Co-ordinates of a Point

Example 1a.

1) Distance between points of coordinates $x_1 y_1$ and $x_2 y_2$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

∴ distance between points (3,2), (5,4).

$$= \sqrt{(5-3)^2 + (4-2)^2}$$

$$= \sqrt{4+4} = \sqrt{8} = 2.8284.$$

$= 2\sqrt{2}$

ii) distance between points (0,0), (1,3)

$$= \sqrt{(1-0)^2 + (3-0)^2}$$

$$= \sqrt{10}.$$

iii) distance between points (−1,2), (2,−1)

$$= \sqrt{(2 - -1)^2 + (-1-2)^2}$$

$$= \sqrt{3^2 + -3^2}$$

$$= \sqrt{18} = 3\sqrt{2}$$

iv) distance between points (3,−1), (−3,−4)

$$= \sqrt{(-3-3)^2 + (-4+1)^2}$$

$$= \sqrt{-6^2 + -3^2}$$

$$= \cancel{\sqrt{36}}\ \sqrt{45} = 3\sqrt{5}$$

v) (0,−2), (3,−1) distance between these points

$$= \sqrt{(3-0)^2 + (-1 - -2)^2}$$

$\sqrt{3^2 + 1^2} = \sqrt{10}$

2 i) distance between points A and B

$$= \sqrt{(2-0)^2 + (3-0)^2}$$
$$= \sqrt{4+9}$$
$$= \sqrt{13}$$

distance between B and C

$$= \sqrt{(1-2)^2 + (4-3)^2}$$
$$= \sqrt{-1^2 + 1^2}$$
$$= \sqrt{2}$$

distance between points A and C

$$= \sqrt{(1-0)^2 + (4-0)^2}$$
$$= \sqrt{17}$$

A is not right angled

ii) distance between points A and B

$$= \sqrt{(1-3)^2 + (7--1)^2}$$
$$= \sqrt{-2^2 + 8^2}$$
$$= \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$$

distance between points B and C

$$= \sqrt{(-7-1)^2 + (5-7)^2}$$
$$= -8^2 + -2^2 = 64 + 4 = \sqrt{68}$$

distance between A and C
$= \cancel{\sqrt{(1-2)^2 + (5--1)^2}}$ $= \sqrt{(-7-3)^2 + (5--1)^2}$
$\cancel{= \sqrt{(-1)^2 + 6^2}}$ $= \sqrt{(-10)^2 + 6^2}$
$\cancel{= \sqrt{37}}$ $= \sqrt{136}$
$= 2\sqrt{34}$

The $\triangle$ is right angled
because $AC^2 = AB^2 + BC^2$ (Pythagoras theorem)
the right angle is $\angle ABC$

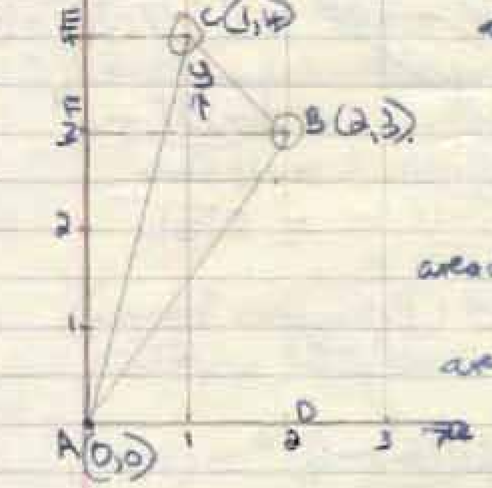
iii). distance between A and B $= \sqrt{(-1-2)^2 + (1--1)^2}$
$= \sqrt{-3^2 + 2^2}$
$= \sqrt{13}$

distance between B and C $= \sqrt{(1--1)^2 + (5-1)^2}$
$= \sqrt{2^2 + 4^2}$
$= \sqrt{20} = 2\sqrt{5}$

distance between A and C $= \sqrt{(1-2)^2 + (5--1)^2}$
$= \sqrt{-1^2 + 6^2}$
$= \sqrt{37}$

The $\triangle$ is not right angled because $AC^2 \neq BC^2 + AB^2$ (Pythagoras theorem)

3i.



~~area of Δ ABC = area trapezium ABCD − area Δ ABD~~

area of Δ ABC = area ABCE − area Δ ECA

area of quad. ABCE = area trap ECBF + area Δ FBA

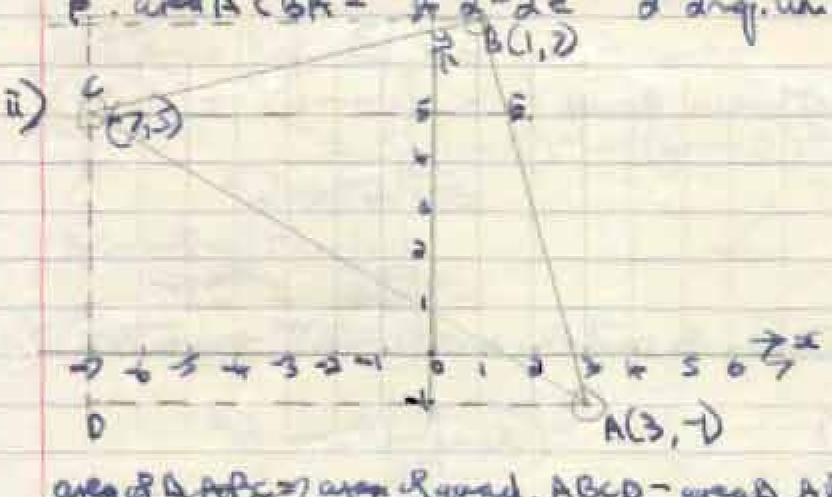
$= \left(\frac{1}{2}(1+2)\times 1\right) + \frac{1}{2}2\times 3$

$= 1\frac{1}{2} + 3 = 4\frac{1}{2}$ sq. units

area Δ ECA = $\frac{1}{2} 1 \times 4 = 2$ sq. units

∴ area Δ CBA = $4\frac{1}{2} - 2 = 2\frac{1}{2}$ sq. units

ii)



~~area of Δ ABC = area of quad. ABCD − area Δ ADC~~

~~area of quad ABCD = area of trap AECD + area Δ CBE~~

~~= ½(10 +~~

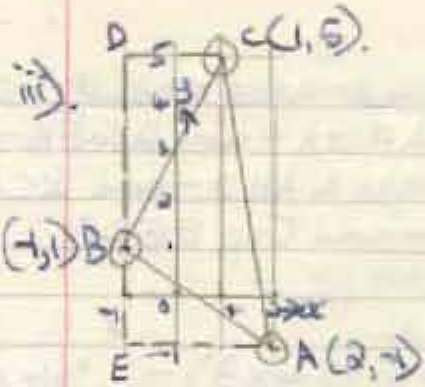
area of Δ ABC = area of trap FBAD − area Δ FBC − area Δ ADC

area of trap. FBAD = $\frac{1}{2}(10+8)\times 8 = 9\times 8 = 72$ sq. units.

area of Δ ADC = $\frac{1}{2} 10\times 6 = 30$ sq units

area of Δ FBC = $\frac{1}{2} 2\times 8 = 8$ sq units

∴ area of Δ ABC = $72 - 38 = 34$ sq. units iii).



area of $\triangle ABCA$ = area trap EACD − area $\triangle DCB$ − area $\triangle BEA$

$= \left(\tfrac{1}{2}(3+2)\times 6\right) - \left(\tfrac{1}{2}2\times 4\right) - \left(\tfrac{1}{2}2\times 3\right)$ sq units.

$= 15 - 4 - 3 = 15 - 7 = 8$ sq units.

4 Length of AB $= \sqrt{(3--3)^2 + (1-2)^2}$

$= \sqrt{6^2 + -1^2}$

$= \sqrt{37}$

Length of BC $= \sqrt{(4-3)^2 + (-1-1)^2}$

$= \sqrt{1^2 + -2^2}$

$= \sqrt{5}$

Length of CD $= \sqrt{(0-4)^2 + (-2--1)^2}$

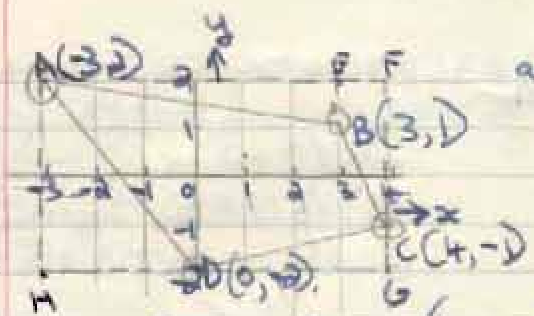
$= \sqrt{-4^2 + -1^2}$

$= \sqrt{17}$

Length of DA $= \sqrt{(0--3)^2 + (-2-2)^2}$

$= \sqrt{3^2 + -4^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$



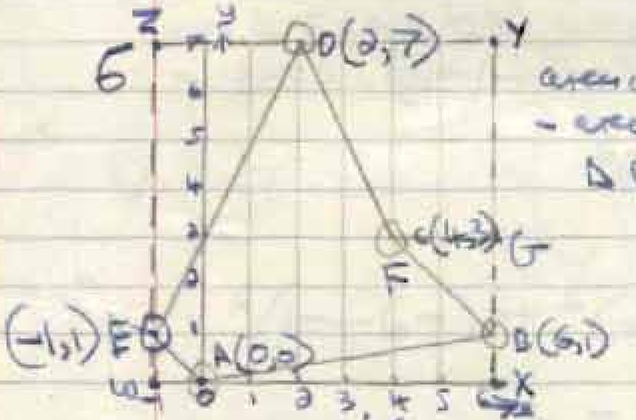
area of quad ABCD = area of rect. AFGH − area △AEB − area △AHD − area △CGD − area Trap EFCB

$= (7\times 4) - (\tfrac12 6\times 1) - (\tfrac12 4\times 3) - (\tfrac12 4\times 1) - (\tfrac12(1+3)\times 1)$ sq units

$= 28 - 3 - 6 - 2 - 2$ sq units

$= 28 - 13 = 15$ sq. units.

6



area of pent. ABCDE = area of rect. WXYZ − area of △ZDE − area △EWA − area △BXA − △FGB − area Trap DYGF

$= (7\times 7) - (\tfrac12 3\times 7) - (\tfrac12 1\times 1) - (\tfrac12 6\times 1) - (\tfrac12 2\times 2) - (\tfrac12(2+4)\times 4)$ sq units

$= 49 - 10\tfrac12 - \tfrac12 - 3 - 2 - 12$ sq units

$= 49 - 28 = \cancel{28}$ ~~sq units~~ 21 sq. units

6. Distance between points (4,4) and (1,0)

$= \sqrt{(1-4)^2 + (0-4)^2}$

$= \sqrt{-3^2 + -4^2}$

$= \sqrt{25} = 5$ units

distance between points $(4,4)$ and $(-1,4)$

$$= \sqrt{(-1-4)^2 + (4-4)^2}$$

$$= \sqrt{-5^2 + 0}$$

$$= \sqrt{25} = 5 \text{ units. } QED$$

7. Distance between (α, β) and $(2, 3)$

$$= \sqrt{(2-\alpha)^2 + (3-\beta)^2}$$

8 Distance of (α, β) from $(0,0)$

$$= \sqrt{(\alpha-0)^2 + (\beta-0)^2}$$

$$= \sqrt{\alpha^2 + \beta^2}$$

Distance of (α, β) from $(3,4)$

$$= \sqrt{(\alpha-3)^2 + (\beta-4)^2}$$

but $\sqrt{\alpha^2+\beta^2} = \sqrt{(\alpha-3)^2 + (\beta-4)^2}$

$\therefore \alpha^2 + \beta^2 = (\alpha-3)^2 + (\beta-4)^2$

$\therefore \cancel{\alpha^2} + \cancel{\beta^2} = \cancel{\alpha^2} - 6\alpha + 9 + \cancel{\beta^2} - 8\beta + 16$

$\therefore 6\alpha - 9 = 16 - 8\beta$

$\therefore 6\alpha + 8\beta = 25.$

9 Distance between points $(\alpha, 0)$ and $(0, \alpha)$

$$= \sqrt{\alpha^2 + -\alpha^2}$$

Distance between points $(1,2)$ and $(-1,3)$

$$= \sqrt{(1 - -1)^2 + (2-3)^2}$$

$$= \sqrt{4+1} = \sqrt{5}.$$

$$\therefore \sqrt{5} = \sqrt{\alpha^2 + \alpha^2}$$

$$\therefore 5 = \alpha^2 + \alpha^2$$

$$\therefore 2a^2 = 5.$$

$$\therefore \alpha^2 = \frac{5}{2}$$

$$\alpha = \pm\sqrt{5/2}.$$

10. Distance of (a, b) from origin, i.e. $(0,0)$

$$= \sqrt{a^2 + b^2}$$

Distance of (a, b) from $(-1, -2)$

$$= \sqrt{(a - -1)^2 + (b - -2)^2}$$

$$= \sqrt{(a+1)^2 + (b+2)^2}$$

But $\sqrt{a^2+b^2} = 2\sqrt{(a+1)^2 + (b+2)^2}$.

$$\therefore a^2 + b^2 = 2\{(a+1)^2 + (b+2)^2\}$$

$$\therefore a^2 + b^2 = 4(a^2 + 2a + 1 + b^2 + 4b + 4)$$

$$\therefore a^2 + b^2 = 4a^2 + 8a + 4 + 4b^2 + 16b + 16$$

$$\therefore 0 = 3a^2 + 8a + 3b^2 + 16b + 20$$

11. ~~Let the Y coordinate (0, a). Let the X coordinate (b, 0)~~ ??

The distance between them is 10"

$\therefore 10'' = \sqrt{(0-b)^2+(a-0)^2}$

but point $(0,a) = \frac{1}{2}$ point $(b,0)$

$10 = \sqrt{b^2+a^2}$ — ①

$2a = b$ — ②

?

$\therefore b^2+a^2 = 10^2$ — ③

$b-2a = 0$ — 4

P28 No's 1, 3, 5, 7, 8. 16-9-'66

1. When lines $3y - x = 2$ and $2y - 5x = 1$ intersect.

$\frac{x+2}{3} = \frac{5x+1}{2}$

$\therefore 2(x+2) = 3(5x+1)$

$2x+4 = 15x+3$

$-13x = -1$

$x = \frac{1}{13}$

If $x = \frac{1}{13}$

$3y - \frac{1}{13} = 2$

$3y = 2\frac{1}{13}$

$y = \frac{27}{13} \times \frac{1}{3}$

$= \frac{27}{39} = \frac{9}{13}$

$\therefore$ line passes thru points $(\frac{1}{13}, \frac{9}{13})$

" is perp. to line $y = 2x+7$

in line $y = 2x+7$.

$m = 2.$

$\therefore$ gradient of line $= 2m' = -1$

$\therefore 2m' = -1$

$m' = -\frac{1}{2}.$

$y-k = m(x-h)$

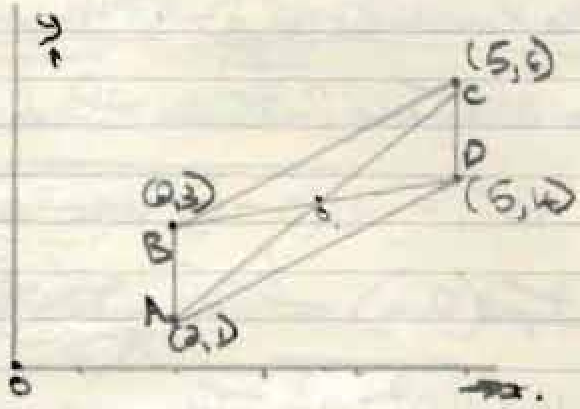
$\therefore y - \frac{9}{13} = -\frac{1}{2}\left(x - \frac{1}{13}\right).$

$y - \frac{9}{13} = -\frac{1}{2}x + \frac{1}{26}$

$y + \frac{1}{2}x = \frac{19}{26}$

Reqd Eq: $26y + 13x = 19.$

3.



~~Gradient~~

gradient of line BC $= m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{6-3}{5-2} = 1.$

gradient of AD $= m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{5-2}{4-1} = 1$

$\therefore$ BC $\parallel$ AD

gradient of CD $= m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{6-4}{5-5} = \infty$

gradient of AB $= m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{3-1}{2-2} = \infty$

$\therefore$ CD $\parallel$ BA.

Both pair

$\therefore$ ABCD is a $\parallel$gm. (opp. sides ~~equal~~ & $\parallel$).

Equation of CA: $\dfrac{y_2-y_1}{x_2-x_1} = \dfrac{y-y_1}{x-x_1}$ $\therefore$ $\dfrac{5}{3} = \dfrac{y-1}{x-2}$

$\therefore 5(x-2) = 3(y-1).$

$\therefore 5x - 10 = 3y - 3.$
$\therefore 3y - 5x = -7.$

Equation of BD is $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y - y_1}{x - x_1}$

$\therefore \dfrac{4-3}{5-2} = \dfrac{y-3}{x-2}$

$\therefore x - 2 = 3(y-3)$

$x - 2 = 3y - 9$

$3y - x = +7.$

when they intersect $\dfrac{5x - 7}{3} = \dfrac{x + 7}{3}$

$3(5x - 7) = 3(x + 7)$

$15x - 21 = 3x + 21$

$12x = 42$

$x = \frac{14}{4} = \frac{14}{4}$

If $x = \frac{14}{4}$

$3y - \frac{14}{4} = +7$

$3y = +\frac{14}{4} + \frac{28}{4} = \frac{42}{4}$

$y = \frac{42}{4} \times \frac{1}{3} = \frac{7}{2}.$

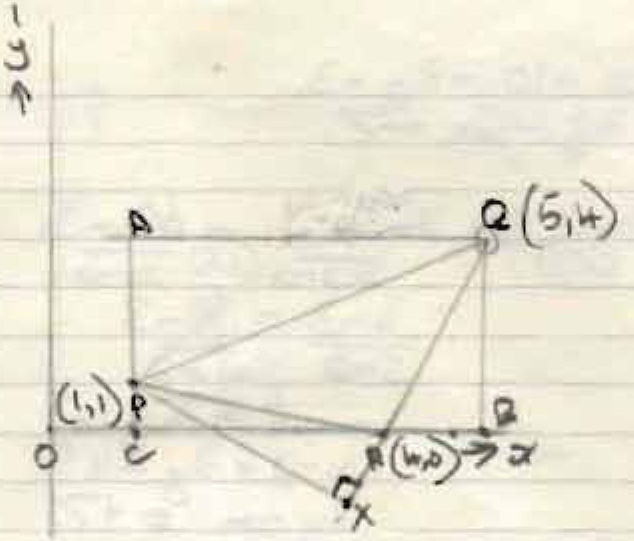
$\therefore$ point of intersection is $\left(\frac{7}{2}, \frac{7}{2}\right)$

$\therefore$ diagonals intersect at point $\left(\frac{7}{2}, \frac{7}{2}\right)$

OR Mid pt of AC is $\left(\frac{5+2}{2}, \frac{6+1}{2}\right)$

$= \left(\frac{7}{2}, \frac{7}{2}\right).$

5.



a) Let gradient of QR and $\therefore$ of QX = m.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-4}{4-5} = \frac{-4}{-1} = 4.$$

Let gradient of PX = m'

because PX $\perp$ QX $\quad mm' = -1$

$$\therefore 4m' = -1$$

$$\therefore m' = -1/4$$

$\therefore$ gradient of PX = m'

If point X = (x, y) and point P = (h, k).

then $y - k = m'(x - h)$

$$\therefore y - 1 = -1/4(x - 1)$$

$$y - 1 = -1/4x + 1/4.$$

$$y + 1/4x = 1\tfrac{1}{4}$$

$$4y + x = 5$$

$\therefore$ equation of PX is $4y + x = 5$

b) equation of QR i.e of QX is $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y - y_1}{x - x_1}$

$$\frac{0-4}{4-5} = \frac{y-4}{x-5}$$

$$\therefore -4(x-5) = -1(y-4)$$
$$-4x + 20 = -y + 4$$
$$y - 4x = -16.$$

when QX and PX intersect.

$$4x - 16 = \frac{5-x}{4}.$$

$$\therefore 4(4x-16) = 5-x$$
$$16x - 64 = 5-x$$
$$17x = 69$$
$$x = 69/17.$$

$$x = \frac{59}{17}$$

$$4y + \frac{59}{17} = \frac{85}{17}$$
$$4y = \frac{85}{17} - \frac{69}{17} = \frac{16}{17}$$
$$\therefore y = \frac{16}{68} = \frac{4}{17}.$$

$\therefore$ coordinates of X are $\left(\frac{69}{17}, \frac{4}{17}\right)$.

c). Length of PX $= \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$

$$= \sqrt{\left(\frac{4}{17}-1\right)^2 + \left(\frac{69}{17}-1\right)^2}$$
$$= \sqrt{\left(-\frac{13}{17}\right)^2 + \left(\frac{52}{17}\right)^2}$$
$$= \frac{\sqrt{169 + 2704}}{17} = \frac{\sqrt{2873}}{17} = \frac{53.6}{17}\text{ units}$$

d). ~~To find length of QX~~

~~Length of QR~~ $= \cancel{\sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}}$

$$\cancel{= \sqrt{\left(4-\frac{4}{17}\right)^2 + \left(5-\frac{69}{17}\right)^2}}$$

No	Log
26.8	1.4281
$\sqrt{17}$	0.6152
	0.8129

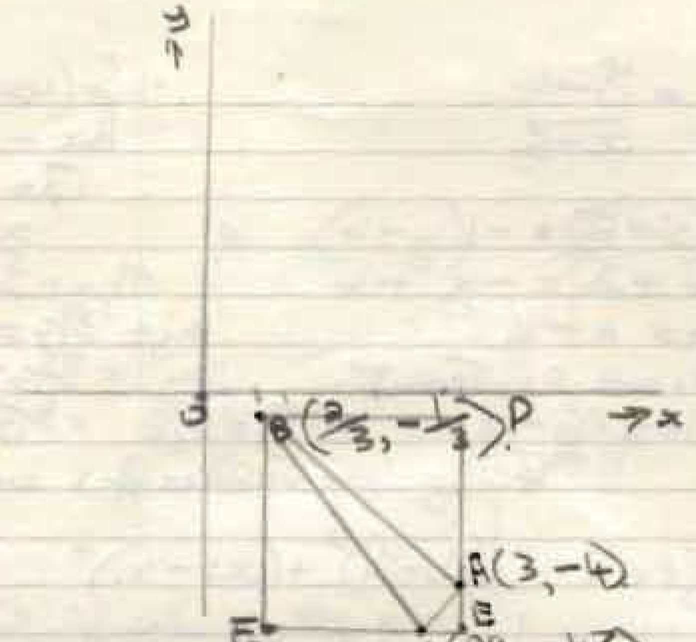
Length of QR $= \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$
$$= \sqrt{(4)^2 + 1^2} = \sqrt{17}$$

$\therefore$ Area of $\triangle$ QPR $= \frac{1}{2} \times \sqrt{17} \times \frac{53.6}{17} = \frac{26.8}{\sqrt{17}} = 6.5$ sq. units

To check.

area of $\triangle$ PQR = area of rect. AQBC − area $\triangle$ QRB − area $\triangle$ AQP − area $\triangle$ PCR $= 16 - (\frac{1}{2} \times 1 \times 4) - (\frac{1}{2} \times 4 \times 3) - (\frac{1}{2} \times 3 \times 1)$
$= 16 - 2 - 6 - 1\frac{1}{2} = 16 - 9\frac{1}{2} = 6\frac{1}{2}$ sq. units. ✓

7.



i) <u>To find line || to $4x + 3y = 8$.</u>

in $4x + 3y = 8$

$$y = \frac{8 - 4x}{3}$$

$\therefore m = -\frac{4}{3}$.

$\therefore$ gradient (m) of line || $4x + 3y = 8 = -\frac{4}{3}$.

Let 2 coordinates of this line be (x, y) and (h, k).

Then $y - k = m(x - h)$.

$\therefore$ this line passes through point $(3, -4)$

$\therefore$ let this point be point (h, k)

$$\therefore y + 4 = -\frac{4}{3}(x - 3)$$

$$y + 4 = -\frac{4}{3}x + 4$$

$$y + \frac{4}{3}x = 0$$

$$3y = -4x.$$

$\therefore$ equation of line || to $4x + 3y = 8$ is $3y = -4x$.

ii). To find line $\perp$ to $4x+3y=8$.

gradient m of $4x+3y=8 = -4/3$.

Let gradient of line $\perp$ to this line be m'

then $mm' = -1$

$\therefore -4/3 m' = -1$

$m' = -1 \times -3/4 = 3/4$.

$\therefore$ gradient of a line $\perp$ to $4x+3y=8 = 3/4$

Let two co-ordinates of this line be (x, y) and (h, k) — Just say

then $y-k = m'(x-h)$ — Eqⁿ line with

this line passes through point $(3, -4)$. — gradient 3/4

Let this point be (h, k) — passing through (3, -4

$\therefore y+4 = 3/4(x-3)$

$y+4 = 3/4x - 9/4$. — $\therefore y+4 = 3/4(x-3)$

$y - 3/4x = -25/4$

$4y - 3x = -25$.

$\therefore$ equation of line $\perp$ to $4x+3y=8$ is $4y - 3x = -25$.

iii) Let point of intersection of $2x+y=1$ and $3y=-4x$ be B.

$\therefore$ at B $\frac{-4x}{3} = 1-2x$

$-4x = 3(1-2x)$ — If $x = 3/2$.

$-4x = 3-6x$ — then $6/2 + y = 1$ — -2

$2x = 3$

$x = 3/2$ — $\therefore$ point B $= (3/2, -2)$

Let point of intersection of $2x+y=1$ and $4y-3x=-25$ be C.

$\therefore$ at C. $1-2x = \frac{3x-25}{4}$

If $x = 29/11$

$4(1-2x) = 3x-25$ — then $58/11 + y = 1$

$4-8x = 3x-25$ — $y = 1 - 58/11$

$-11x = -29$ — $= -47/11$

$x = 29/11$ — $\therefore$ point C $= (29/11, -47/11)$.

area $\triangle ABC = \frac{1}{2}\left[y_1(x_2 - x_3) + y_2(x_3 - x_1) + y_3(x_1 - x_2)\right]$

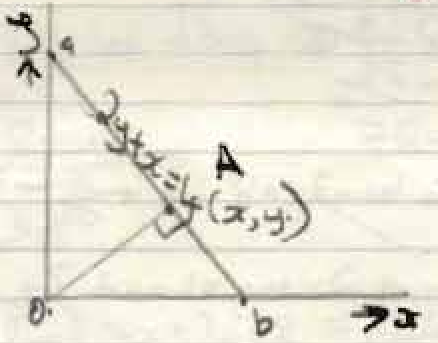
$= \frac{1}{2}\left[-4\left(\frac{29}{11} - \frac{3}{2}\right) + \frac{-47}{11}\left(\frac{3}{2} - 3\right) + -2\left(3 - \frac{29}{11}\right)\right]$

$= \frac{1}{2}\left[\frac{-116}{11} + 6 - \frac{141}{22} + \frac{141}{11} - 6 + \frac{58}{11}\right]$

$= \frac{1}{2}\left(\cancel{\frac{29}{11}} - \cancel{\frac{373}{22}} - \frac{373}{22} + \frac{199}{11}\right)$

$= \frac{1}{2}\left(-\frac{373}{22} + \frac{398}{22}\right) = \frac{1}{2} \times \frac{25}{22} = \frac{25}{44}$

$\therefore$ area $\triangle ABC = \frac{25}{44}$ sq. units

8\. 

Sn $2y + x = 4$

$y = \frac{4 - x}{2}$

$\therefore$ gradient m of the line $= -\frac{1}{2}$.

Let gradient of AO $= m'$

Then $mm' = -1$

$\therefore -\frac{1}{2}m' = -1$

$m' = 2$.

Let point $A = (x, y)$ and $O = (h, k)$

then $y - k = m(x - h)$

this line passes through the point O which is $(0,0)$

$\therefore \quad y = 2x$.

This is equation of line AO.

when $y=2x$ and $2y+x=4$ intersect at A

$$2x = \frac{4-x}{2}$$

$$4x = 4-x$$

$$5x = 4 \quad x = 4/5.$$

If $x = 4/5$

then $y = 8/5$

$\therefore$ co ordinates of AO are $(4/5, 8/5)$ and $(0,0)$.

$$\text{length of AO} = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$

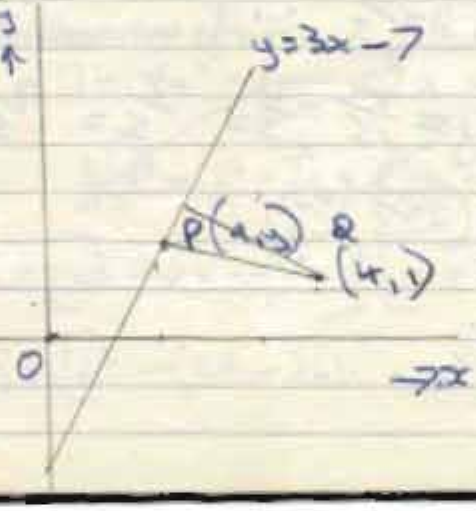
$$= \sqrt{(8/5)^2+(4/5)^2}$$

$$= \sqrt{\frac{64}{25}+\frac{16}{25}} = \frac{\sqrt{80}}{5} = \frac{4\sqrt{5}}{5} = \frac{4}{\sqrt{5}}$$

$\therefore$ length of perp. from origin to line $2y+x=4$ is $\frac{4}{\sqrt{5}}$ units.

20/20 ✓ Good but don't cram your answers into half the page

P 90 No 14, 19



x coordinate of $P = a$.
$\therefore y$ coordinate $= 3a - 7$

Length of $PQ = \sqrt{(a-4)^2 + (3a-7-1)^2}$

$\therefore PQ^2 = (a-4)^2 + (3a-8)^2$

$= a^2 - 8a + 16 + 9a^2 - 48a + 64.$

$= 10a^2 - 56a + 80.$

Let $PQ^2 = x$.

$\therefore \quad x = 10a^2 - 56a + 80.$

$\frac{dx}{da} = 20a - 56. \; \cancel{= 10(2a - 28)}$
$= 2(10a - 28)$

for turning values of a.

$2(10a - 28) = 0$
$4(5a - 14) = 0.$
$a = \frac{14}{5} = 2\frac{4}{5}.$

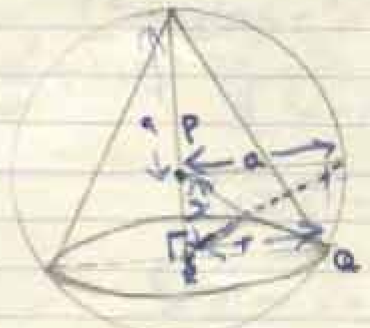
$\frac{d^2x}{da^2} = 20. \; \therefore$ when $a = 2\frac{4}{5}$ $\frac{d^2x}{da^2} > 0 \; \therefore$ point is a minimum

$\therefore x$ coordinate is $2\frac{4}{5}$ $\because$ for minimum value of PQ^2, $PQ \perp y = 3x$

If $x = 2\frac{4}{5}$ $y = \frac{42}{5} - \frac{35}{5} = \frac{7}{5} = 1\frac{2}{5}$

$\therefore$ coordinates are $(2\frac{4}{5}, 1\frac{2}{5})$

19.



$r^2 = (a^2 - x^2)$

Vol. of cone $= \frac{1}{3}\pi r^2 h$.

$= \frac{1}{3}\pi r^2 (a+x)$

In ΔPQR $\angle PRQ = 90°$.

let $h = a+x$

$x = h - a$.

$a = h - x$

∴ By pythagoras theorem

$PQ^2 = PR^2 + RQ^2 = x^2 + r^2$

$\therefore a^2 = x^2 + r^2$

$r^2 = a^2 - x^2$

$\therefore$ Vol of cone $= \frac{1}{3}\pi (a^2 - x^2)(a+x)$.

Let Vol of cone $= V$.

$\therefore V = \frac{1}{3}\pi (a^2 - x^2)(a+x)$

$= \cancel{\frac{1}{3}\pi (a^3 - x^2 a + xa^2 - x^3)}$

$= \frac{1}{3}\pi (a^3 - x^2 a + xa^2 - x^3)$

let a (constant) $= h - x$

$\therefore x = h - a$

$\cancel{\therefore V = (a^3 - (h-a)^2 a + (h-a)a^2 - (h-a)^3)\frac{1}{3}\pi}$

$\cancel{\frac{1}{3}\pi (a^3 - (h^2 - 2ha + a^2)a + a^2 h - a^3 - (h^3 - 3ah^2 + 3a^2 h - a^3))}$

$\therefore V = \frac{1}{3}\pi [a^2 - (h-a)^2][a + h - a]$

$= \frac{1}{3}\pi h [a^2 - (h^2 - 2ah + a^2)]$ ✓

$$= \tfrac{1}{3}\pi h\left[a^2 - h^2 + 2ah - a^2\right]$$

$$= \tfrac{1}{3}\pi h\left[2ah - h^2\right]$$

$$= \tfrac{1}{3}\pi 2ah^2 - \tfrac{1}{3}\pi h^3$$

$$\therefore \frac{dV}{dh} = \tfrac{4}{3}\pi ah - \pi h^2.$$

$$\frac{d^2V}{dh^2} = \tfrac{4}{3}\pi a - 2\pi h.$$

for limiting values of h.

$$\frac{dV}{dh} = 0.$$

$$\therefore \tfrac{4}{3}\pi ah - \pi h^2 = 0.$$

$$\therefore \pi h\left(\frac{4a}{3} - h\right) = 0.$$

either $h = 0$ (h can't be 0 because then there would be no cone).

or $h = \dfrac{4a}{3}$

when $h = \dfrac{4a}{3}$, $\dfrac{d^2V}{dh^2} < 0.$

∴ this value is a maximum.

∴ ~~maximum~~ value of height of cone when its volume is a maximum is $\dfrac{4a}{3}$.

Good. $\frac{10}{10}$

P107.

1. $y = \frac{3x^2}{2}$; $y = \frac{3x^2}{2} + 1$; $y = \frac{3x^2}{2} + 2$.

2 i) $\frac{dy}{dx} = 1 - \frac{1}{x^2}$.

$\therefore y = x + \frac{1}{x}$.

ii) $y = x \cancel{+c} + \frac{1}{x} + c$

3 $\frac{dy}{dx} = 2x^2$

$\therefore y = \frac{2x^3}{3.} + C$

when $x = 1$, $y = 2/3$ $\therefore$ $\frac{2}{3} = \frac{2}{3} + C$

$\therefore C = 0$.

$\therefore$ solution is $y = \frac{2x^3}{3} \cancel{+ \frac{1}{3}}$?

4. $\frac{dy}{dx} = 4x^3 - 4x$

$y = x^4 - 2x^2 + C$

when $x = 2$, $y = 0$. $0 = 16 - 8 + C$ $= 8 + C$.

$y \cancel{= 16 - 8 = 8}$ $y = 0$ $\therefore C = -8$

$\therefore y = 0$

then $y = x^4 - 2x^2 - 8$

5. $\frac{ds}{dt} = 3 - t^2$

$\therefore s = 3t - \frac{t^3}{3.} + C$

When $t=0, s=0$ $\therefore$ $s = 3t - \frac{t^3}{3} + C$

$\therefore s=5.$ when $t=0$ $s=0$ $\therefore C=0$

then $s = 3t - \frac{t^3}{3} + 5$ $\therefore s = 3t - \frac{t^3}{3}.$

When $t=2$ when $t=2$

$s = 6 - \frac{8}{3} + 5$ $s = 3.2 - \frac{2^3}{3}$

$= 11 - \frac{8}{3} = 8\frac{1}{3}$ $= 6 - 2\frac{2}{3}$

$= 3\frac{1}{3}$

---

6. $\frac{dp}{dv} = 2v - \frac{v^3}{2}$

$\therefore p = v^2 - \frac{v^4}{8} + C$

When $v=0, p=0.$ $\therefore C=0.$

$\therefore$ When $v=1$

$p = 1 - \frac{1}{8} = \frac{7}{8}.$

---

7. at point (x, y)

$\frac{dy}{dx} = 2x - 1$ ✓

$\therefore y = x^2 - x + C$ ①

When $x=3, y=4.$ $\therefore$ $4 = 3^2 - 3 + C$

When $x=3$ sub in ① $\therefore C = 4 - 6 = -2.$

$y = 6.$ $\therefore y = x^2 - x - 2$

$\therefore$ equation of curve is $y = x^2 - x - 2.$

---

8 $\frac{dy}{dx} = 2x^2 + 3x - 1$

$\therefore y = \frac{2x^3}{3} + \frac{3x^2}{2} - x + C$

When $x=0, y=0$. at point $(0,0)$ $\therefore C=0$

$\therefore$ ~~$0=0+$~~

$\therefore$ equation of curve is $y=\frac{2x^3}{3}+\frac{3x^2}{2}-x$

9i) $y=\frac{x^5}{5}$; ii) $y=\frac{x^{15}}{15}$; iii) $y=\frac{x^{-2}}{-2}=-\frac{1}{2x^2}$;

iv) $y=\frac{x^{5/2}}{5/2}=\frac{2x^{5/2}}{5}$; v) $y=x^4$; vi) $y=-\frac{2}{x}$.

vii) $\frac{dy}{dx}=x^{2/3}$, $\therefore y=\frac{3x^{5/3}}{5}$; viii) $y=\frac{x^{1.2}}{1.2}$ ix) $y=x^4-\frac{x^3}{3}$.

x) $y=2x+\frac{1}{2x^2}$; xi) $\frac{dy}{dx}=\sqrt[3]{x}-\frac{1}{\sqrt[3]{x}}$ $\therefore \frac{dy}{dx}=x^{\frac{1}{3}}-\frac{1}{x^{\frac{1}{3}}}=x^{\frac{1}{3}}-x^{-\frac{1}{3}}$

$\therefore y=\frac{3x^{4/3}}{4}-\frac{3x^{2/3}}{2}$; xii) $\frac{dy}{dx}=(3x-1)(1-x)=(3x-1)(-x+1)$

$=-3x^2+4x-1=4x-3x^2-1$ $\therefore y=2x^2-x^3-x$.
$=x(2x-x^2-1)$

10: $\int x^7 dx$; ~~$\therefore y$~~ $=\frac{x^8}{8}+c$.

ii. $\int(x^2+x)dx$; ~~$\therefore y$~~ $=\frac{x^3}{3}+\frac{x^2}{2}+c$

iii) $\int\left(t+\frac{1}{t}\right)^2 dt$. ~~$dx$~~ $\int\left(t^2+2+\frac{1}{t^2}\right)dt$

Only thing is $s \neq y$ if it $\div$ needed.

$\therefore s = \frac{t^3}{3} + 2t - \frac{1}{t} + c$

iii) $\int (1-3x)(1+x)\,dx; = \int (1-2x-3x^2)\,dx.$

$\therefore \cancel{y} = x - x^2 - x^3 + c.$ ✓

v) $\int \frac{x^3-1}{x^2}\,dx = \int \frac{x^3}{x^2} - \frac{1}{x^2}\,dx = \int x - \frac{1}{x^2}\,dx.$

$\therefore \cancel{y} = \frac{x^2}{2} + \frac{1}{x} + c.$ ✓

11a) $v = 3t - 1$

$\cancel{\frac{dv}{dt}} \therefore \frac{ds}{dt} = 3t - 1$ ✓

$\therefore s = \frac{3t^2}{2} - t. + C$

---

b. $v = t^2 + t^3$

$\therefore \frac{ds}{dt} = t^2 + t^3$

$\therefore s = \frac{t^3}{3} + \frac{t^4}{4} + C$

c. $v = 1 - 2t - 3t^2$

$\therefore \frac{ds}{dt} = 1 - 2t - 3t^2$

$\therefore s = t - t^2 - t^3 + C$

---

12 If velocity after passing O be v ft/sec

$\therefore v = 2 + 3t^2$

$\therefore \frac{ds}{dt} = 2 + 3t^2.$

$\therefore s = 2t + t^3 + c$

when $t = 2$ $\quad 12 = 2\times 2 + 2^3 + c \quad \therefore c = 0$

$s = 4 + 8 = 12$ ft.

---

13. $v = 4t - 2t^2.$

$\therefore \frac{ds}{dt} = 4t - 2t^2$

$\therefore s = 2t^2 - \frac{2t^3}{3} + c$ (where c is a constant)

but as body is at origin when $t=0$, $s = 0$ when $t=0$.

$\therefore c = 0.$ ✓

$\therefore s = 2t^2 - \frac{2t^3}{3}$

when $t = 1$ sec.

$s = 2 - \frac{2}{3} = 1\frac{1}{3}$ ft ✓

when $t = 2$ secs

$s = 8 - \frac{16}{3} = 2\frac{2}{3}$

$\therefore$ dist. moved in 2nd sec. $=$ ~~[illegible]~~ $1\frac{1}{3}$ ft. ✓

---

14 $\frac{d^2y}{dx^2} = 4x$

$\therefore \frac{dy}{dx} = 2x^2 + C$

$\therefore \frac{dy}{dx} = 2x^2$ $\quad$ when $x=3$, $y=0$

when $x = 0$

$\frac{dy}{dx} = 0 \therefore C = 0 \quad \therefore y = \frac{2x^3}{3} + cx$ ~~$+2$~~ $, \frac{dy}{dx} = 2x^2$

15a). $g = 2t$.

$g = \frac{dv}{dt} = 2t$. ✓

$\therefore \frac{ds}{dt} = t^2 + c$ — COG constants.

$\frac{t^3}{3} + ct + c_1$

$\therefore s = \frac{t^3}{3}$ (where c is a constant)

but when $s = 0$, $t = 0$ — when $t=0$ $\frac{ds}{dt} = 0 \therefore c = 0$. ✓

$\therefore c_1 = 0$.

$\therefore s = \frac{t^3}{3} = \frac{t^3}{3}$

$s=0$, $t=0$, $\frac{ds}{dt}$

b.) $g = \frac{dv}{dt} = 1 - t^2$.

$\therefore \frac{ds}{dt} = t - \frac{t^3}{3} + c$ (where c is a constant)

but when $s=0$, $t=0$, $v=0$ $\therefore c = 0$.

$\therefore \frac{ds}{dt} = t - \frac{t^3}{3}$

$\therefore s = \frac{t^2}{2} - \frac{t^4}{12} + c$ (where c is a constant) — when $t=0$, $s=0$ $\therefore c=0$.

~~but when s=0,~~ $c_1 = 0$

$\therefore s = \frac{t^2}{2} - \frac{t^4}{12}$ ✓

c) $g = 2t - t^3 = \frac{dv}{dt}$ ✓

$\therefore \frac{ds}{dt} = t^2 - \frac{t^4}{4} + c$ (where c is a constant) ✓

$v=0$

then when $s=0$, $t=0$ $\therefore c=0$

$\therefore \frac{ds}{dt} = t^2 - \frac{t^4}{4}$

$\therefore s = \frac{t^3}{3} - \frac{t^5}{20} + c$. (where c is a constant)

$t=0, s=0 \therefore c=0$

$f = \text{const}$

$\therefore \frac{dv}{dt} = f$

$\therefore v = ft + C$

~~Let~~ $c = 0$.

$\therefore s = \frac{t^3}{3} - \frac{t^5}{20}$.

Important N.B. $\boxed{f = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = v\frac{dv}{dx}}$

$\frac{d}{dx}(\tfrac{1}{2}v^2) = \frac{d}{dv}(\tfrac{1}{2}v^2)\cdot\frac{dv}{dx} = v\frac{dv}{dx}$

$\therefore$ it follows that ~~here~~ $\int v\frac{dv}{dx}\,dx = \tfrac{1}{2}v^2 + c$

to obtain the formula $v^2 = u^2 + 2fs$ where algebra or $v\frac{dv}{ds} = f$.

integrating wrt s

$\tfrac{1}{2}v^2 = fs + c$ (constant).

when s is 0 $v = u$

$\therefore \tfrac{1}{2}u^2 = c$

$\therefore \tfrac{1}{2}v^2 = fs + \tfrac{1}{2}u^2$

$\therefore v^2 = u^2 + 2fs$.

Easter Term. 1962.

Powers of the Natural numbers and Mathematical Induction

Ex. 21d 21e

22 v) $u_n = 5$.

$\therefore$ sum of n ~~terms~~ Terms $= \sum_{r=1}^{n} u_r = \sum_{r=1}^{n} 5$

$= 5\sum_{r=1}^{n} 1 = 5n$.

viii) $u_n = 3n^2 + 2n + 1$

$\therefore$ Sum to n Terms $= \sum_{r=1}^{n}(3n^2 + 2n + 1)$.

$$= 3\sum_{r=1}^{n} n^2 + 2\sum_{r=1}^{n} n + \sum_{r=1}^{n} 1.$$

$$= \tfrac{1}{2} n(n+1)(2n+1) + n(n+1) + n$$

$$= n\left(\tfrac{1}{2}(2n^2+3n+1) + n+1+1\right).$$

$$= n\left[n^2 + \tfrac{5n}{2} + \tfrac{5}{2}\right]$$

$$= \tfrac{1}{2} n\left(2n^2 + 5n + 5\right).$$

26vi) $1^2 + 4^2 + 7^2 + 10^2 + 13^2 + \ldots\ldots + n$th Term

nth Term $= (1 + (n-1)3)^2$

$$= (3n-2)^2 = 9n^2 - 12n + 4.$$

$\therefore$ Sum to n Terms $= \sum_{r=1}^{n}(9n^2 - 12n + 4)$

$$= 9\sum_{r=1}^{n} n^2 - 12\sum_{r=1}^{n} n + 4\sum_{r=1}^{n} 1$$

$$= \frac{3n}{2}(n+1)(2n+1) - 6n(n+1) + 4n.$$

$$= \cancel{\tfrac{1}{2}n\left[3(2n^2+3n+1) - 24n\right.}$$

$$= n\left[\tfrac{3}{2}(2n^2+3n+1) - 6n - 6 + 4\right].$$

$$= n\left(3n^2 + \tfrac{9n}{2} + \tfrac{3}{2} - 6n - 6 + 4\right).$$

$$= n\left(3n^2 - \tfrac{3n}{2} - \tfrac{1}{2}\right)$$

$= \frac{n}{2}(6n^2-3n-1)$

b) Sum to $2n$ terms $= \sum_{r=1}^{2n}(9r^2-12r+4)$

$$= 9\sum_{r=1}^{2n} r^2 - 12\sum_{r=1}^{2n} r + 4\sum_{r=1}^{2n} 1$$

$$= \frac{9}{6}(2n)(2n+1)(4n+1) - \frac{12}{2}(2n)(2n+1) + 4(2n)$$

$$= n\left[24n^2+18n+3-24n-12+8\right]$$

$$= n\left(24n^2-6n-1\right)$$

Example 8. To Prove.

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\dots+\frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}.$$

Proof.

When $n=1$ L.H.S. R.H.S. $= \frac{1}{3+1} = \frac{1}{4}$ which is true.

If the result is true when $n=k$.

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\dots\dots+\frac{1}{(3k-2)(3k+1)} = \frac{k}{3k+1}$$

adding next term of series to each side

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\dots+\frac{1}{(3k-2)(3k+1)}+\frac{1}{[3(k+1)-2][3(k+1)+1]}$$

$$= \frac{k}{3k+1}+\frac{1}{[3(k+1)-2][3(k+1)+1]}$$

$$= \frac{k}{3k+1}+\frac{1}{(3k+1)(3k+4)}.$$

$$= \frac{(3k+4)k+1}{(3k+1)(3k+4)}$$

$$= \frac{(3k^2+4k+1)}{(3k+1)(3k+4)} = \frac{\cancel{(3k+1)}(k+1)}{\cancel{(3k+1)}(3k+4)}$$

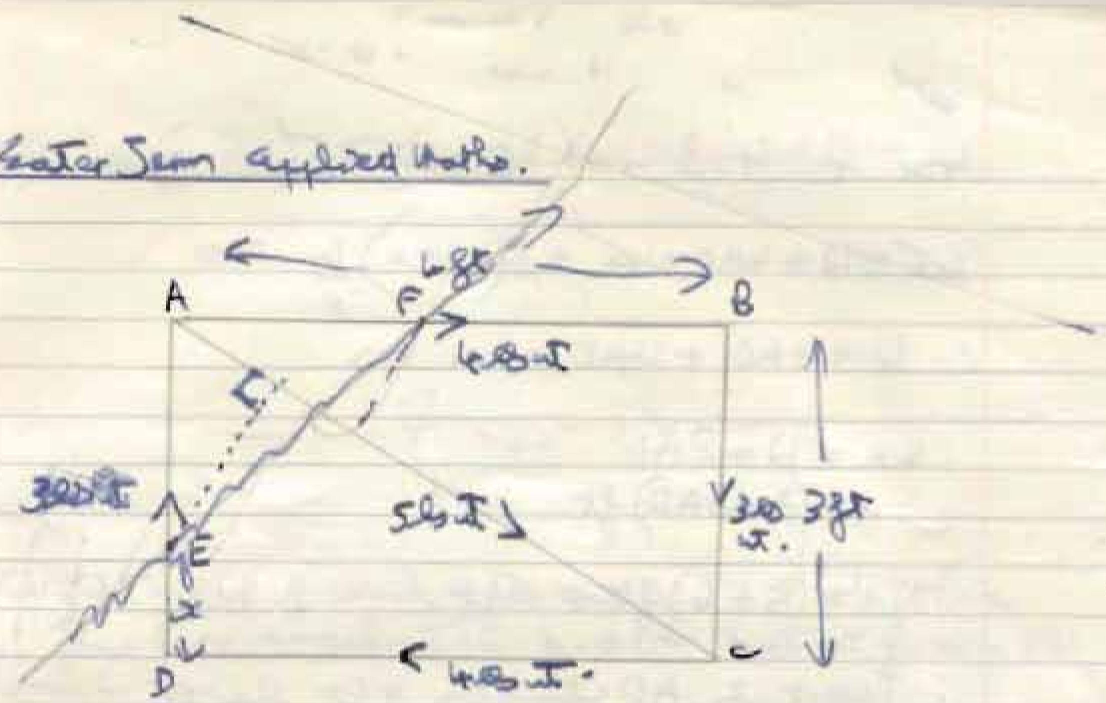
$$= \frac{k+1}{3k+4} = \frac{k+1}{3(k+1)+1} = \frac{n}{3n+1} \quad \text{let } n = k+1$$

If the result is true for $n = k$, it is also true for $n = k+1$ and $n = 1, 2, 3, 4, 5$ etc., i.e. all positive integral values of n.

See me about integration

Easter Exam applied Maths.

7.



If the system is equivalent to a single force there is a resultant force which is a single force and not a couple.
Let the horizontal compont of the resultant force be X
" " vert. " " " " " Y

$$\angle ACD = \tan^{-1} \tfrac{3}{4}$$

Resolving horizontally

$$4 + 5\cos\angle ACD - 4 = X$$

$$\therefore X = 5\cos\angle ACD \text{ lbs wt} = \frac{\cancel{5}\cdot 4}{\cancel{5}} = 4 \text{ lbs wt.}$$

Resolving vertically $Y = 3 - 3 + 5\sin\angle ACD$

$$= \cancel{5}\cdot\frac{3}{\cancel{5}} = 3 \text{ lbs wt.}$$

$\therefore$ resultant force $= \sqrt{X^2+Y^2} = \sqrt{3^2+4^2} = 5$ lbs wt.

Let the resultant cut AD and AB at the points E and F.
$\therefore$ moments about these two pts will add up to zero.
Taking moments about AE if DE = x ft

36 = 4x x = 9
12 = 4x + 12 - 3x

$$4x + 4.3 + 4(AD - x) + 5(AD - x)\sin E\hat{A}C = 0.$$

$$\therefore \cancel{4x} + 12 + 4AD - \cancel{4x} + \cancel{5}(AD - x)\frac{4}{\cancel{5}} = 0$$

$$\therefore 12 + 4AD + 4AD - 4x = 0$$

$$4x = 12 + 8AD$$
$$x = (3 + 2AD)\text{ ft}$$
$$= (3 + 6)\text{ ft} = 9\text{ ft}.$$

$\therefore$ Resultant cuts AD externally 9 ft from D.

Taking moments about F,
If AF = x ft
$$3(4 - x) + 12 + 3x = 5x \sin B\hat{A}C$$

$$\therefore 12 - \cancel{3x} + 12 + \cancel{3x} = 3x$$
$$24 = 3x$$
$$x = 8.$$
$\therefore$ Resultant cuts AB externally 8 ft from A on the side remote from B

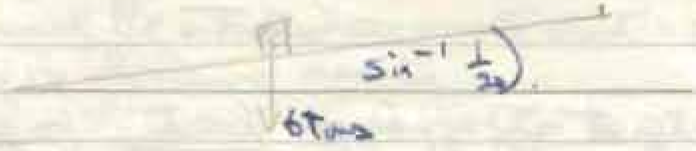
The magnitude of ~~...~~ of each of the forces is $2\frac{1}{2}$ lbs wt acting at an angle of $\tan^{-1}\frac{Y}{X}$ to AB or DC

$$= \tan^{-1}\frac{3}{4} \text{ to } AB$$

These two forces are equivalent to the resultant force of 5 & 5 acting parallel to these forces at the same angle to AB and DC; they are not a couple because the system is equivalent

To a single resultant force. Asked to find these.

5.



Constant speed of coach = 30 m.p.h. = 44 ft/sec.
∴ There is no acceleration and the resultant force on the coach is zero.
∴ resistance of ground = Tractive force − comp. of truck up the ~~sl~~

∴ If R is resistance of ground, T the tractive force (in lb wt)

$$R + \frac{6\times\cancel{2240}^{112}}{\cancel{20}} = T \quad - ①.$$

If the horsepower of the truck is H.
then $H = \frac{T\,44}{\cancel{550}}$ ft lbs wt/sec.

$$\therefore T = \frac{H}{44} \text{ lbs wt}$$

$$\therefore \text{in ① } R + \frac{6\times\cancel{2240}^{112}}{\cancel{20}} = \frac{H}{44} \quad — ①$$

When travelling on level ground
resistance = new tractive force
new tractive force = R ~~..~~ = $\frac{H}{88}$ — ②.

$\therefore$ in ① $\cancel{\frac{44}{88}}\ R + 112\times 6 = \overset{2}{\cancel{\frac{88}{44}}}R$

$\therefore\ R = 112\times 6\ \cancel{\text{cwt}}$

$= 6\text{ cwt.}$

$\therefore$ R per ton wt of coach $= \dfrac{6\text{ cwt}}{20\times 6\text{ cwt}} = \dfrac{1}{20}$ ~~of 6 tons~~

R is 6 cwt for 6 tons
$\therefore$ R is 1 cwt for 1 ton

$= 112$ lb per ton

$\therefore$ Horse power of coach $= 88\times 112$ ft lb wt/sec

$= \dfrac{\overset{4}{\cancel{88}}\times 112}{\cancel{550}}$ H.P.

$= \dfrac{448}{25}$ H.P.

60 40 88 44 = 60.

$= 17{\cdot}9$ H.P.

When the coach is running at 45 m.p.h.
By Newton's ~~law of motion~~ 2nd law
to coach is on the level

$32\times\dfrac{88\times 112}{66} = 2240\times 6\times f$ (f is accel in ft/sec²)

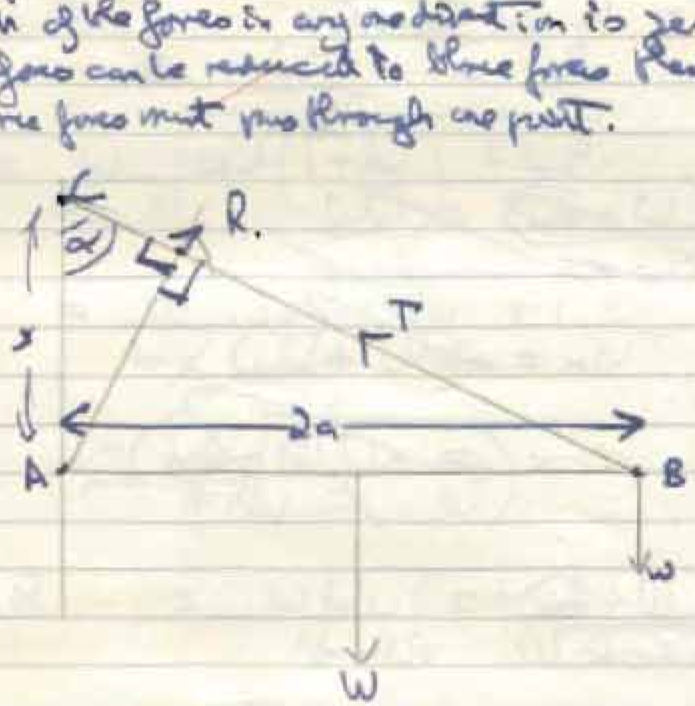
$\therefore\ f = \dfrac{32\times 88\times 112}{2240\times 6\times 66}$

$$= \frac{32 \times \cancel{88} \times \cancel{112}}{\cancel{2240} \times \cancel{5} \times \cancel{66}} = \frac{32}{90} \text{ ft}/\text{sec}^2 \text{ ans}$$

$$= \frac{16}{45} \text{ ft}/\text{sec}^2.$$

3.) The sum of the anticlockwise moments about a point is equal to the sum of the clockwise moments.
2) The sum of the forces in any one direction is zero.
3) If the forces can be reduced to three forces then for equilibrium these three forces must pass through one point.

The sum of couples in two directions must be zero



Let the tension in the string CB be T ~~...~~
Let the reaction at A be R. ~~...~~
Let AC = x.
~~...~~
Taking moments about A by the Principle of Moments
aW + 2aw = T x Sin∠ACB

$$\therefore x(W+2w) = T \times \frac{2x}{\sqrt{4a^2+x^2}}$$ — (1).

~~...~~ $\therefore T = \frac{\sqrt{4a^2+x^2}\,(W+2w)}{2x}$

Taking moments about C. by the Principle of Moments

$$Wa + 2aw = R\,x\cos\alpha \quad \text{(as in diagram)}$$

$$\therefore R = \frac{a(W+2w)}{x\cos\alpha} \quad \text{— (1)}$$

Taking moments about B.

$$Wa = R\,2a\sin\alpha.$$

$$\therefore R = \frac{W}{2\sin\alpha} \quad \text{— (2)}$$

$$\cancel{\therefore \frac{a(W+2w)}{x\cos\alpha} = \frac{W}{2\sin\alpha}}$$

$$\cancel{\therefore Wx = a(W+2w)}$$

$$\cancel{\therefore x = \frac{a(W+2w)}{W}}$$

$$\therefore \frac{a(W+2w)}{x\cos\alpha} = \frac{W}{2\sin\alpha}$$

$$\therefore 2a\sin\alpha\,(W+2w) = x\cos\alpha\,W$$

$$\therefore x = \frac{2a\tan\alpha\,(W+2w)}{W}.$$

$$= \frac{2a}{x}\cdot\frac{2a\,(W+2w)}{W}.$$

$$\therefore x^2 = 4a^2\left(\frac{W+2w}{W}\right) = 4a^2\left(1+\frac{2w}{W}\right)$$

$$\therefore x = 2a\sqrt{\left(1+\frac{2w}{W}\right)}$$

$$\therefore AC = 2a\sqrt{\left(1+\frac{2w}{W}\right)}$$

$$R = \frac{W}{2\sin\alpha} = \frac{W\sqrt{AC^2+4a^2}}{2.2a}$$

$$= \frac{W}{4a}\sqrt{4a^2\left(1+\frac{2w}{W}\right)+4a^2}$$

$$= \frac{W}{4a}.2a\sqrt{1+\frac{2w}{W}+1}$$

$$= \frac{W}{2}\sqrt{2\left(1+\frac{w}{W}\right)}$$

$$\therefore \text{Maximum } R = W\sqrt{\frac{1}{2}\left(1+\frac{w}{W}\right)}$$

8. Let the mass of the first ball be m
the mass of the 2nd ball is 2m
~~Consider a ball of mass m~~
~~$\cancel{\ldots - u^3 = 2\ldots}$~~
considering ball of mass m.
vel of ball at greatest height is 0 ft/sec
eq $v = u + ft$
$32t = 160$
Time $(t_1) = \frac{160}{32} = 5$ secs

by $v^2 - u^2 = 2fs$.

$$160^2 = 2 \times 32 \times s$$

$$\therefore \text{ht } (s) = \frac{\cancel{160}^{5} \times \cancel{160}^{80}}{\cancel{2 \times 32}} = 400 \text{ ft}$$

✓

<u>Considering ball of mass m.</u>

by $v^2 - u^2 = 2fs$.

$$\text{vel}^2 (V_1) = 2 \times 32 \times 64$$

$$\therefore \text{vel } v_1 = \sqrt{2 \times 32 \times 64}$$

$$= 64 \text{ ft/sec.}$$

✓

By the law of conservation of momentum.
If V be vel of m mass if V_1 be vel of 2m mass
If u be new vel of m mass and u_1 be new vel of 2m mass

$$64 (2\cancel{m}) = \cancel{m} \times u + 2\cancel{m} \times u_1$$

$$\therefore 128 = u + 2u_1 \quad -②$$

2m
64
m

By Newton's Exp. Law. if e be the coeff of elasticity

$$\frac{64}{u_1 - u_1} = -e \qquad \frac{u - u_1}{64} = -e$$

✓

$$\therefore 64 = \cancel{2}(u_1 - u)$$

$\therefore u - u_1 = -32$

$$\therefore 128 = u_1 - u.$$

adding

$$256 = 3u_1$$

$\therefore u_1 = \cancel{8}\,\frac{256}{3}$ ft/sec

$\cancel{128 = u + 2u_1 - ①}$

$\cancel{\text{adding } ② \quad 256 = 2u_1 - 2u - 8 \quad -2u + 2u_1}$

$\cancel{\text{subtracting } 128 = -3u.}$

$\therefore u = -\frac{128}{3}$ ft/sec.

$\therefore u - u_1 = -32$

$u_1 - u = 32 \quad ②$ ✓ $\qquad 2u + u = 128$ $\quad 2u_1 - 2u = 64$

$2u_1 + u = 128. \quad ①$

adding $3u_1 = 160$

$\therefore u_1 = \frac{160}{3}$ ft/sec

sec

subtracting $\quad -u = 64$

$\therefore u = -64$ ft/sec.

$\therefore$ vel of train $m = 64$ ft/sec.

dist to travel $= 400$ ft.

accelerat $m = t$.

by $s = ut + \frac{1}{2}gt^2$

$25\ \cancel{100}\ \cancel{400} = \cancel{16}\ \cancel{64}t + \cancel{16}\,t^2 \qquad t^2 + 4t - 25 = 0.$

$\therefore 25 = 4t + t^2.$

$\therefore t = \frac{-4 \pm \sqrt{16+100}}{4}$

$= \frac{-4 \mp \sqrt{116}}{2} = \frac{10.77-4}{4} = \frac{6.77}{2} = 1.69$ secs

$= 3.38$ secs

Sender moves

by $S = ut + \frac{1}{2}gt^2$

$400 = \frac{160}{3}t + 16t^2$

$\therefore 100 = \frac{40}{3}t + 4t^2$

$0 = \frac{10}{3}t + t^2 - 25$

$\therefore t = \frac{-\frac{10}{3} \pm \sqrt{\frac{100}{9} + 100}}{2}$

$= \frac{-3\frac{1}{3} \pm \sqrt{111\frac{1}{9}}}{2} = \frac{-3.33 + 10.54}{2}$

$= \frac{7.21}{2} = 3.61$ secs

$\therefore$ difference $= 0.37$ secs + that delay he waits to take up after second actuation to

$0 = \frac{160}{3}t - 16t^2$

$16t^2 - \cancel{160}t = 0$. $\therefore$ difference in time $= 0.37$ sec.

~~$t^2 - \frac{10t}{3} = 0$~~

~~$t\left(t - \frac{10}{3}\right) = 0$~~

1. By $v^2 - u^2 = 2fs$.

If f = retardation of train

$$f = \frac{88 \times 88}{2 \times 968 \times 3} \text{ ft/sec}^2 = \frac{44 \times 88}{968 \times 3} = \frac{4}{3} \text{ ft/sec}^2$$

If final vel is 15 m.p.h. = 22 ft/sec

time is given by $v = u + ft$.

$88 \times 3 = \left(\frac{88 + 22}{2}\right) t$. $\quad 88 = 22 + \frac{4t}{3}$

$\therefore t = \quad 66 = \frac{4t}{3}$

$t = \frac{66 \times 3}{4}$ secs

If acceleration is uniform when initial vel is 88 ft/sec.

By $S = \left(\frac{v+u}{2}\right) t$.

$1232 \times 3 = \left(\frac{88 + 22}{2}\right) t \quad \therefore t = \frac{1232 \times 3}{55}$ secs

$= \frac{112 \times 3}{5}$ secs

$\therefore$ Total time $= \frac{66 \times 3}{4} + \frac{112 \times 3}{5}$ secs

If train had not retarded

at 15 m.p.h dist covered already

$= \frac{(v+u)}{2} t$ ft

$= 55 \times \frac{66 \times 3}{4}$ ft.

$\therefore$ Total dist travelled $= \left(55 \times \frac{66 \times 3}{4} + 1232 \times 3 \right)$ ft

$\therefore$ time taken $= \left(55 \times \frac{66 \times 3}{4} + 1232 \times 3 \right) \frac{1}{88}$ secs

$= \frac{\overset{5}{\cancel{55}}}{\underset{8}{\cancel{88}}} \times \frac{\overset{33}{\cancel{66}} \times 3}{\underset{2}{\cancel{4}}} + \frac{\overset{112}{\cancel{1232}} \times 3}{\underset{8}{\cancel{88}}}$ secs

$= \frac{99 \times 5}{16} + \frac{112 \times 3}{8}$ secs

$\therefore$ time lost $= \left(\frac{66\times3}{4} + \frac{112\times3}{5}\right) - \left(\frac{99\times5}{16} + \frac{\overset{14}{\cancel{112}}\times3}{8}\right)$

$= \frac{3\cdot99}{2} + \frac{336}{5} - \frac{99\times5}{16} - 14\times3$

$= \frac{99}{2} + \frac{336}{5} - \frac{99\times5}{16} - 42$

$= 49\frac{1}{2} + 67\frac{1}{5} - \frac{495}{16} - 42$

$= 116\frac{2}{10} - 30\frac{1}{16} - 42$

$= 116\frac{2}{10} - 72\frac{1}{16}$

$= 44\frac{112}{160} - \frac{10}{160}$

$= 44\frac{102}{160}$ secs.

$= \frac{3}{4}$ min. approx.

~~[illegible]~~

time taken for ball to hit floor

vertically ~~$s = \left(\frac{u+v}{2}\right)$~~

$s = ut + \frac{1}{2}gt^2 =$

$\therefore 4 = 16t^2 \quad \therefore t^2 = \frac{1}{4} \quad t = \frac{1}{2}$ sec.

~~Ex~~ 2nd Lec Train moves

If ball be dropped at the beginning of the deceleration period. $f = \frac{4}{3}$. $v = 88$ ft/sec

$$t = \frac{1}{2} \text{ sec}$$

By $v = u + ft$

$$88 = u + \frac{2}{3}$$

$$\therefore u = 88 - \frac{2}{3} = 87\tfrac{1}{3} \text{ ft/sec.}$$

$$\cancel{\therefore \text{by } s = \left(\frac{v+u}{2}\right)t}$$

$$\cancel{s = \left(\frac{88 + 87\frac{1}{3}}{2}\right)\frac{1}{2}} = \cancel{\frac{175\frac{1}{3}}{2} \times \frac{1}{2}}$$

$$= \cancel{\frac{526}{12} \text{ ft}}$$

$$\cancel{= 4}$$

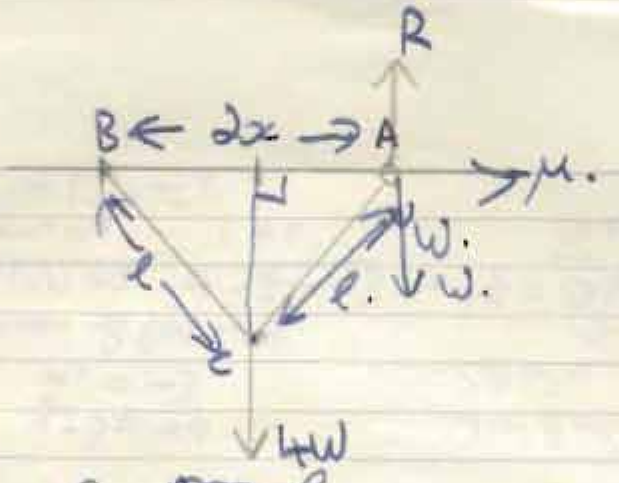
by $s = ut + \frac{1}{2}ft^2$.

$$s = 44 - \frac{1}{6} \cdot \frac{1}{4} = 43\tfrac{5}{6} \text{ ft.}$$

$\therefore$ ball will land $43\frac{5}{6}$ ft down the floor of the carriage if ball was dropped from at. But ball is travelling at 88 ft/sec $\therefore$ in $\frac{1}{2}$ sec it travels $43\frac{5}{6}$ ft horizontally

$\therefore$ ball lands ~~44 ft~~ at the same place vertically beneath the place it is dropped from.

4.



~~Resolving~~ let the normal reaction at A be R.

$\therefore$ Frictional force $= \mu W$.

~~Taking moments about B.~~ Taking moments about B

~~4Wx~~ $4Wx + 2xW = 2x R$.

$\therefore R = \frac{3W}{\cancel{x}}$

?!

~~Taking moments about C.~~

$$Wx + \mu R\sqrt{l^2-x^2} = Rx$$

$$\therefore \cancel{Wx} + \frac{3W}{\mu}\sqrt{l^2-x^2} = \frac{3Wx}{\cancel{x}}$$

$$x + \frac{3\sqrt{l^2-x^2}}{\mu} = \frac{3x}{\cancel{x}}$$

$$\frac{3\sqrt{l^2-x^2}}{\mu} = x\left(\frac{3}{\mu} - 2\right) \quad 2x$$

$\left(\frac{3}{\mu}-1\right)\left(\frac{3}{\mu}-1\right)$

$$\therefore \frac{9(l^2-x^2)}{\mu^2} = 4x^2\left(\frac{9}{\mu^2} - \frac{6}{\mu} + 1\right).$$

on

TRIGONOMETRY

P. 194 Nos 8 – 12

8. $2\cos^2\theta + \sin\theta = 1$ — ①

$\cos^2\theta + \sin^2\theta = 1$ — ②

from ① $\sin\theta = 1 - 2\cos^2\theta$.

$\therefore$ in ②. $\cos^2\theta + (1 - 2\cos^2\theta)^2 = 1$

$\cos^2\theta + \cancel{1} - 4\cos^2\theta + 4\cos^4\theta = \cancel{1}$

$4\cos^4\theta - 3\cos^2\theta = 0$.

$\cos^2\theta(4\cos^2\theta - 3) = 0$.

either $\theta = 90°$

or $\cos\theta = \pm\sqrt{\frac{3}{4}}$

$\theta = 30°, 150°, 90°$

θ cannot be $30°$ or $150°$ $\because$ equation ① does not hold.

$\therefore \theta = 90°$.

9. $2\operatorname{cosec}^2\theta = 3\cot\theta + 1$

~~$2\operatorname{cosec}^2\theta$~~ ~~$3/c$~~

$2(1 + \cot^2\theta) = 3\cot\theta + 1$.

$2 + 2\cot^2\theta = 3\cot\theta + 1$

$2\cot^2\theta - 3\cot\theta + 1 = 0$.

$(2\cot\theta - 1)(\cot\theta - 1) = 0$.

either $\cot\theta = \frac{1}{2}$ i.e. $\theta = 63° 26'$

or $\cot\theta = 1$, i.e. $\theta = 45°$.

10. $\cos^2\theta + \cos\theta = \sin^2\theta$.

$\cos^2\theta + \cos\theta = 1 - \cos^2\theta$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$.

either $\cos\theta = \frac{1}{2}$, $\theta = 60°$

or $\cos\theta = -1$, $\theta = 180°$

11. $\tan^2\theta = \sec\theta + 1$

$\sec^2\theta - 1 = \sec\theta + 1$

$\sec^2\theta - \sec\theta - 2 = 0.$

$(\sec\theta - 2)(\sec\theta + 1) = 0$

either $\sec\theta = 2$, $\theta = 60°$

or $\sec\theta = -1$ $\theta = 180°.$

12 $1 + \sin\theta\cos^2\theta = \sin\theta.$

$1 + \sin\theta(1 - \sin^2\theta) = \sin\theta.$

$1 + \cancel{\sin\theta} - \sin^3\theta = \cancel{\sin\theta}$

$1 - \sin^3\theta = 0$

$\sin^3\theta = 1$

$\sin\theta = 1$

$\theta = 90°$

P 201 Nos 15(ii, iii), 13(i, ii, iii, iv)

15(ii) $\cos C + \cos B\cos A = \sin A\sin B$

$\therefore \cancel{\cos C} \cos B\cos A - \sin B\sin A = -\cos C$

$\therefore \cancel{\cos B} \cancel{\cos C}$ $\cos(B+A) = -\cos C$

But $A+B+C = 180°$

$\therefore B = 180° - A + C$

$\therefore \cos B = -\cos(A+C)$

$C = 180° - (A+B)$

$C = -\cos(A+B)$

$\therefore$ L.H.S. = R.H.S

iii) $\sin A - \cos B\sin C = \sin B\cos C$

$\sin A = \sin B\cos C + \cos B\sin C$

$= \sin(B+C)$

$\boxed{\text{But } A+B+C = 180°}$

$= \sin(180° - \cancel{(B+A)})$

$= \sin A$

= R.H.S.

3 i) $y = x$, $y = 2x$

$\therefore m = 1$, $m_1 = 2$.

Let angle between lines be θ

$\tan\theta = \dfrac{m \sim m_1}{1 + m_2 m_1}$

$= \dfrac{1 \sim 2}{1 + 2} = \dfrac{-1}{3}$

~~i.e. θ = ...°~~

i.e. $\theta = 18^\circ 26'$

or $180^\circ - 18^\circ 26' = 161^\circ 34'$

ii) $y = 2x + 1$, $y = 3x - 2$

$m = 2$ $m_1 = 3$

Let the angle between the lines be θ.

$\tan\theta = \dfrac{m_1 \sim m_2}{1 + m_1 m_2} = \dfrac{3 - 2}{1 + 6} = \frac{1}{7} = 0.1429$

$\theta = 8^\circ 8'$

or $171^\circ 52'$

iii) $2y - x = 1$ $3y = x - 1$

$m = \frac{1}{2}$ $m_1 = \frac{1}{3}$

Let the angle between the lines be θ.

$\tan\theta = \dfrac{m_1 \sim m_2}{1 + m_1 m_2} = \dfrac{\frac{1}{2} - \frac{1}{3}}{1 + \frac{1}{2}\cdot\frac{1}{3}} = \frac{1}{6}\cdot\frac{6}{7} = \frac{1}{7} = 0.1429$

$\therefore \theta = 8^\circ 8'$ or $\theta = 171^\circ 52'$

iv) $y + 2x = 0$, $y + 3x = 0$

$m = -2$ $m_1 = -3$

Let the angle between the lines be θ.

$\therefore \tan\theta = \dfrac{m_1 \sim m_2}{1 + m_1 m_2} = \dfrac{-2 - -3}{1 + 6} = \frac{1}{7} = 0.1429$

$\therefore \theta = 8^\circ 8'$ or $171^\circ 52'$