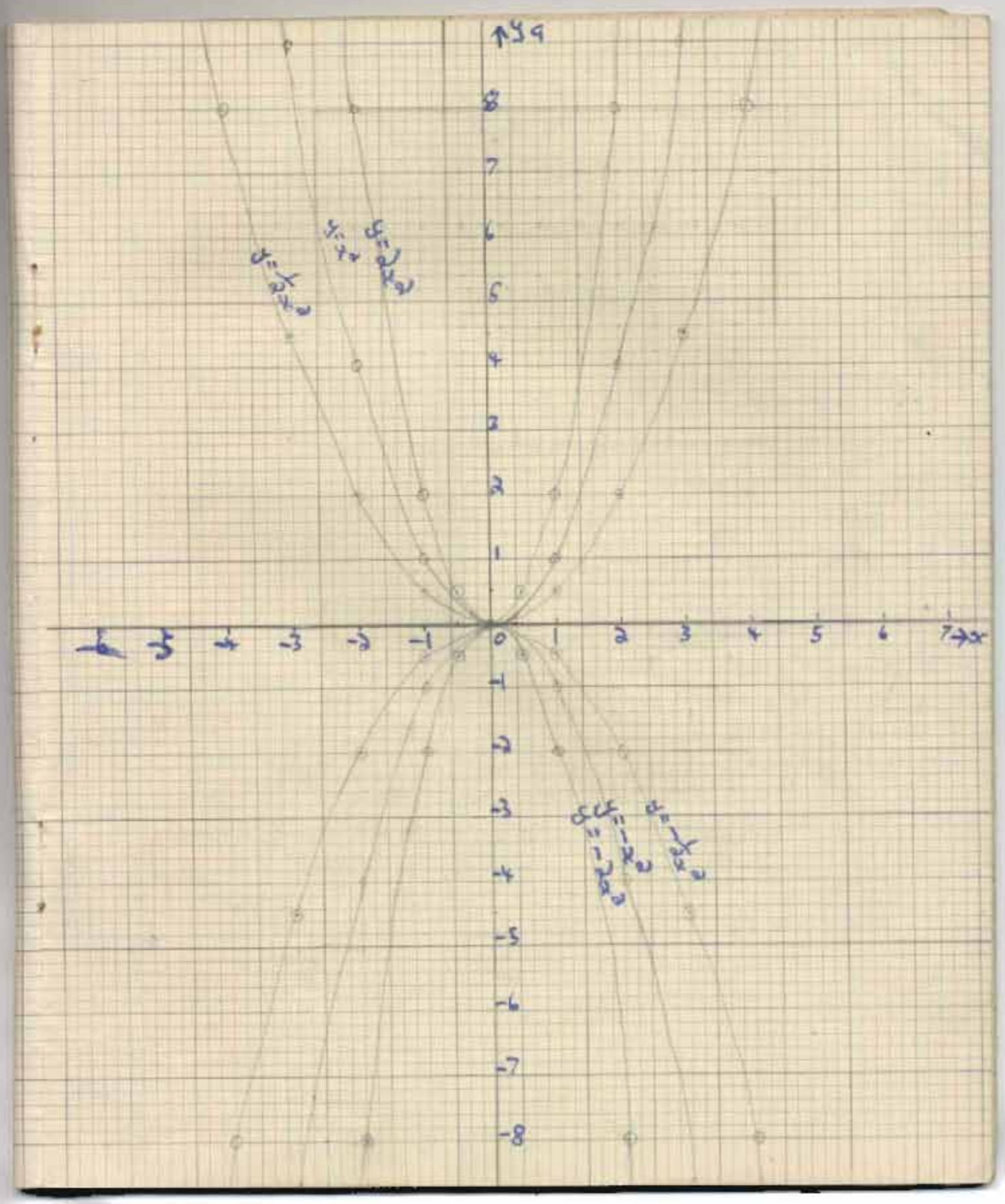
x	-3	-2	-1	0	1	2	3	4
$y=x^2$	9	4	1	0	1	4	9	16

x	-3	-2	-1	0	1	2	3	4
x^2	9	4	1	0	1	4	9	16
y	-9	-4	-1	0	-1	-4	-9	

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$\frac{1}{2}x^2$	8	$4\frac{1}{2}$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2	$4\frac{1}{2}$	8
y	8	$4\frac{1}{2}$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2	$4\frac{1}{2}$	8

x	-4	-3	-2	-1	0	1	2	3	4
$\frac{1}{2}x^2$	8	$4\frac{1}{2}$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2	$4\frac{1}{2}$	8
$-\frac{1}{2}x^2$	-8	$-4\frac{1}{2}$	-2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-2	$-4\frac{1}{2}$	-8
y	-8	$-4\frac{1}{2}$	-2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-2	$-4\frac{1}{2}$	-8

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$2x^2$		8	2	0	2	8	
$y=-2x^2$		-8	-2	0	-2	-8	

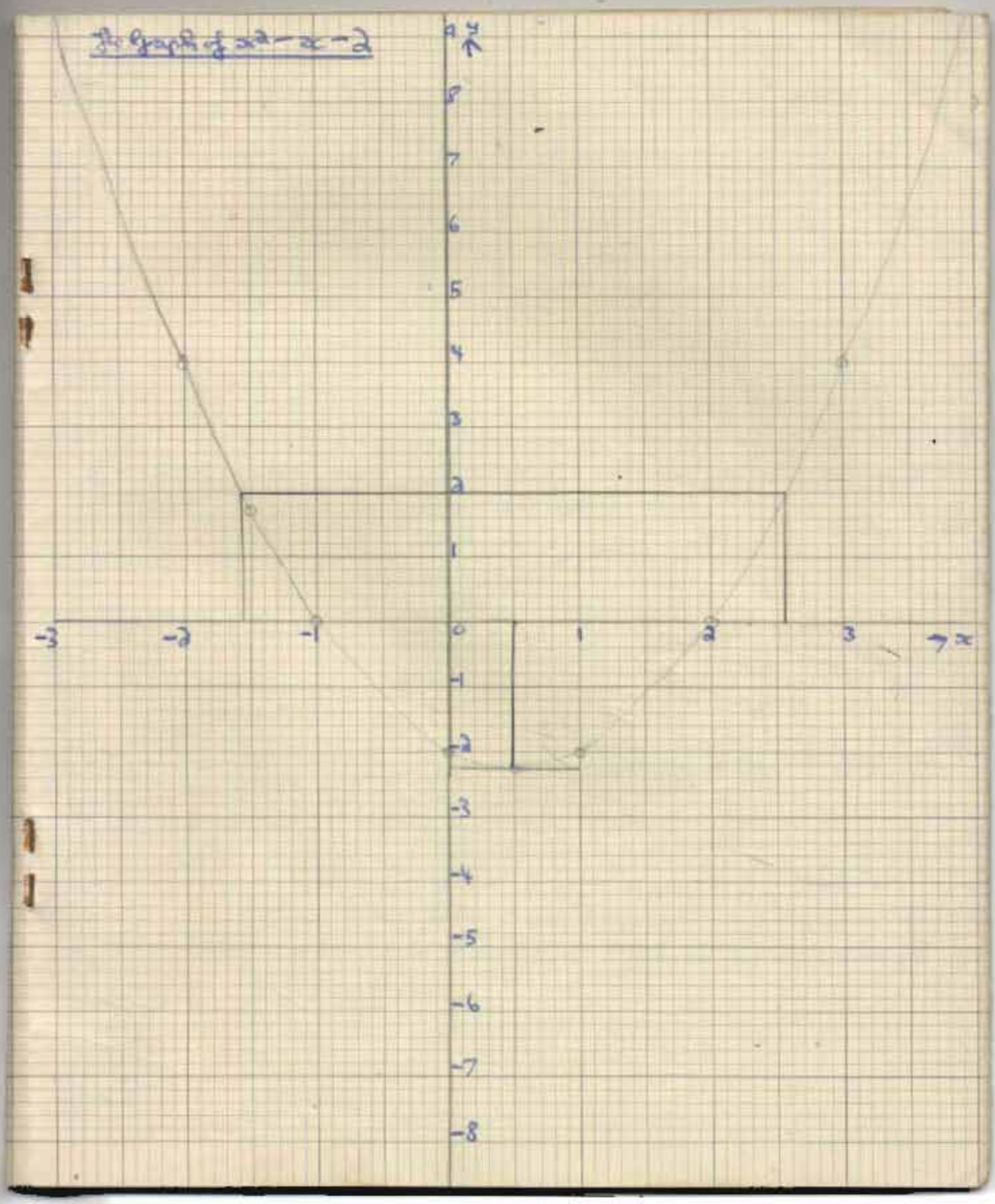

y
x
$y = x^2$
$y = 2x^2$
$y = -x^2$
$y = -2x^2$

Draw the graph of $y = x^2 - x - 2$ from $x = -3$, to $x = 3$

x	-3	-2	-1	0	1	2	3	$\frac{1}{2}$
x^2	9	4	1	0	1	4	9	$\frac{1}{4}$
$-x$	3	2	1	0	-1	-2	-3	$-\frac{1}{2}$
-2	-2	-2	-2	-2	-2	-2	-2	-2
y	10	4	0	-2	-2	0	4	$-2\frac{1}{4}$

1. what is the least value of y?
   (ie what is the minimum value of y?)
2. what value of x gives the minimum value?
3. For what value of x is $y = 0$?
4. For what values of x is $y = 2$?
5. For what range of values of x is y positive?

1. The least value of y is $-2\frac{1}{4}$
2. The value of x which gives the minimum value is $\frac{1}{2}$ ✓
3. $y = 0$ when $x = -1$ and $x = 2$
4. $y = 2$ when $x = -1.56$ and 2.56 ✓
5. y positive from the range of x -1 to -3 and from 2 to 4 ✓
   $x < -1, x > 2$


y
x
-3
-2
-1
0
1
2
3
-8

P.219. No 1.

Draw the graph of $y = x^3 - x - 5$ from $x = -3$ to $x = 4$.

x	-3	-2	-1	0	1	2	3	4	$\frac{1}{4}$	$-2\frac{1}{2}$
x^3	9	4	1	0	1	4	9	16	$\frac{1}{4}$	6.25
$-x$	3	2	1	0	-1	-2	-3	-4	$-\frac{1}{4}$	2.5
-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5.00
y	7	1	-3	-5	-5	-3	+1	7	$-5\frac{1}{4}$	3.75

i. The least value of y is $-5\frac{1}{4}$.

(ii) The value of x that gives this is $\frac{1}{2}$

(iii) The values of x for which $y = 0$ are $x = -1.8$ and $x = 2.8$

(iv) The values of x for which $x^3 - x - 5 = 0$ i.e. $y = 0$ are -1.791 and 2.791.

This is a check

Your answer is $x = -1.8$ or $x = 2.8$

by formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{1 \pm \sqrt{1 - 4(1)(-5)}}{2(1)}$

$= \frac{1 \pm \sqrt{1 + 20}}{2}$

$= \frac{1 \pm 4.583}{2}$

$\therefore$ either $x = \frac{5.583}{2}$ i.e. $x = 2.791$ or $x = \frac{-3.583}{2}$ i.e. $x = -1.7$

(v) The range of values of x for which y is increasing are $x \geq \frac{1}{2}$ [and $x < \frac{1}{2}$ it decreases here.]

(vi) The value of y for which $x = 1.7$ is -3.8 ✓

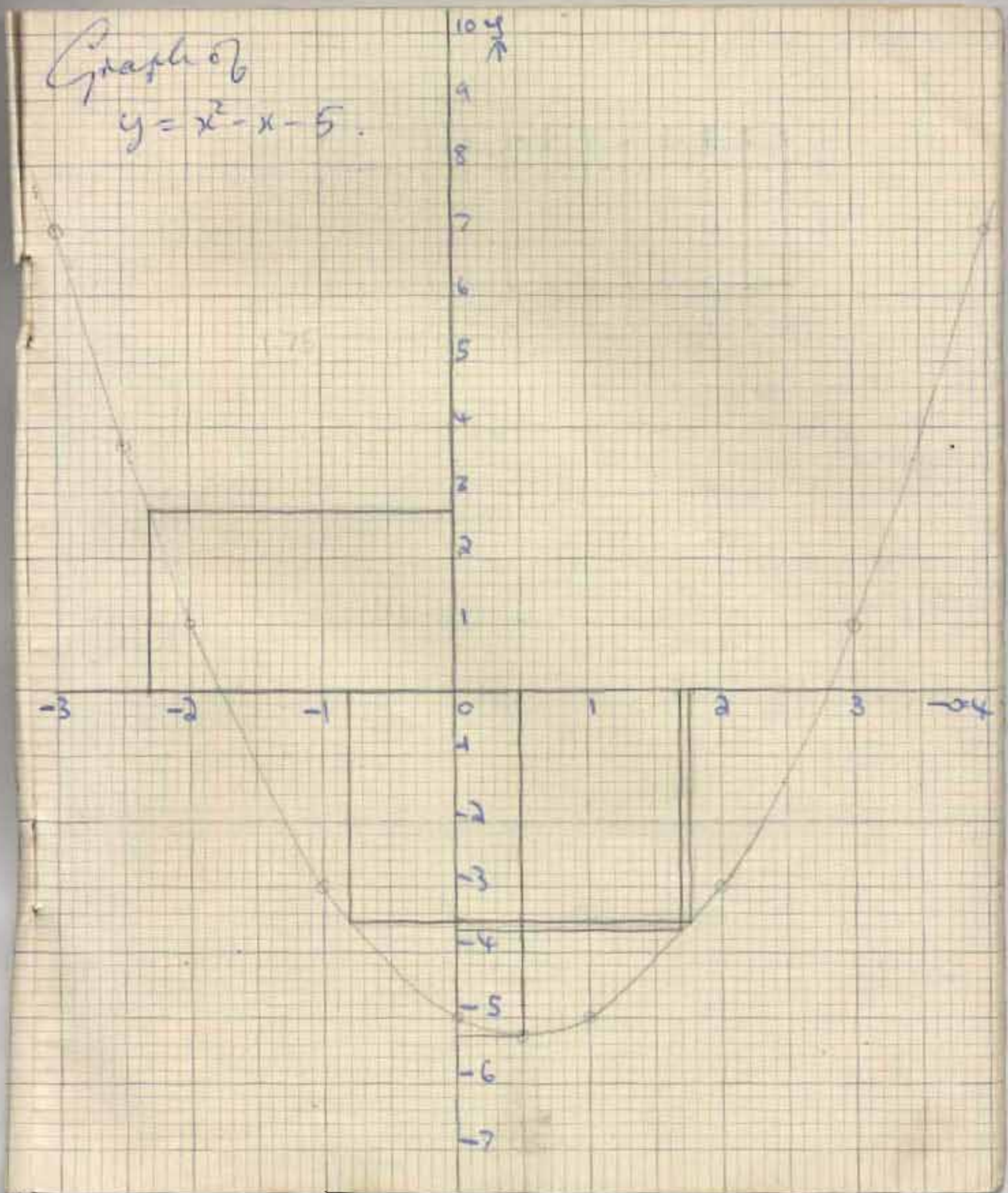
The value of y for which $x = -2.3$ or 2.6 ✓

(vii) The values of x for which $y = 1$ are $x = -2, x = 3$ ✓

The values of x for which $y = -3.5$ are $x = -2.45$ and $x = 3.45$.

✓

$9\frac{1}{2}$/10

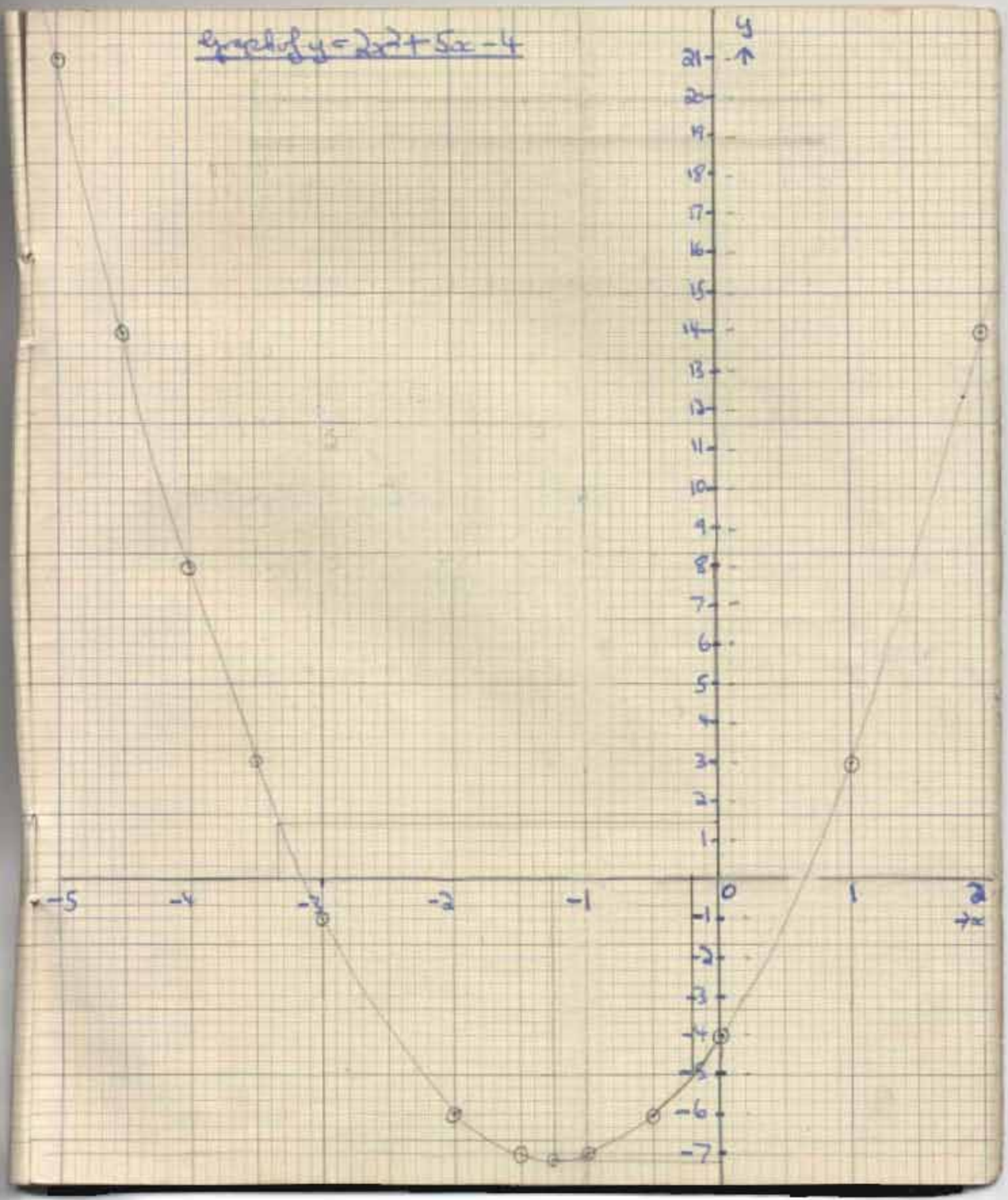

Графік
$y = x^2 - x - 5$.
y
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-3
-2
-1
1
2
3
4

Draw the graph of $y = 2x^2 + 5x - 4$.

x	-5	-4	-3	-2	-1	0	1	2	-4.5	-0.5	-3.5	-1.5	-1.25
x^2	25	16	9	4	1	0	1	4	20.25	0.25	12.25	2.25	1.562
$2x^2$	50	32	18	8	2	0	2	8	40.5	0.5	24.5	4.5	3.126
$+5x$	-25	-20	-15	-10	-5	0	5	10	-22.5	-2.5	-17.5	-7.5	-6.25
-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4
y	21	8	-1	-6	-7	-4	3	14	14	-6	3	-7	-7.124

1. The least value of $y = -7.124$
2. The value of x that gives this least value is $= -1.25$
3. The values of x for which $y = 0$ are $x = 0.65$
4. The roots of the equation $2x^2 + 5x - 4 = 0$ are $x = -3.15$ $x = 0.65$ or $x = -3.15$
5. The range of values of x for which y is decreasing are $x < -1.25$
6. The values of y for which $x = 2.4$ and $x = -0.2$ are $y = 19.52$, $y = -5.0$ respectively
7. The values of x for which $y = 1.5$ and $y = -4$ are $x = -3.35$ and $x = 0$ respectively

Graph of $y = 2x^2 + 5x - 4$



Draw the graph of $y = -x^2 + 3x + 3$

x	-2	-1	0	1	2	3	4	5	1·5	-1·5	4·5	-0·5		
x^2	4	1	0	1	4	9	16	25	2·25	2·25	20·25	0·25		
$-x^2$	-4	-1	0	-1	-4	-9	-16	-25	-2·25	-2·25	-20·25			
$+3x$	-6	-3	0	3	6	9	12	15	4·5	-4·5	13·5	-0·25		
									3	3	2	-1·5		
$+3$	3	3	3	3	3	3	3	3				3		
y	-7	-1	3	5	5	3	-1	-7	5·25	-3·75	3	-24·75		

1) The greatest value of $y = 5·25$ ✓

2) The value of x which gives the greatest value $= 1·5$ ✓

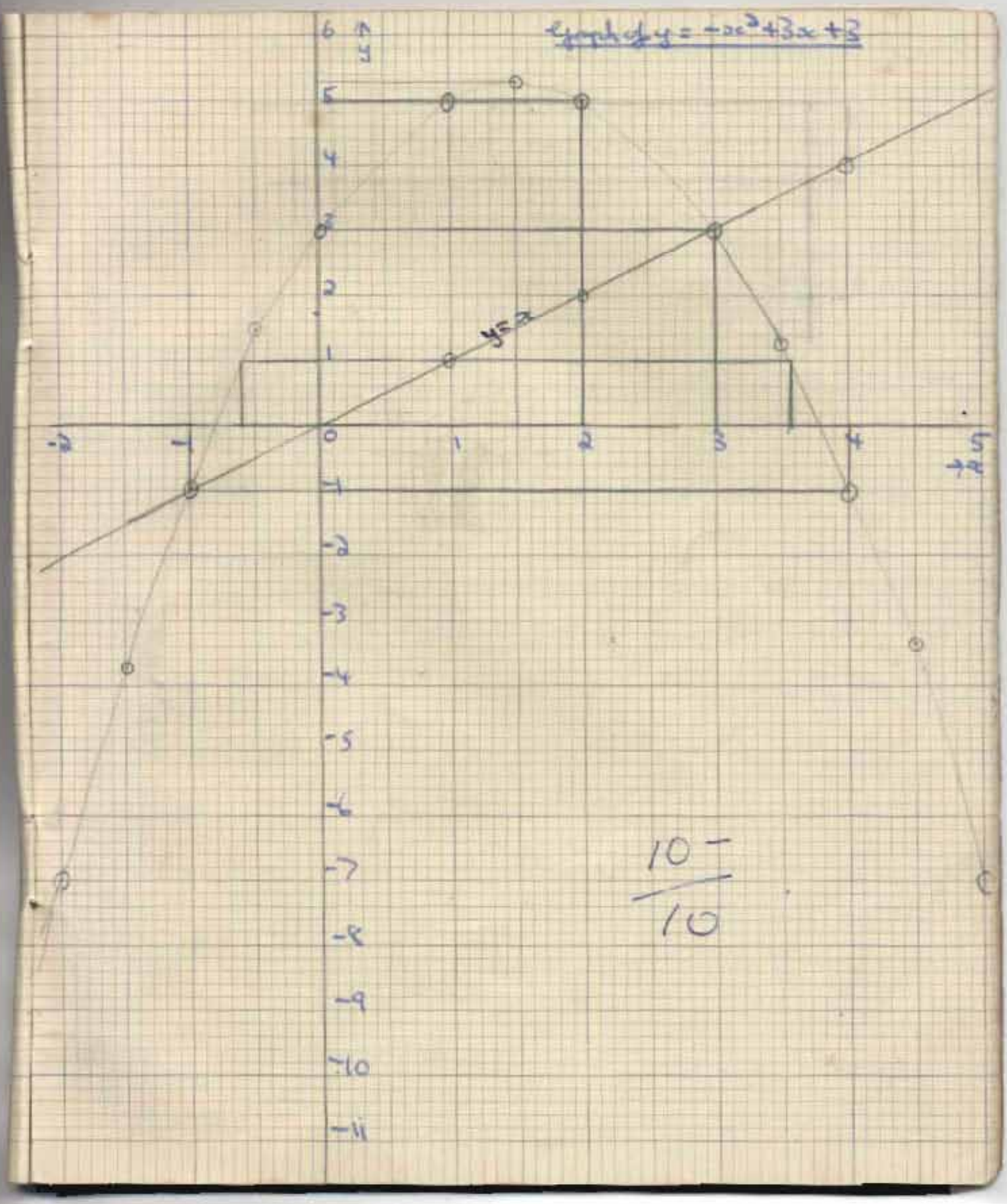
3) The roots of the equation $3 - x^2 + 3x = 0$ are $x = -0·8$ or $x = 3·79$ ✓

4) The range of values of x for which $1 < y$ are from $x = -0·6$ to $x = 3·59$ ✓
The range of values of x for which $y < 3$ are from $x = 0$ to $x = 3$ ✓

5) The range of values of y for which $x > 2$ is $y < 5$ ✓
The range of values of y for which $x < 4$ is $y > -1$ ✓

6) The values of x that make $x = y$ are $x = -1$ and $x = 3$ ✓

// Ans:
-0·6 <
0 3 <



10 / 10

Draw the graph of $y = (4-x)(2+x)$ from $x = -3$ to $x = 5$.

x	-3	-2	-1	0	1	2	3	4	5	-1.5				
$(4-x)$	7	6	5	4	3	2	1	0	-1	5.5				
$(2+x)$	-1	0	1	2	3	4	5	6	7	.5				
y	-7	0	5	8	9	8	5	0	-7	2.75				

(i) the value of x that gives the greatest value of y is $x = 1$ ✓

(ii) the roots of the equation $x^2 - 2x - 8 = 0$

~~$= (x-4)(x+2)$~~

$= (4-x)(2+x)$ are $x = -2$ or $x = 4$ ✓

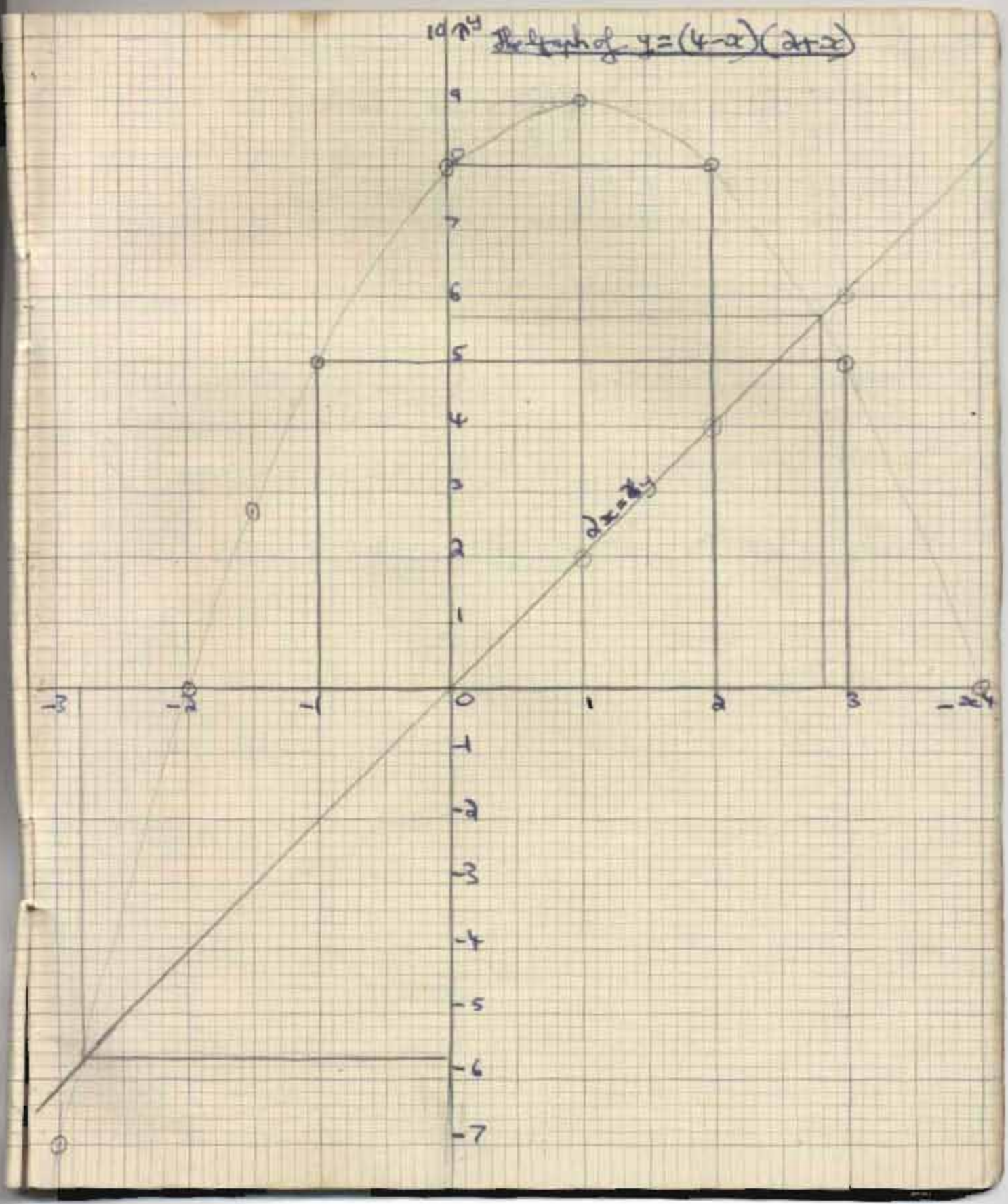
(iii) the range of values of x for which $5 \leq y \leq 8$ are $x = -1$ to $x = 0$ ✓

and $x = 2$, to $x = 3$ ✓

(iv) the values of x that make $2x = y$ are $x = -2.81$ and $x = 0.81$ ✓

$y = 2x$ and $y = 8 + 2x - x^2$ meet when $-2x = 8 + 2x - x^2$ i.e. $0 = 8 - x^2$

From graph solution of the equation is $x = \pm 2.83$


The graph of $y=(4-x)(2+x)$
$2x=3y$
y
x
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-3
-2
-1
0
1
2
3
4

Draw the graph of $y=(2-x)(5+x)$ from $x=-6$ to $x=3$

x	-6	-5	-4	-3	-2	-1	0	1	2	3	-1.5
$(2-x)$	8	7	6	5	4	3	2	1	0	-1	3.5
$(5+x)$	-1	0	1	2	3	4	5	6	7	8	3.5
y	-8	0	6	10	12	12	10	6	0	-8	12.25

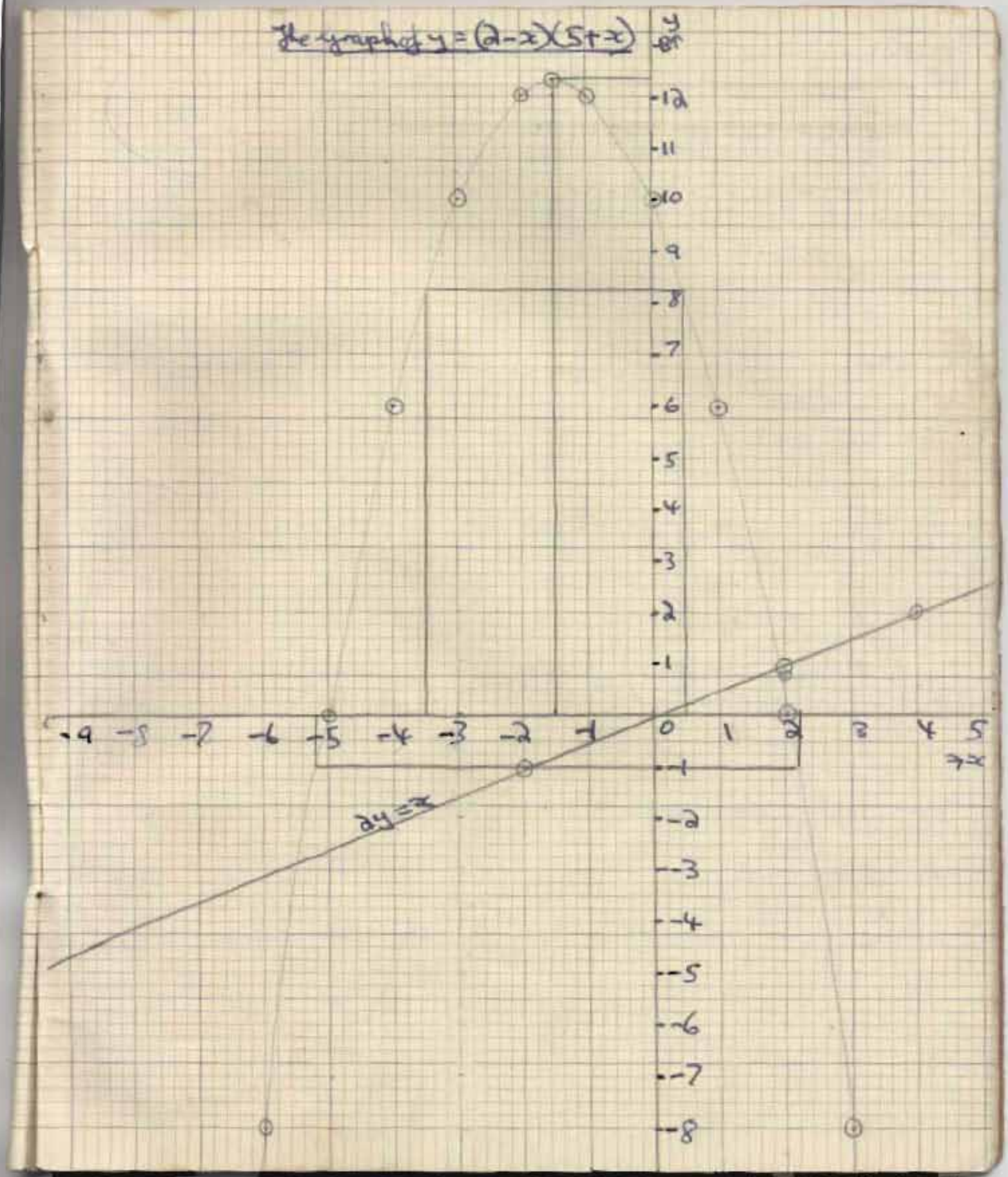
(i) The value of x that gives the greatest value of $10-3x-x^2$ is $x=-1.5$
$=(2-x)(5+x)$

(ii) The range of values of x for which $-1 \leq y \leq 8$ are $x=-5.2$ to $x=-3.5$ and $x=0.5$ to $x=2.2$

(iii) The roots of the equation $x^2+3x-10=0$ are $x=-5$, and $x=2$
$=(2-x)(5+x)=0$

(iv) The value of x that makes $x=2y$ is $x=1.9, x=-5.5$

The graph of $y = (2-x)(5+x)$



P.223 3191 No.1

Draw the graph of $y = x^2 - 3x - 5$.

x	-2	-1	0	1	2	3	4	5	1·5
x^2	4	1	0	1	4	9	16	25	2·25
$-3x$	6	3	0	-3	-6	-9	-12	-15	-4·5
-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
y	+5	-1	-5	-7	-7	-5	-1	05	7·25

(i) When $x^2 - 3x - 5 = 0$, $y = 0$ ∴ answer is $y^x = -1·15, \overset{x}{y} = 4·15$

(ii) $x^2 - 3x - 6 = 0$

$x^2 - 3x - 5 = 1$

∴ $y = 1$

From graph $y = 1, x = -1·34, 4·34$

(iii) $x^2 - 3x - 4 = 0$

$x^2 - 3x - 5 = -1$

∴ $y = -1$

∴ From graph $y = -1, x = -1, x = 4$

(iv) $x^2 - 3x + 1 = 0$

$\therefore x^2 - 3x - 5 = -6$

∴ $y = -6$

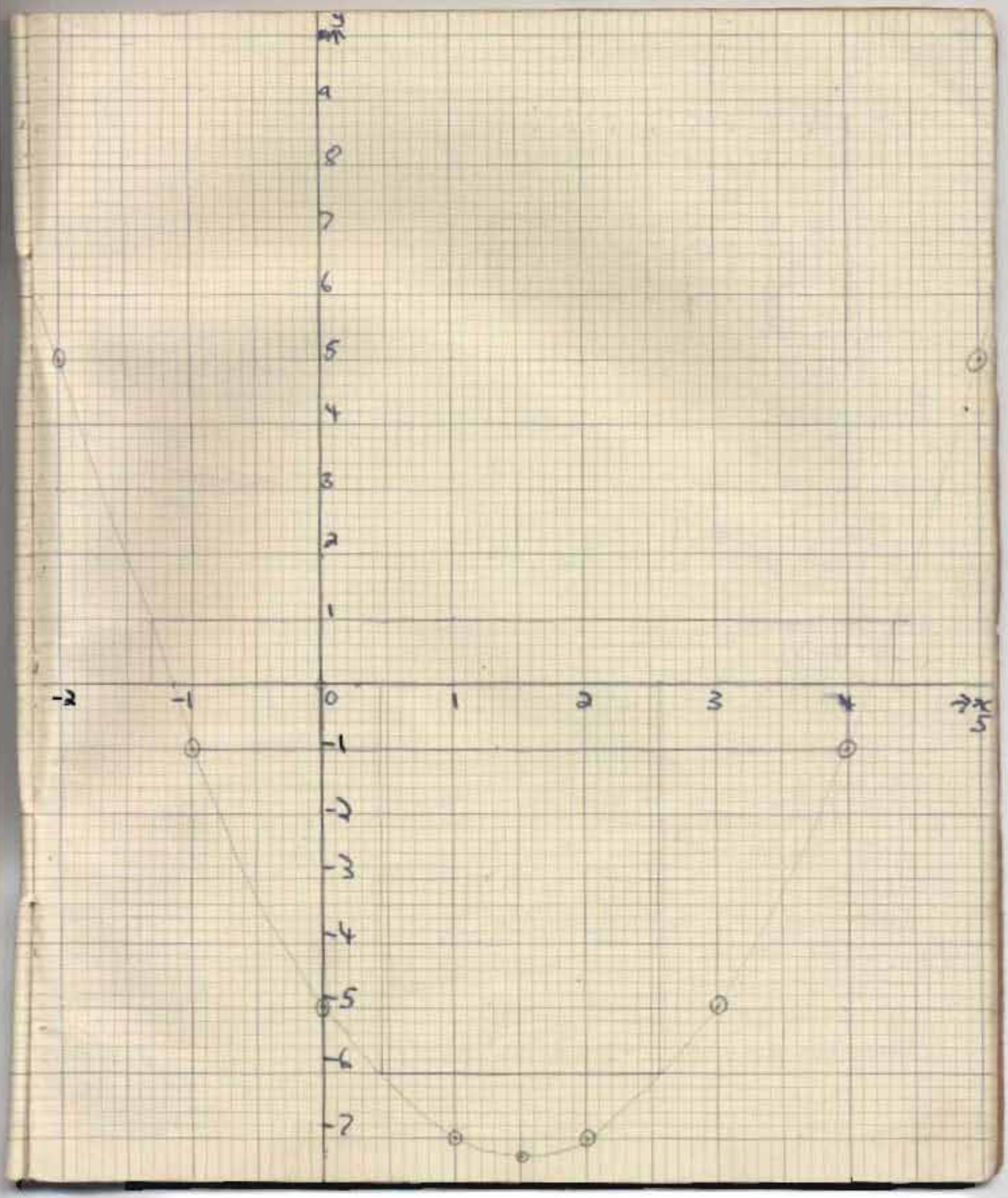
From graph $y = -6$ ∴ $x = 0·44, x = 2·55$

$x^2 - 3x + 5 = 0$

$x^2 - 3x - 5 = -10$

∴ $y = -10$

∴ the line $y = -10$ does not cut the graph ∴ the roots are imaginary


y
9
8
7
6
5
4
3
2
1
-2
-1
0
1
2
3
4
x
5
-1
-2
-3
-4
-5
-6
-7

5. Draw the graph of $y = 7 - 3x - x^2$ from $x = -5$ to $x = 3$

x	-5	-4	-3	-2	-1	0	1	2	3	-1·5
7	7	7	7	7	7	7	7	7	7	7
$-3x$	15	12	9	6	3	0	-3	-6	-9	+4·5
$-x^2$	-25	-16	-9	-4	-1	0	-1	-4	-9	+4·5
y	-3	+3	7	9	9	7	3	-3	-11	9·25

(i) $7 - 3x - x^2 = 0$
i.e. $y = 0$ ✓
From graph $y = 0$, $\therefore x = -4·55, 1·55$

(ii) $5 - 3x - x^2 = 0$
$7 - 3x - x^2 = 2$
i.e. $y = 2$ ✓
From graph $y = 2$ $\therefore x = -4·19, 1·2$

(iii) $x^2 + 3x = 4$
$\therefore$ ~~$x^2 + 3x - 4 = 0$~~
$\therefore 0 = 4 - 3x - x^2$
$3 = 7 - 3x - x^2$
i.e $y = 3$ ✓
From graph $y = 3 \therefore x = -4, x = 1$

(iv) $x^2 + 3x = 6$
$0 = 6 - 3x - x^2$
$\therefore 1 = 7 - 3x - x^2$
i.e. $y = 1$ ✓
From graph $y = 1$, $\therefore x = -4·38, 1·38$

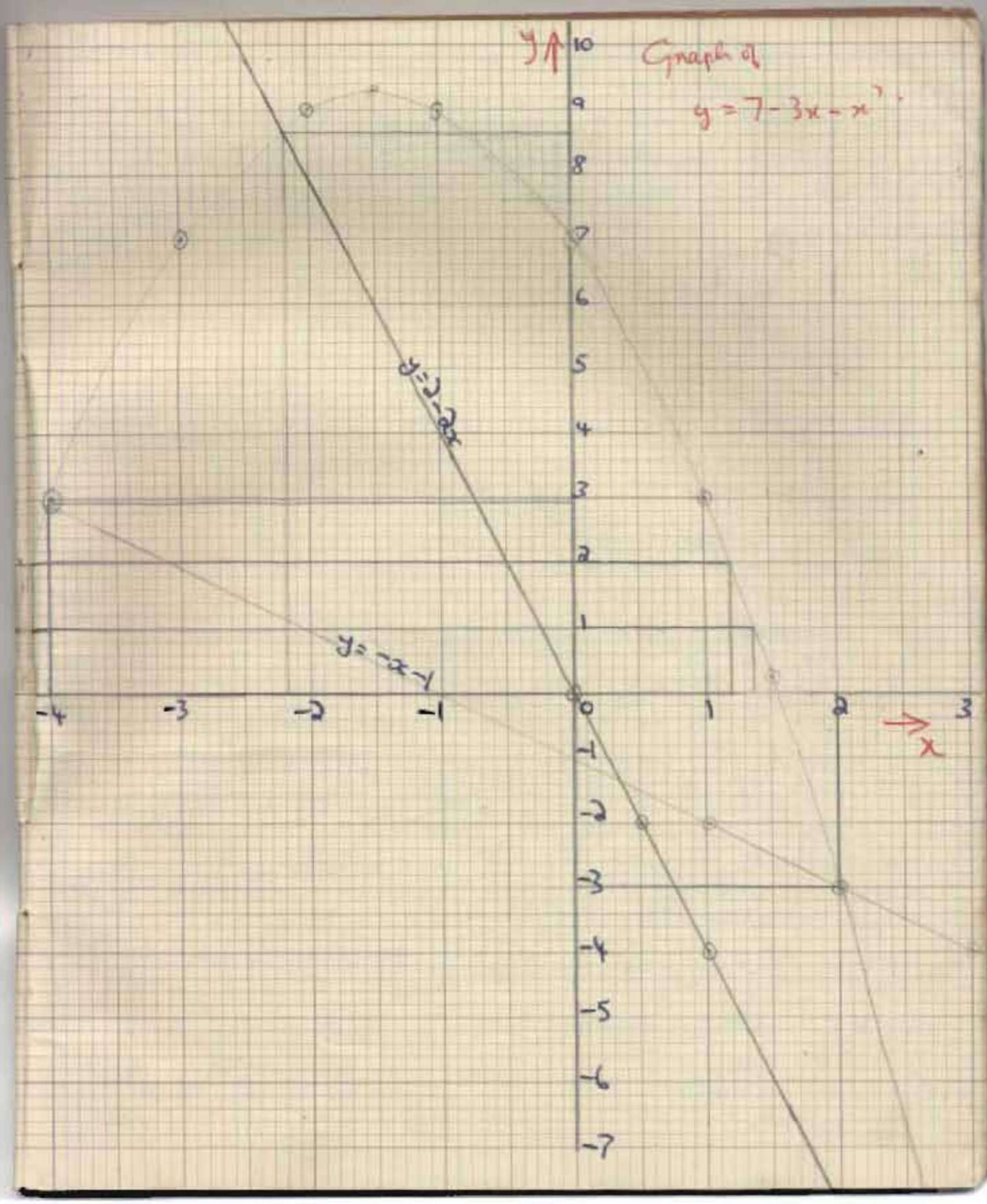
(v) $5 - x - x^2 = 0$
$\therefore 7 - 3x - x^2 = 2 - 2x$ ✓
i.e $y = 2 - 2x$
From graph $y = 2 - 2x$, $x = -2·19, 3·19$

(vi) $8 - 2x - x^2 = 0$
$7 - 3x - x^2 = -x - 1$ ✓
i.e $y = -x - 1$
From graph $y = -x - 1$ $\therefore x = -4, 2$

But wrong graph drawn

-5

9½/10


Graph of
$y = 7 - 3x - x^2$.
y
x
y=3-2x
y=-x-1
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-4
-3
-2
-1
1
2
3

Draw on the same diagram the graphs of $y = \frac{1}{4}(8+5-2x^2)$ and $2y = x+2$
Take 1" as unit on both axes.
Use your graphs to find the maximum value of $y = \frac{1}{4}(8+5-2x^2)$
The x co-ordinates of the points of intersection of the two graphs. Show without using that these co-ordinates are the roots of the equation $2x^2 - 3x = 4$

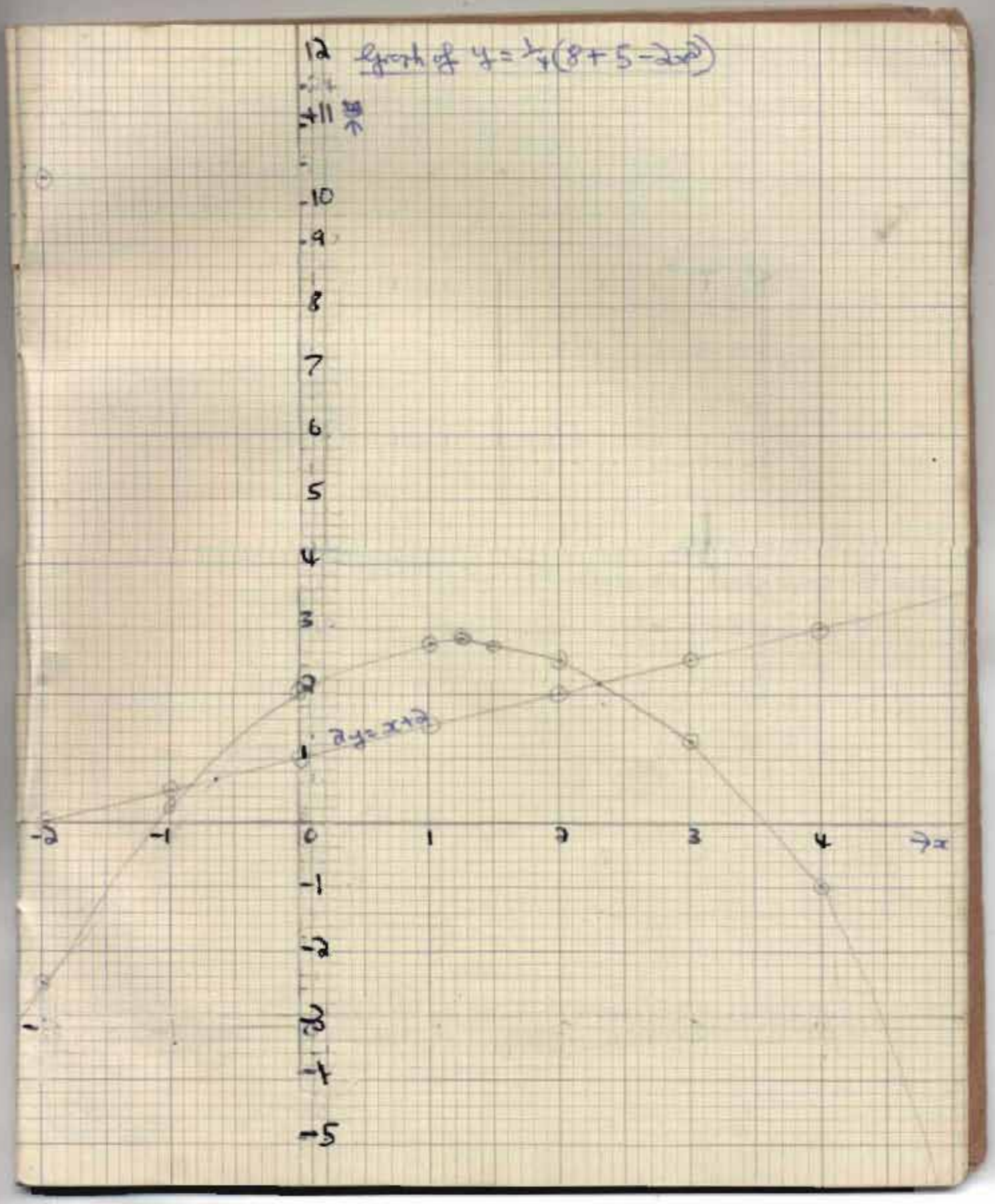
x	-2	-1	0	1	2	3	4	1·5	1·25
8	8	8	8	8	8	8	8	8	8
5x	-10	-5	0	5	10	15	20	7·5	6·25
$-2x^2$	-8	-2	0	-2	-8	-18	-32	-4·5	-3·125
4y	-10	+1	8	11	10	5	-4	11	11·1875
y	-2·5	0·25	2	2·75	2·5	1·25	-1	2·75	2·7815

x	-2	-1	0	1	2	3	4
$2x^2$	8	2	0	2	8	18	32
$-3x$	6	3	0	-3	-6	-9	-12
-4	-4	-4	-4	-4	-4	-4	-4
y	10	1	-4	-5	-2	$\frac{23}{5}$	16

x	-2	-1	0	1	2	3	4
x	-2	-1	0	1	2	3	4
2	2	2	2	2	2	2	2
2y	0	1	2	3	4	5	6
y	0	0·5	1	1·5	2	2·5	3

The maximum value of $y = \frac{1}{4}(8+5-2x^2)$ is $y = 2·7815$
The x co-ordinates of the points of intersection of the graph are $x = -0·82, 2·31$
If these co-ordinates are the roots of the equation $2x^2 - 3x - 4 = 0$
Then $\frac{8+5x-2x^2}{4} - \frac{(x+2)}{2} = 0$ must be equal to $2x^2 - 3x = 4$

$\therefore 8 + 5x - 2x^2 - 2x - 4 = 0$
$\therefore 4 + 3x - 2x^2 = 0$
$4 = -3x + 2x^2$
$\therefore 2x^2 - 3x = 4$


Graph of $y = \frac{1}{4}(8 + 5 - 2x^2)$
12
11
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-2
-1
0
1
2
3
4
x
2y = x + 2

Draw the graph of $y = 3 + 8x - x^3$ from $x = -3$ to $+3$ taking 1" as the unit for values of x and 0·2" as the unit for values of y. With the same scales and axes draw the graph of $y = 8x$ from $x = 0$ to $x = 2$. Find the x co-ordinate of the point of intersection of the line with the graph and explain why it should give an approximate value of the $\sqrt[3]{3}$

x	-3	-2	-1	0	1	2	3	-1·5
3	3	3	3	3	3	3	3	3
$8x$	-24	-16	-8	0	8	16	24	-12·0
$-x^3$	27	8	1	0	-1	-8	-27	3·375
y	6	-5	-4	3	10	11	0	

x	0	1	2
$8x$	0	8	16
y	0	8	16

x co-ordinate = 1·45

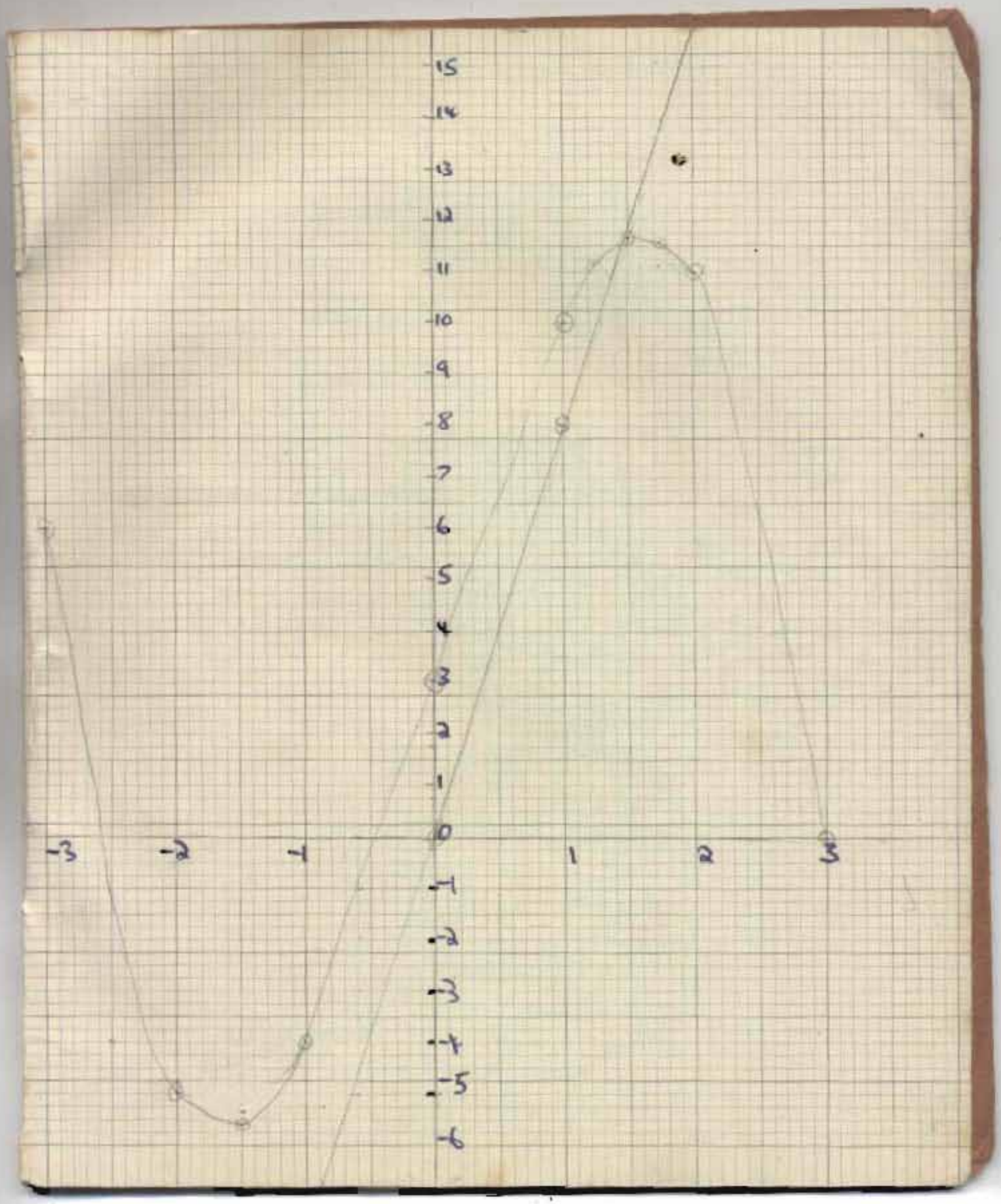
When the graph gives an approximate value of the $\sqrt[3]{3}$

$8x = 3 + 8x - x^3$

$\therefore x^3 = 3$

$x = \sqrt[3]{3}$

$= 1·45$


15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-3
-2
-1
1
2
3

P.223 No.4

$y = 5x - x^2 + 2$

(i) $5x - x^2 + 2 = 0$, $y = 0$

(ii) $5x - x^2 + 3 = 0$
$5x - x^2 + 2 = -1$
$\therefore y = -1$

(iii) $5x - x^2 = 1$
$\therefore 5x - x^2 - 1 = 0$
$5x - x^2 + 2 = 3$
$\therefore y = 3$

(iv) $x^2 = 5x + 1$
~~$x^2 - 5x + 1 = 0$~~
~~$\therefore x^2 - 5x + 2 =$~~
$\therefore 5x - x^2 + 1 = 0$
$5x - x^2 + 2 = 1$
$\therefore y = 1$

(v) $4x - x^2 - 1 = 0$
$5x - x^2 + 2 = x + 3$
$\therefore y = x + 3$

vi) $x^2 - 5x - 2 = 0$
$\therefore 0 = 5x - x^2 + 2$
$\therefore y = 0$

P.223 No.5.

$y = x^2$

(i) $x^2 = 4x + 1$
$x^2 = y$
$\therefore y = 4x + 1$

(ii) $x^2 = 5x - 2$
$x^2 - 5x + 2 = 0$
$\therefore x^2 = +5x - 2$

(iii) $x^2 - 2x - 1 = 0$
$\therefore x^2 = 2x + 1$
$\therefore y = 2x + 1$

(iv) $x^2 + 3x - 5 = 0$
$\therefore x^2 = 5 - 3x$
$\therefore y = 5 - 3x$

(v) $0 = x^2 + 2x - 3$
$3 - 2x = x^2$
$\therefore y = 3 - 2x$

(vi) $x^2 + 5x + 4 = 0$
$\therefore x^2 = -5x - 4$
$\therefore y = -5x - 4$

(vii) $2x^2 - 4x - 1 = 0$
$\therefore x^2 = x^2 + 4x + 1$
$\therefore y = 4x + 1 - x^2$

(viii) $3x^2 - 2x - 9 = 0$
$\therefore x^2 = -2x^2 + 2x + 9$
$\therefore y = 9 + 2x - 2x^2$

(ix) $3x^2 - x - 6 = 0$
$\therefore x^2 = -2x^2 + x + 6$
$\therefore y = 6 + x - 2x^2$

(x) $4x^2 - 8x - 3 = 0$
$\therefore x^2 = -3x^2 + 8x + 3$
$\therefore y = 3 + 8x - 3x^2$

$y = x^2 - 5x + 4$

(i) $x^2 - 5x + 2 = 0$
$x^2 - 5x + 4 = 2$
$\therefore y = 2$

(ii) $x^2 - 5x + 5 = 0$
$x^2 - 5x + 4 = -1$
$\therefore y = -1$

(iii) $x^2 - 6x + 4 = 0$
$x^2 - 5x + 4 = x$
$\therefore y = x$

(iv) $x^2 - 7x + 2 = 0$
$x^2 - 5x + 4 = 2x + 2$
$\therefore y = 2x + 2$

(v) $x^2 + x - 3 = 0$
$x^2 - 5x + 4 = -6x + 7$
$\therefore y = 7 - 6x$

(vi) $x^2 - 2x + 1\frac{1}{2} = 0$
$x^2 - 5x + 4 = -3x + 2\frac{1}{2}$
$\therefore y = 2\frac{1}{2} - 3x$.

* (xi) $\frac{1}{2}x^2 - 3x + 2 = 0$
$\therefore x^2 = \frac{1}{2}x^2 + 3x - 2$
$\therefore y = \frac{1}{2}x^2 + 3x - 2$