Grammar School,

PONTARDAWE.

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Form: VI b

Subject: Mathematics (applied)

E.N.S., LONDON.

P16 Examples III nos 2, 5, 16

2. Vel B rel A + vel A = Vel B.

∴ Vel B rel A = Vel B − Vel A.

Set ab rep vel A in mag. + dir.

" ac " " B in " " "

Then cb rep " B rel A in mag + dir

∴ Vel A_1 = $\sqrt{10^2 + 15^2}$

$= \sqrt{100 + 225}$

$= \sqrt{325}$ = 18.028 m.p.h at ∠ $\tan^{-1}$ ~~...~~ $\frac{3}{2}$ to dirⁿ of b...

This is 2nd eqn

Sequels A & B reversed.

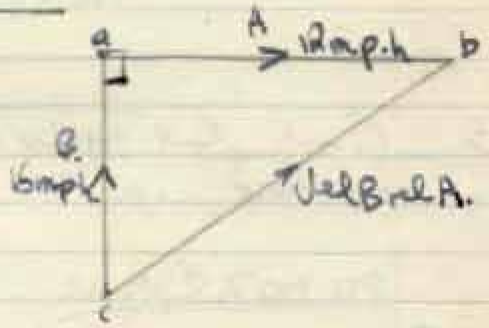
5. Vel B rel A + vel A = Vel B.

∴ Vel B rel A = Vel B − Vel A

Set ab rep vel A in mag + dir

" ac " " B in " " "

Then cb " " B rel A in mag + dir.



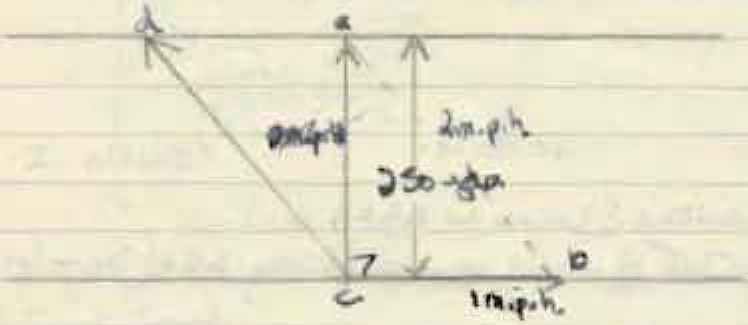
∴ Vel B rel A $= \sqrt{12^2 + 16^2}$

$= \sqrt{144 + 256} = \sqrt{400}$ = 20 m.p.h.

at ∠ $\tan^{-1} \frac{12}{16}$

= ∠ $\tan^{-1} \frac{3}{4}$ to vel of boat heading due north

16.



Result of man's vel 2 m.p.h in dirⁿ dc with vel of x m in dirⁿ cb is vel of ~~...~~ ab. at ∠ acb to bank cb of given

∴ ab = vel of man in mag + dir.

cb = vel of river in magt dir
then ac = resultant vel. in magt. r dir.
$ac = \sqrt{ab^2 - bc^2}$ m.p.h
$= \sqrt{4-1}$
$= \sqrt{3}$ = 1.7321 m.p.h

dist = 250 yds.

$= \frac{250}{1760}$ yds

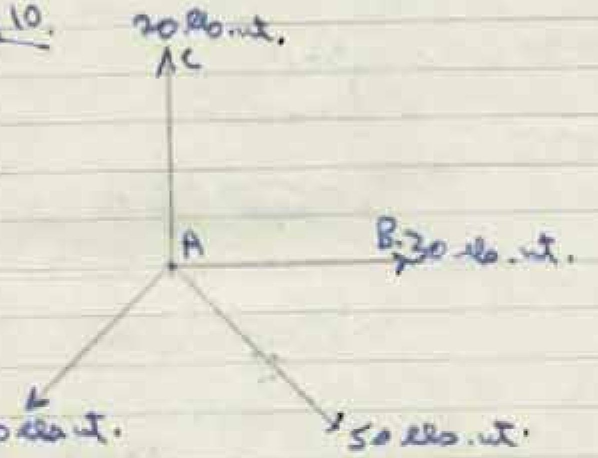
$\therefore$ time taken $= \frac{25 \times 60}{176 \times 1.7321}$ mins. = 4.921 mins

No	Log
25	1.3979
60	1.7782
	3.1761
176	2.2455
1.732	0.2385
	2.4840
	0.6921.

10/10. Good. but rather than say Vel A vel B say vel boy vel boat etc.

P17 Nos 7, 8, 9, 10.

7.



Let resultant of forces in dir AB = X lbs wt
Resultant of sum of components along AB = (30 − 40 cos 45 + 50 cos 45) lbs. wt.

$= 30 + 5\sqrt{2}$.
$= 30 + 5 \times 1.414$
$= 37.07$.

lbs wt.

Resultant of sum of components along AC $= (20 - 40 \sin 45 - 50 \sin 45)$ lbs. wt.

$= 20 - 45\sqrt{2}$.

$= -43.63$.

$\therefore$ resultant force at A $= \sqrt{\left\{(30 - 5\sqrt{2})^2 + (20 - 45\sqrt{2})^2\right\}}$

$= \sqrt{\{37.07^2 + 43.63^2\}}$.

$= \sqrt{3278}$

$= 57.25$ lbs. wt.

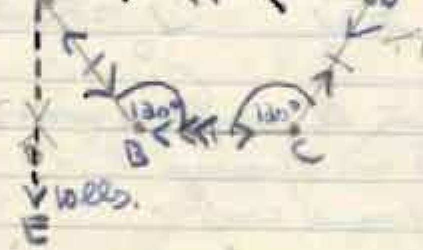
No	Log
43.63	1.6398
37.07	1.5690.
	0.0708

at an angle of $\tan^{-1} + \dfrac{43.63}{37.07}$ to AB

$=$ ~~[illegible]~~ $49° 39'$ to AB S

$\therefore$ direction of resultant is $49° 39'$ S of ~~[illegible]~~ (because AC is negative resultant of components along AC is in the opposite direction, but equal to it as A is in equilibrium).

8.



Let tension be T lbs. wt. acting on pegs in diagram

As the pegs are smooth the tension is the same throughout the string and equal to 10 lbs. wt.

$\therefore$ Force on peg B is the resultant of 10 lbs wt acting in dir^n AB and 10 lbs wt in dir^n BC.

If Force on peg B = P lbs wt.

Then $P^2 = 2.10^2 + 2\,10^2 \cos 120^\circ$

$= 200 - 100$

$= 100$

$\therefore P^2 = 100.$

$\therefore$ Resultant force on peg B = 10 lbs wt.

Similarly resultant force on peg C = 10 lbs wt

If the resultant force on peg C = X lbs wt.

Then $x^2 = 2\,10^2 + 2\,10^2 \cos 60^\circ$

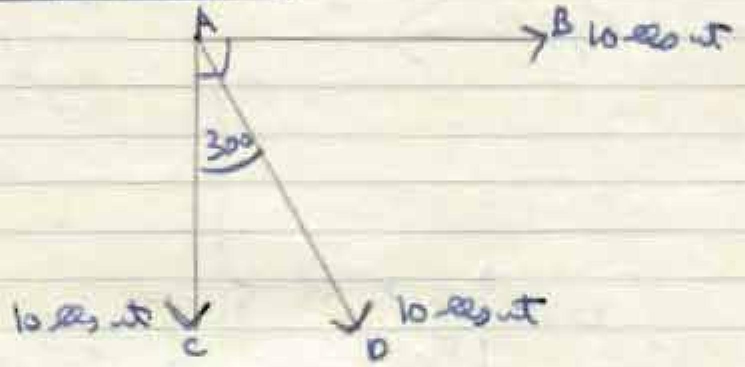
$= 200 + 100$

$= 300.$

$\therefore$ force on peg C = $\sqrt{300}$ lbs wt or $10\sqrt{3}$ lbs wt.

Mention dirn of Result. (bisecting angle in above cases)

<u>To find result. force on A.</u>



If $\overrightarrow{AC}$ reps. 10 lbs wt, the weight in magn + dirn

If $\overrightarrow{AB}$ " horizontal arm of hexagon " " " "

If $\overrightarrow{AD}$ " one of the diagonal arms " " " "

resultant sum of components along AB

$= (10 + 10\cos 60)$ lbs wt $= 15$ lbs wt.

Result sum of components in dir^n perp AB

$$= (-10 - 10\sin 60)\text{ lbs wt.}$$

$$= \left(-10 - \frac{\sqrt{3}}{2}.10\right)\text{ lbs wt.}$$

$$= -18.66\text{ lbs wt.}$$

$\therefore$ Resultant force on A $= \sqrt{\{15^2 + (-18.66)^2\}}$ lbs wt

$$= \sqrt{\{225 + 348.2\}}\text{ lbs wt.}$$

$$= \sqrt{573.2}\text{ lbs wt}$$

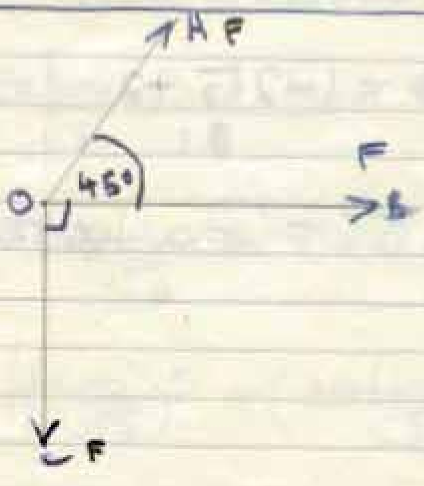
$$= 23.941\text{ lbs wt.}$$

No	Log
18.66	1.2709
15	1.1761
	0.0948

at an angle of $\tan^{-1}\frac{18.66}{15}$ to AB

$= 51°\ 12'$ to AB. (i.e. E. 51° 12' S. of AB)

9.



Resultant sum of components along OB $= F + F\cos 45°$

$$= F\left(1 + \frac{1}{\sqrt{2}}\right)$$

resultant sum of components $\perp$ OB $= F\sin 45^\circ - F$

$$= F(\sin 45^\circ - 1) = F\left(\frac{1}{\sqrt{2}} - 1\right)$$

$\therefore$ resultant force $= \sqrt{\left\{\left(F\left(1+\frac{1}{\sqrt{2}}\right)\right)^2 + \left(F\left(\frac{1}{\sqrt{2}}-1\right)\right)^2\right\}}$

$$= \sqrt{\left\{F^2\left(1+\frac{\cancel{2}}{\cancel{\sqrt{2}}}+\frac{1}{2}\right) + F^2\left(\frac{1}{2}-\frac{\cancel{2}}{\cancel{\sqrt{2}}}+1\right)\right\}}$$

$$= \sqrt{3F^2} = \sqrt{3}\,F$$

Dir^n to OB $= \tan^{-1} \dfrac{\cancel{F}(\sin 45^\circ - 1)}{\cancel{F}(\cos 45^\circ + 1)} = \dfrac{\sin 45^\circ - 1}{\cos 45 + 1}$

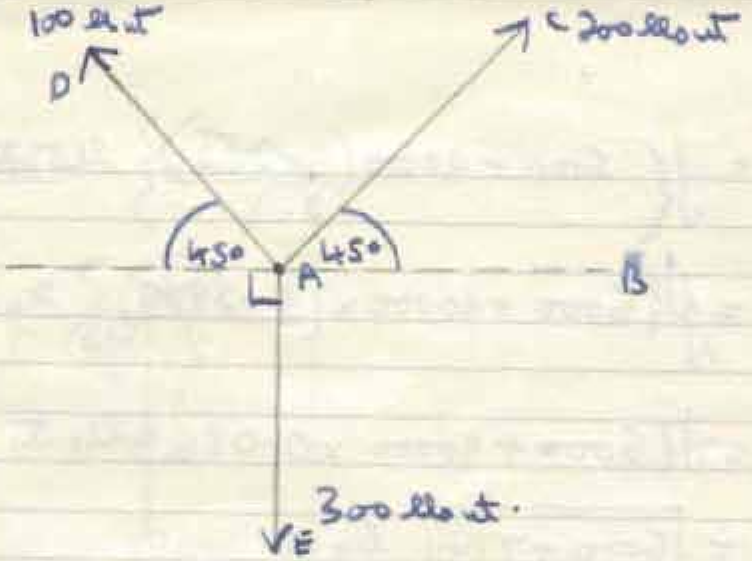
$$= \left(\frac{1}{\sqrt{2}} - 1\right) \div \left(\frac{1}{\sqrt{2}} + 1\right)$$

$$= \frac{1-\sqrt{2}}{\cancel{\sqrt{2}}} \times \frac{\cancel{\sqrt{2}}}{1+\sqrt{2}} = \frac{1-\sqrt{2}}{1+\sqrt{2}}$$

$$= \frac{1-2\sqrt{2}+2}{1} = (3-2\sqrt{2})$$

$\therefore$ result force is $\sqrt{3}\,F$ at an angle $\tan^{-1}(3-2\sqrt{2})$ to OB

10



Resultant sum of components in direction AB

$$= (200 \cos 45° - 100 \cos 45°) \text{ lbs wt.}$$

$$= \frac{100}{\sqrt{2}} \text{ lbs wt.}$$

Resultant sum of components in dirⁿ $\perp$ AB

$$= (200 \sin 45° + 100 \sin 45° - 300) \text{ lbs wt.}$$

$$= \frac{300}{\sqrt{2}} - 300 \text{ lbs wt.}$$

$$= \frac{300(1-\sqrt{2})}{\sqrt{2}} \text{ lbs wt.}$$

$$\therefore \text{Resultant force} = \sqrt{\left\{\left(\frac{100}{\sqrt{2}}\right)^2 + \left(\frac{300(1-\sqrt{2})}{\sqrt{2}}\right)^2\right\}} \text{ lbs wt}$$

$$= \sqrt{\left\{\frac{10000}{2} + \frac{90{,}000(1-2\sqrt{2}+2)}{2}\right\}} \text{ lbs wt.}$$

$$= \sqrt{\left\{ 5000 + \frac{90000}{2}\left(3 - 2\sqrt{2}\right) \right\}} \text{ lbs wt}$$

$$= \sqrt{\left\{ 5000 + 90000 \times \left(\frac{3 - 2.828}{2}\right) \right\}} \text{ lbs wt}$$

$$= \sqrt{\left\{ 5000 + 90000 \times 0.086. \right\}} \text{ lbs wt.}$$

$$= \sqrt{5000 + 7740} \text{ lbs wt}$$

$$= \sqrt{12740} \text{ lbs wt.}$$

$$= 112.87 \text{ lbs wt. ans.}$$

at angle of $\tan^{-1} \dfrac{\cancel{300}^{\,3}\left(1 - \sqrt{2}\right)}{\cancel{\sqrt{2}}} \times \dfrac{\sqrt{2}}{100}$

$$= 3\left(1 - \sqrt{2}\right) \text{ to AB}$$

$$\cancel{=} 3\left(1 - \sqrt{2}\right) \text{ N } \cancel{E}$$

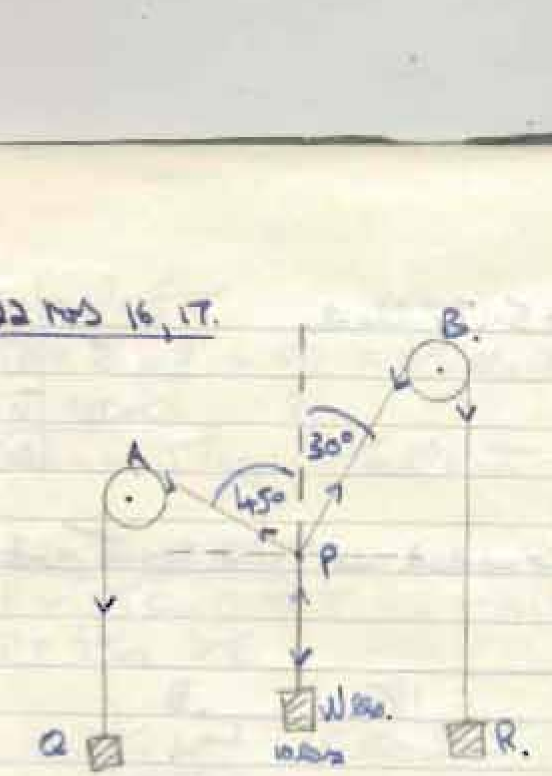
$$= \text{E } 3\left(\sqrt{2} - 1\right) \text{ S.}$$

$\therefore$ resultant force is 112.87 lbs wt. at an angle $\tan^{-1}\left(\text{E}.3\left(\sqrt{2}-1\right)\text{S}\right)$.

19/20

P22 Nos 16, 17.

16



Since the pulleys are assumed ~~weight~~ frictionless the weights Q and R can be represented by the tension in the strings PA and PB.

∴ If Q, and R are the weights respectively
then Q, and R also act as the tensions in the strings AP and PB respectively
the weight W acts through the string PW.

The point P is in equilibrium under the action of the three forces Q, R and W acting through P.

∴ By Lami's theorem

$$\frac{R}{\sin\angle APW} = \frac{Q}{\sin\angle BPW} = \frac{W}{\sin\angle APB}.$$

If a 10 lb wt is added to W., corresponding adjustments must be made to the forces Q and R in accordance to Lami's theorem as applied above

$$\therefore \frac{R}{\sin 135^\circ} = \frac{Q}{\sin 150^\circ} = \frac{W}{\sin 75^\circ} = \frac{10}{\sin 75}$$

$\sin 135^\circ = \sin 45^\circ = \frac{1}{\sqrt{2}}$.

$\sin 150^\circ = \sin 30^\circ = \frac{1}{2}$

$\sin 75^\circ =$ ~~[illegible]~~

$$\therefore R = \frac{\sin 45^\circ \times 10}{\sin 75^\circ}$$

$= 7.321$ lbs wt.

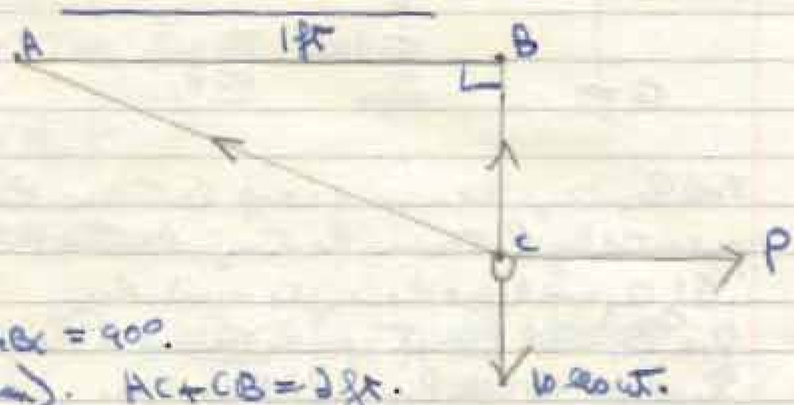
No	log
Sin45	$\bar{1}.8495$
10	1.0000
	0.8495
Sin75°	$\bar{1}.9849$
	0.8646

$Q = \frac{10 \times \frac{1}{2}}{\sin 75} = \frac{5}{\sin 75} = 5.177$ lbs wt.

No	Log
5	0.6990
Sin 75°	$\bar{1}.9849$
	0.7141

∴ additional weights to be added are 7.321 lbs wt and 5.177 lbs wt respectively

17.



In Δ ABC $\angle$ ABC = 90°.
AB = 1ft (given). AC + CB = 2 ft.
∴ AC + CB = 2 — ①
$AC^2 = 1^2 + BC^2$ (By Pythagoras' Theorem) — ②
From ① AC = 2 − CB.
In ②. $(2 - CB)^2 = 1 + BC^2$
$4 - 4CB + \cancel{CB^2} = 1 + \cancel{CB^2}$
$4CB = 3$
$CB = \frac{3}{4}$ ft. = 9"
In ① if CB = 9", AC = 2' − 9" = 1' 3" = $1\frac{1}{4}$ ft.

Since the string is assumed smooth, and also the ring, the tension is the same throughout the string
∴ resolving vertically, If $\overrightarrow{CB}$ rep tension in string CB in mag + dir
If $\overrightarrow{CA}$ " " " " CA " " "
then $\overrightarrow{CA} = \overrightarrow{CB} = T$.
∴ 10 = T cos 0 + T cos $\angle$ BCA
10 = T + T × $\frac{3}{4} \times \frac{4}{5}$

$\therefore 10 = 1\frac{3}{5}T.$

$T = 10 \times \frac{5}{8} = \frac{50}{8} = 6\frac{1}{4}$ lbs wt.

$\therefore$ Tension in string is $6\frac{1}{4}$ lbs wt.

<u>Resolving horizontally</u>

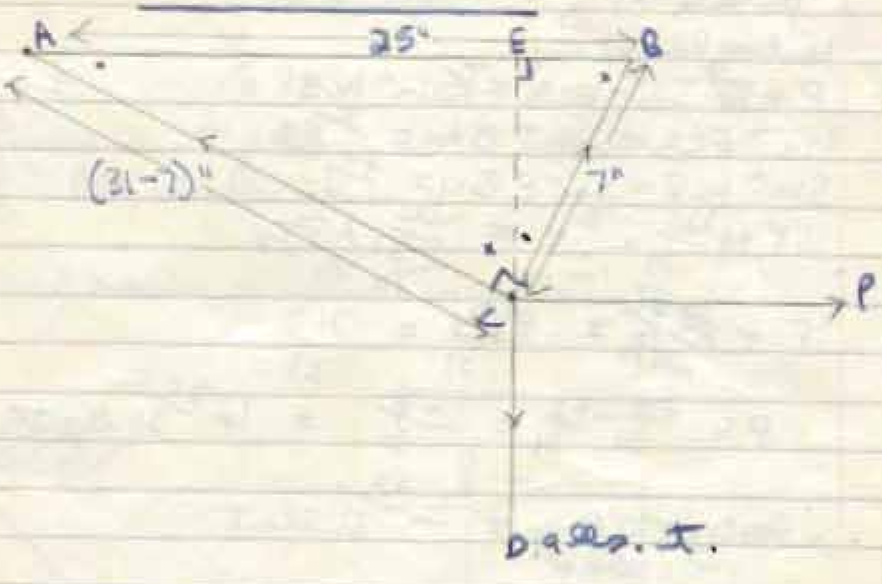
$P + T\sin\theta = T\sin\angle ACB.$

$P = 6\frac{1}{4} \times \frac{4}{5}$

$= \frac{\cancel{25}^{5}}{4} \times \frac{4}{\cancel{5}_{1}} = 5$ lbs wt.

$\therefore$ Horizontal force required is 5 lbs wt.

18.



9/10

Let P be horizontal force required.

Let T be tension in string, which is equal in strings CB and CA because the ring is assumed frictionless.

Let A, B be points from which string is suspended. Let CE be ⊥ AB.

In Δ ABC $AB^2 = AC^2 + BC^2 = 24^2 + 7^2 = 625.$

$\therefore$ By converse of Pythagoras theorem $\angle ACB = 90°$

o real

ed 9 any-

AC>BC

~~In △ABC Sin∠ABC = 24/25 = 0.96.~~
~~∴ ∠ABC = 73° 44'~~
∠BAC = ∠ECB (similar △s)
Cos∠BAC = 24/25.
∴ Cos∠ECB = 24/25.
∠ACE = ∠EBC
Cos∠EBC = 7/25.
∴ Cos∠ECA = 7/25
Resolving horizontally

$$9 \text{ lbs wt} = T \times \tfrac{7}{25} + T \times \tfrac{24}{25}$$

$$9 = T\,\tfrac{31}{25}$$

$$T = \frac{25 \times 9}{31} = \frac{225}{31} = 7\tfrac{8}{31} \text{ lbs wt.}$$

∴ Tension in string = $7\frac{8}{31}$ lbs wt.
Resolving vertically

$$P + T \sin\angle ECB = T \sin\angle ACE.$$

$$\sin\angle ECB = \sin\angle BAC = \tfrac{7}{25}$$

$$\sin\angle ACE = \sin\angle EBC = \tfrac{24}{25}$$

$$\therefore P + \frac{225}{31} \times \frac{7}{25} = \frac{225}{31} \times \frac{24}{25}$$

$$P + \frac{63}{31} = \frac{9 \times 24}{31} = \frac{212}{31}$$

$$P = \frac{212}{31} - \frac{63}{31} = \frac{149}{31} = 4\tfrac{25}{31} \text{ lbs wt}$$

∴ Horizontal force = $4\frac{25}{31}$ lbs wt.

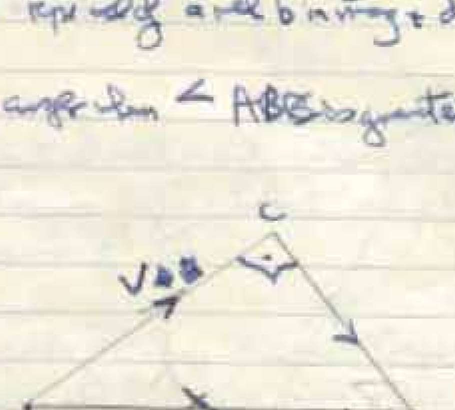
1958.

1. By definition.

vel a rel b = vel a − vel b.

Let $\overrightarrow{AB}$ repr. vel of a in mag. + dir.
Let $\overrightarrow{AC}$ repr. vel of b in mag + dir
Then CB repr vel of a rel b in mag + dir.

Inf. with the angle then ∠ ABC is greatest in Δ ACB



$\frac{u}{\sin C} = \frac{v}{\sin B}$

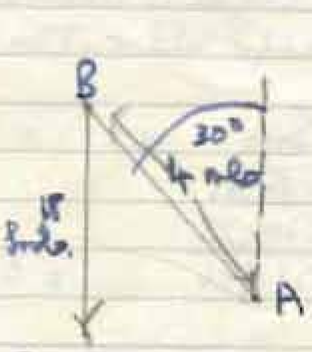
$\sin B = \frac{v}{u}\sin C$

∴ B is a max^m when
sin C is a max^m
i.e. C = 90°

This is because when ∠ ACB is 90° (∵ v < u).
The sin of ∠ ABC when this is so is $\frac{v}{u}$.

∴ direction of maximum inclination is $\sin^{-1}\frac{v}{u}$

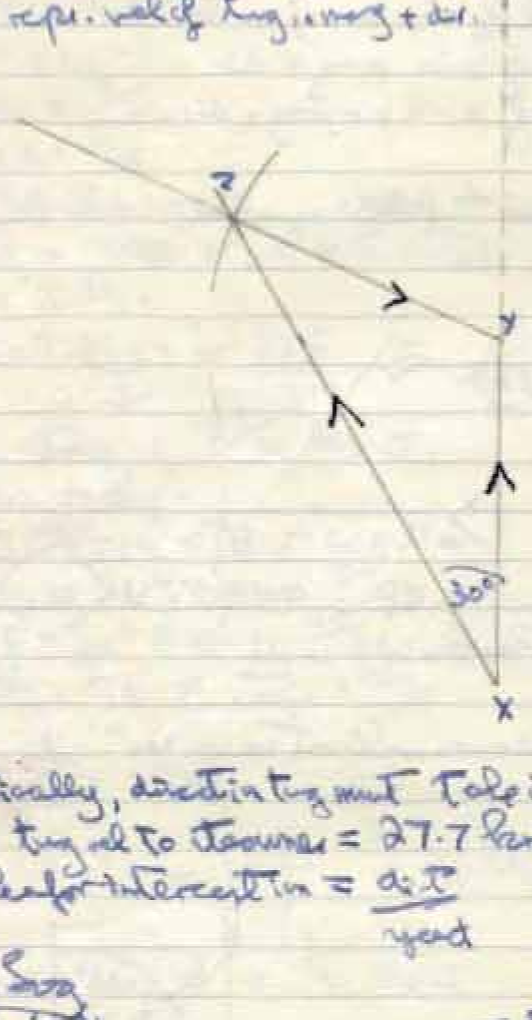
ii ~~By definition~~
~~vel~~



Let tug first be at A when steamship is at B.
∴ in order to intercept the steamship the vel of tug rel to ship must be along AB.

By definition
vel of tug rel to ship + vel of ship = vel of tug.
Let $\overrightarrow{XY}$ repr vel of ship in mag + dir.
Let $\overrightarrow{YZ}$ repr. vel of tug rel to ship in mag + dir.
$\therefore$ $\overrightarrow{XZ}$ repr. vel of tug in mag + dir.

Scale 1" ≡ 10 knots



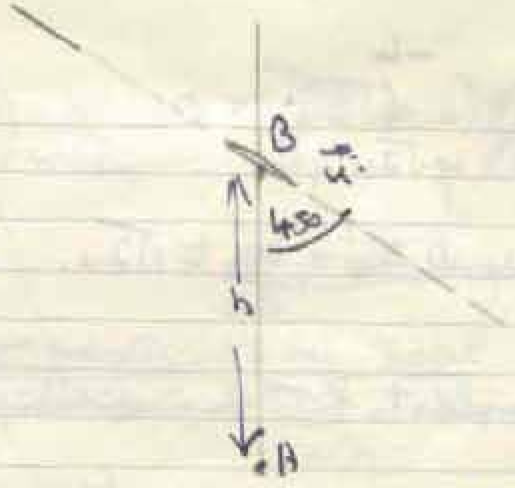
$\therefore$ graphically, direction tug must take is N. 66° W.
Speed of tug rel to steamer = 27.7 knots
Time taken for interception $= \dfrac{\text{dist}}{\text{speed}} = \dfrac{4}{27.7}$ hrs.

No	Log
4	0.6021
27.7	1.4425
	$\bar{1}$.1596

$= 0.1444$ hrs.

$= 0.1444 \times 60 = 8.664$ mins.

1438



Let A, B be initial positions of submarine and centre of ship.
~~Big obj~~ If object of torpedo is to intercept ship then vel torpedo rel to ship amidships is along AB.

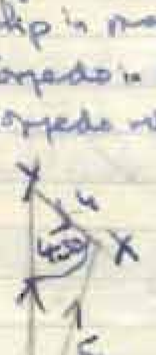
By definition

vel of torpedo rel ship = vel of torpedo − vel of ship

Let $\vec{XY}$ repr. vel of ship in mag & dir

Let $\vec{XZ}$ repr vel of torpedo in mag & dir

∴ Let $\vec{ZY}$ repr vel of torpedo rel ship in mag & dir.



In △XYZ By Co. Rule. Let $\vec{ZX} = a$.

$$25u^2 = u^2 + a^2 - \frac{2ua}{\sqrt{2}}$$

$$\therefore 24u^2 = a^2 - \frac{2ua}{\sqrt{2}}$$

$$\therefore \sqrt{2}\,24u^2 = \sqrt{2}\,a^2 - 2ua.$$

$$\therefore \sqrt{2}a^2 - 2ua - 24\sqrt{2}u^2 = 0.$$

$\therefore (\sqrt{2}a + 4u)(a - 4\sqrt{2}u) = 0.$

$\therefore a = \frac{4u}{\sqrt{2}} \quad 4\sqrt{2}u.$

$\therefore$ vel of torpedo rel ship $= \frac{4u}{\sqrt{2}}$. $4\sqrt{2}u$.

time taken for torpedo to hit ship amidships $= \frac{\text{dist}}{\text{speed}} = \frac{\sqrt{2}h}{4u\sqrt{2}}$

during that time ship travels $u . \frac{\sqrt{2}h}{4u\sqrt{2}} = \frac{\sqrt{2}h}{3u} . \frac{\sqrt{2}u^2}{4} \frac{h}{4\sqrt{2}}$.

position of stern of ship is $\frac{\sqrt{2}h}{4\sqrt{2}} - \frac{h}{2}$ from A.

distance travelled by torpedo ~~rel vel~~ to ship $= h$.

If then ship is accelerated ~~ship~~ ship ~~travels~~ travels a dist more than h rel to torpedo then torpedo will miss ship

when ship is accelerated by $\frac{32bu^2}{h^2}$, if new vel $= V$

using $v = u + ft$

new vel $= u + \frac{32bu^2}{h^2} . \frac{h}{4u\sqrt{2}} = u + \frac{32\sqrt{2}bu}{4} \quad \frac{32bu}{4\sqrt{2}}$

$= u + \frac{8\sqrt{2}bu}{\sqrt{2}}$.

new dist travelled by ship in time taken by torpedo $= \left(u + \frac{8\sqrt{2}bu}{\sqrt{2}}\right) . \frac{\sqrt{2}h}{4u} \neq \frac{h}{4\sqrt{2}u}$.

$= \frac{\sqrt{2}hu}{3u} + \frac{16bxh}{4x} \quad \frac{\sqrt{2}h}{3u}$ ~~+ 4bh~~ $> h$

$$\left(\frac{u + 8bu}{\sqrt{2}}\right)\cdot\frac{h}{4\sqrt{2}u} = \frac{hu}{4\sqrt{2}u} + \frac{8buh}{8u} \Bigg\rangle h.$$

$$= \frac{h}{4\sqrt{2}} + bh \Bigg\rangle h$$

$\therefore$ ~~ship must accelerate by~~ $\frac{32bu^2}{h^2}$.

Steamer must move $b/2$ in time $\frac{h}{4\sqrt{2}u}$.

using $s = ut + \frac{1}{2}gt^2$.

$$\frac{b}{2} = \frac{h}{4\sqrt{2}u} + \frac{1}{2}g\frac{h^2}{32u^2}.$$

$$\frac{+gh^2}{32u^2} = \frac{2\sqrt{2}b - h}{4\sqrt{2}}.$$

$$g = \frac{32bu^2}{h^2} - \frac{16hu^2}{\sqrt{2}h^2}$$

But $\frac{16hu^2}{\sqrt{2}h^2} = 0$ because vel of ship rel to torp is 0.

$$\therefore g = \frac{32bu^2}{h^2}.$$

Relative to torpedo vel. of steamer is zero $\therefore$ the ship must accelerate through dist. $\frac{b}{2}$ in time $\frac{h}{4\sqrt{2}u}$ rel.^{n} torpedo

$\therefore$ using $s = ut + \frac{1}{2}ft^2$ with $u = 0$

$$\frac{b}{2} = \frac{1}{2}f\frac{h^2}{32u^2}$$

$$\therefore f = \frac{32u^2 b}{h^2}$$

$\therefore$ Accel^n $= \frac{32u^2b}{h^2}$ units of vel / sec.

P 32 Nos 11, 13, 21 ~~Uniform~~ motion shows

11. ~~Considering~~ $v = u + ft$.

accelerat.^{ion} $g = \frac{v-u}{t} = \frac{28-16}{3} = 4$ ft/sec/sec. (16 ft/sec)

$$g = \frac{v-u}{t} = \frac{52-28}{6} = \frac{24}{6} = 4 \text{ ft/sec/sec.}$$

$\therefore$ ~~acceleration~~ is uniform.

Thus motion is consistent with supposition

what are you considering here?

Velocity at end of 10th. second = 52 ft/sec − 4 ft/sec = 48 ft/sec
Velocity at first second = 16 ft/sec − 4 ft/sec = 12 ft/sec
~~using S = ut + ½ft²~~
~~∴ dist s travelled in 10 secs~~

$$\cancel{= 120 + \tfrac{1}{2} \times 4 \times 10^2 = 120 + 200 = 320\text{ ft.}}$$

using $v = u + ft$.
$52 = u + 44$
$u = 8$ ft/sec
∴ initial vel = 8 ft/sec

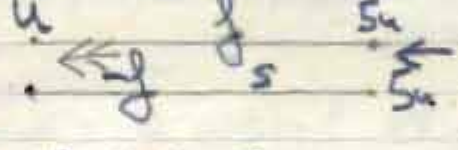
using $s = ut + \frac{1}{2}ft^2$. $\qquad s = \left(\dfrac{u+v}{2}\right)t.$

$\cancel{s = 8 \times 10 + \tfrac{1}{2} \times 4 \times 10^2}$ $\qquad = \left(\dfrac{8+52}{2}\right) \times 10$

$\cancel{= 280\text{ ft.}}$ $\qquad = 300$ ft.

∴ distance travelled = 300 ft.

13.



<u>To find final velocity</u>
Considering motion with uniform acceleration in f.
using $v_1^2 - u^2 = 2fs$ $\qquad$ (here $v_1 = 5u$).
$25u^2 - u^2 = 2fs$,
$24u^2 = 2fs$ — (1).

considering motion with uniform acceleration +f in other direction
final vel = v.
$v^2 - u^2 = 2fs$.

$\therefore v^2 - 25u^2 = 2fs$ — ②.

but $2fs$ is similar in both cases (as f's are same, s's are same)

$\therefore v^2 - 25u^2 = 24u^2$

$\therefore v^2 = 49u^2$

$v = \pm 7u$.

$\therefore$ final velocity = $7u$. (− sign is in answer because of opposite direction of velocity)

21

$\vec{u}$ (10 m.p.h) $\qquad \overrightarrow{20 \text{ m.p.h}} \qquad v$

← 1 ml. → ← 1 ml →

Considering motion with uniform acceleration (f) over first mile.

using $s = \frac{(v+u)}{2} t$.

$1 = \frac{30}{2} \times t$.

$t = \frac{1}{15}$ hr = 4 mins.

using $v = u + ft$.

$f = \frac{v-u}{t} = 10 \times 15 = 150$ mls/hr^2

considering motion over last mile also.

using $v^2 - u^2 = 2fs$

$v^2 - 20^2 = 300$

$v^2 = 700$

$v = \sqrt{700} = 10\sqrt{7}$ m.p.h.

using $v = u + ft$

$10\sqrt{7} = 20 + 150t$. $\quad t = \frac{10\sqrt{7} - 20}{150}$ hrs.

$$= \frac{10\sqrt{7} - 20}{\frac{\cancel{60}}{5}} \times \cancel{60}^{2} \text{ min}$$

$$= \frac{26.46 - 20}{5} \times 2$$

$$= \frac{6.46 \times 2}{5} = \frac{12.92}{5} = 2.58 \text{ mins}$$

∴ Time taken for successive miles are 4 mins and 2.58 mins. Velocity after passing 3rd milestone = $10\sqrt{7}$ m.p.h.

$s_n = un + \frac{1}{2} f n^2$ $\qquad$ s_{n-1} is dist travelled in $(n-1)$ secs

$s_{n-1} = u(n-1) + \frac{1}{2} f (n-1)^2$

P51 Nos 16, 15.

16. Initial velocity $u = 128$ ft/sec

Final vel = v.

acceleration $g = -32$ ft/sec^2.

Let distance be s ft.

Let time be t secs

using $s = ut + \frac{1}{2} f t^2$

$s = 128 \times 5 - \frac{1}{2} \times 32 \times 5^2$.

$$s = 5(128 - 80)$$

$$= 5 \times 48$$

$$= 240 \text{ ft.}$$

∴ ball is 240 ft from the ground.

when $v = 0$ ball is at maximum height from ground.

using $v^2 - u^2 = 2fs$

$-(128^2) = 2 \times -32 \times s.$

$$s = \frac{128 \times \cancel{128}^{\cancel{64}\,2}}{1\cancel{2} \times \cancel{32}_1} = 256 \text{ ft.}$$

$\therefore$ Total distance travelled $= 256 + (256 - 240)$

$= 256 + 16$

$= 272$ ft.

final velocity $= -128$ ft/sec.

When stone begins to fall down well
initial velocity $u = -128$ ft/sec.
let time be t_1 secs
distance $s_1 = 120$ ft.
using $s = ut + \frac{1}{2}gt^2$

$120 = -128t + \frac{1}{2} \times 32 \times t^2$

$120 = -128t + 16t^2$

$16t^2 - 128t - 120 = 0$

$2t^2 - 16t - 15 = 0$

$\therefore t = \dfrac{16 \pm \sqrt{256 + 4 \cdot 2 \cdot 15}}{4}$

$= \dfrac{16 \pm \sqrt{376}}{4}$

$\therefore t = \dfrac{16 + 19.391}{4} = \dfrac{35.391}{4} = 8.84775$ secs.

$\therefore$ It hits the bottom after 8.84775 secs.

---

15. Consider a stone of body falling.
Init. vel. $= 0$.
Let final vel be v.
acceleration $= g$.
Let dist. travelled be s_1 ft.
using $v^2 - u^2 = 2fs$
$v^2 = 2gs_1$

Considering motion of body thrown up.
Let initial velocity be u. Let distance travelled be s2
using $v^2 - u^2 = 2gs$

$v^2 - u^2 = -2gs_2$.

$\therefore v^2 = u^2 - 2gs_2$.

$\therefore$ as $v = v$

$2gs_1 = u^2 - 2gs_2$

~~Let $u^2 = v^2 + 2gs_2$.~~

~~$\therefore 2gs_1 = v^2 + 2gs_2$~~

Let t_2 be time taken by object thrown up to hit the object.
using $v = u + ft$.

$v = u - gt_2$.

$\therefore u = v + gt_2$.

$\therefore 2gs_1 = (v + gt_2)^2 - 2gs_2$.

$\therefore v^2 = (v + gt_2)^2 - 2gs_2$

$\cancel{v^2} = \cancel{v^2} + 2vgt_2 + g^2 t_2^2 - 2gs_2$.

$2gs_2 = 2vgt_2 + g^2 t_2^2$

$2gs_2 = gt_2(2v + gt_2)$.

$\therefore s_2 = \dfrac{\cancel{g}t_2(2v + gt_2)}{2\cancel{g}} = \dfrac{t_2(2v + gt_2)}{2}$.

Let t_1 be the time for the missile dropped to travel then using $s = ut + \frac{1}{2}ft^2$.

$$s_1 = \frac{1}{2}gt_1^2$$

$$\therefore \frac{s_1}{s_2} = \frac{\frac{1}{2}gt_1^2 \cdot 2}{2\, t_2 2v_1 + gt_2^2}$$

$$\therefore \frac{s_2}{s_1} = \frac{t_2 2v + gt_2^2}{gt_1^2} = \frac{t_2 2v}{gt_1^2} + \frac{t_2^2}{t_1^2}$$

But these times are the same

$$\therefore t_1 = t_2 = t$$

$$\therefore \frac{s_2}{s_1} = \frac{2v}{gt} + 1$$

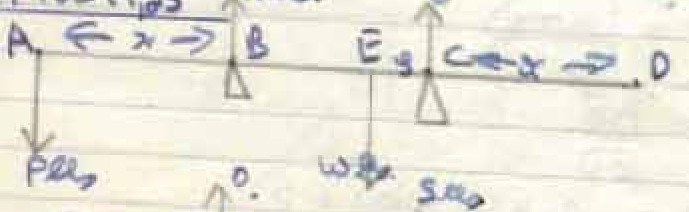
But $v = gt$ (using $v = u + ft$ for 1st particle.)

$$\therefore \frac{s_2}{s_1} = 2 + 1 = 3.$$

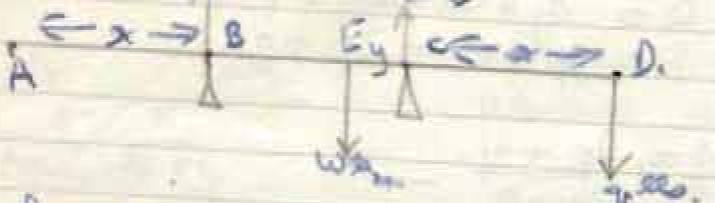
$8\frac{1}{2}/10$

P47 Nos (19, 23)

19.



Fig(i)



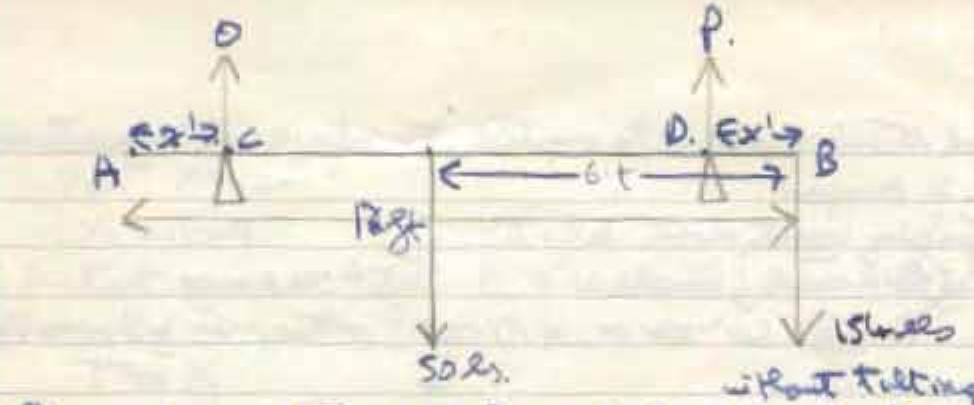
Fig(ii)

When the respective beams in fig(i) and fig(ii) be on the point of tipping reaction at C and B is respectively zero.

Let $x = AB = BC = CD$ in both diagrams
~~Where~~ Let the weight of the lever be W lbs.
Let the ~~G.~~ C.G. of the lever be at E, y units of distance from C
Considering first diagram
The system is in equilibrium when the lever is just on the point of tilting
$\therefore$ By the Principle of Moments
considering moments about B.
$$px = W(x-y) \quad —①.$$
In second diagram
By the Principle of Moments
considering moments about ~~[illegible]~~ C.
$$qx = Wy \quad —②.$$
from ① $$px = Wx - Wy$$
adding $$x(p+q) = Wx$$
$$\therefore W = p+q.$$
$\therefore$ weight of lever $= (p+q)$ lbs.

ii) Taking moments about A in ~~fig~~ fig (i), By the Principle of Moments.
$$xR = (2x-y)W \quad —① \quad (\text{where } R \text{ is the reaction at } B).$$
In fig ii), Taking moments about D by the Principle of Moments.
$$xS = (x+y)W \quad —② \quad (\text{where } S \text{ is the reaction at } C)$$
$\therefore$ dividing $$\frac{S}{R} = \frac{(x+y)}{(2x-y)}.$$
but $$\frac{x+y}{2x-y} = \frac{AE}{ED}.$$
In fig (i) $R = q + W = 2q + p$
In fig (ii) $S = p + W = 2p + q$.
$$\therefore \frac{AE}{ED} = \frac{2p+q}{2q+p}.$$
$\therefore$ the C.G. of the lever divides it in the ratio $2p+q : 2q+p$.

23.



The maximum distance man B can be from D without tilting is when the beam will just start to tilt. When this is so reaction at C is just zero.

∴ When this is so the system is just in equilibrium.

∴ Taking moments about D as in diagram. (Let the man be x' from D on AD produced)

By the Principle of Moments

$154x = 50(6-x)$

$154x = 300 - 50x$

$204x = 300$

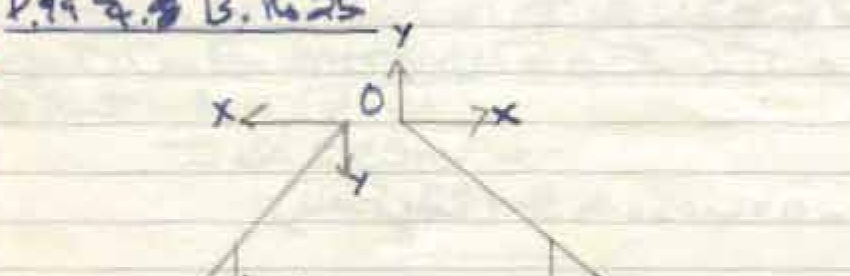
$x = \frac{300}{204} = \frac{100}{68} = \frac{25}{17} = 1\frac{8}{17}$ ft.

∴ The maximum dist of DB ~~from~~ man to maintain equilibrium is $1\frac{8}{17}$ ft from D on AD produced. (assumed man stands at very end of beam beyond D)

Easter Term 1967.

P.99 Ex. B. No 25

Good $\frac{19}{20}$



By symmetry, the reactions at A and B are equal. Let the length of

and vertical components of the reaction at O be X and Y respectively, acting as in diagram.
Taking moments about A for AO.
By the Principle of Moments. Let $\angle OAB = \alpha$

$$10 \cdot \frac{65}{8} \cdot \cos\alpha + Y \cdot \frac{65}{4} \cos\alpha = X \sin\alpha \cdot \frac{65}{4}.$$

$$\therefore (10+2Y)\cos\alpha = 2X\sin\alpha. \quad — (1)$$

Taking moments about B for OB.

$$10 \cdot \frac{65}{8} \cos\alpha = Y \cdot \frac{65}{4} \cos\alpha + X \cdot \frac{65}{4} \sin\alpha.$$

$$\therefore (10-2Y)\cos\alpha = 2X\sin\alpha \quad — (2)$$

$$\therefore (10-2Y)\cos\alpha = (10+2Y)\cos\alpha.$$

$$\therefore Y = 0.$$

In (1) if $Y=0$. $\quad 2X = \frac{10}{\tan\alpha}$

$$\therefore 2X = \frac{10 \cdot 10\frac{1}{5}}{\sqrt{\left[16\frac{1}{4}^2 - 10\frac{1}{5}^2\right]}} = \frac{102}{\sqrt{160}} = \frac{102}{12.65} = 8.063$$

$$\therefore X = 4.0312 \text{ lbs wt.}$$

$$\therefore \text{Resultant reaction at } O = \sqrt{Y^2+X^2}$$

$$= 4.0312 \text{ lbs wt.}$$

$\therefore$ Pressure on O = 4.0312 lbs wt. (horizontally).

Let the horiz. + vert. components of the reactions at ~~A~~ A and B be X_1 and Y_1 respectively.
~~By taking moments about~~ The horiz. components X_1 are cancelled by the tension T in the string AB.
$\therefore X_1 = 0$.

Resolving vertically for rod AO.

$Y_1 = Y + 10$

$\therefore Y_1 = 10$ lbs.

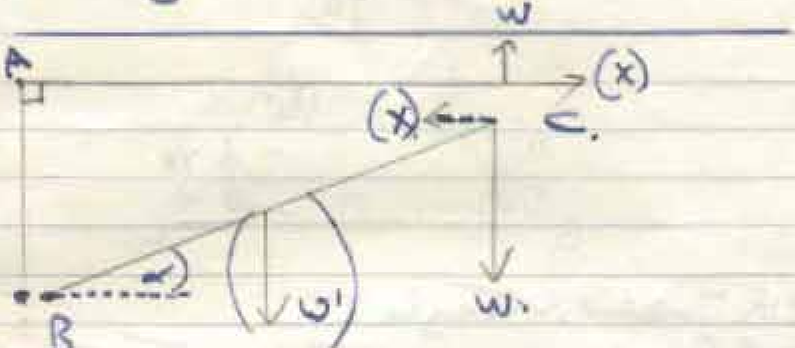
$\therefore$ Resultant pressure at A (action B) = 10 lbs wt (vertically).

Also resolving horizontally for rod AO.

$T = X$.

$= 4.0312$ lbs wt.

$\therefore$ Tension in string = 4.0312 lbs wt.

15. 

The vertical components of the reaction at C is W. The horizontal component ~~[illegible]~~ acts along the direction of the rod.
Taking moments about B for BC. If l = length of BC.

$Wl\cos\alpha = Xl\sin\alpha$.

$\therefore X = W\cot\alpha$

$\therefore$ Tension in AC = $W\cot\alpha$ (as vert. compt. of tension does not affect it as tension is in horizontal direction)

& Thrust in BC = $\sqrt{X^2+W^2}$

$$= \sqrt{W^2\cot^2\alpha + W^2}$$

$$= W\sqrt{\operatorname{cosec}^2\alpha} = W\operatorname{cosec}\alpha.$$

If W' is hung from mid point of BC. If l = length of BC

Taking moments about B for BC

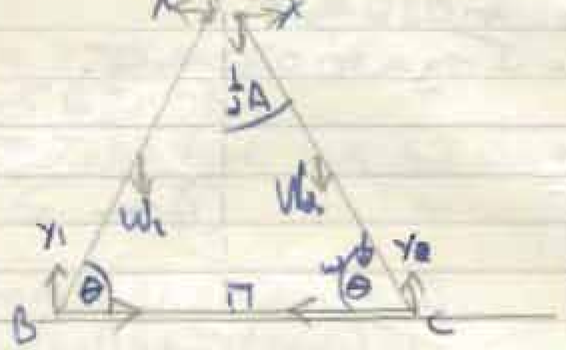
$\frac{1}{2}W'l\cos\alpha + Wl\cos\alpha = Xl\sin\alpha$.

$W'\cos\alpha + 2W\cos\alpha = 2X\sin\alpha$.

$\therefore X = \dfrac{\frac{1}{2}(2W + W')}{\tan\alpha}$

$\therefore$ Tension in AC $= \frac{1}{2}\cot\alpha\,(2W + W')$.

22



Let the tension in the string be T.

Let the horiz. + vert. components of the reaction at A be X and Y resp.

Taking moments about B for AB. If $AB = AC = 2l$

$(W_1 l\cos\theta) = 2lX\sin\theta + 2lY\cos\theta$.

$\therefore 2X\tan\theta = W_1 - 2Y$

$2X = (W_1 - 2Y)\tan\frac{1}{2}A$ — ①

Taking moments about C for AC

$\frac{1}{2}2l\sin\frac{1}{2}A\,w + W_2 l\sin\frac{1}{2}A + 2Yl\sin\frac{1}{2}A = 2lX\cos\frac{1}{2}A$.

$\therefore \tan\frac{1}{2}A(\frac{1}{2}w + W_2 + 2Y) = 2X$ — ②

$\therefore \frac{1}{2}w + W_2 + 2Y = W_1 - 2Y$.

$\therefore 4Y = W_1 - W_2 - \frac{1}{2}w$

$\therefore Y = \frac{1}{4}(W_1 - W_2 - \frac{1}{2}w)$

Resolving horizontally for AC or BA $X = T$.

$\therefore$ in ②. $2T = \tan\frac{1}{2}A\left(\frac{1}{2}w + W_2 + \frac{1}{2}W_1 - \frac{1}{2}W_2 - \frac{1}{4}w\right)$

$= \tan\frac{1}{2}A\left(\frac{1}{4}w + \frac{1}{2}W_2 + \frac{1}{2}W_1\right)$

$\therefore T = \frac{1}{4}\left[\frac{1}{2}w + W_2 + W_1\right]\tan\frac{1}{2}A.$

$\therefore$ Tension in string $= \frac{1}{4}\left[\frac{1}{2}w + W_2 + W_1\right]\tan\frac{1}{2}A.$

Pure Mathematics, Easter Term, 1967

1 i). $\sin 2A = \frac{1}{2}$.

$\therefore$ between 0 and 360°

Want A between 0° & 360°

$\therefore$ Need 2A between 0° & 720°

i) $2A = 30°, 150°$ ii). $\therefore A = \cancel{30° \text{ or } 60°}$.

390°, 510°.   15°, 75°, 195°, 255°.

2) $3A = 90°, 180°$. $\therefore 3A = 45°, 225°$.

ii). To proof. $\operatorname{cosec}\theta + \cot\theta = \cot\frac{1}{2}\theta$.

$$L.H.S. = \frac{1}{\sin\theta} + \frac{1}{\tan\theta}.$$

$$= \frac{\tan\theta + \sin\theta}{\sin\theta.\tan\theta} = \frac{\cos\theta(\tan\theta + \sin\theta)}{\sin^2\theta}.$$

$$= \frac{\cos\theta\tan\theta + \cos\theta\sin\theta}{\sin^2\theta} = \frac{\sin\theta + \cos\theta\sin\theta}{\sin^2\theta}.$$

$$= \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}$$

$$\tan^2\theta = \frac{2\tan\frac{1}{2}\theta}{1-\tan^2\frac{1}{2}\theta}.$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

$$\therefore \cot\theta = \frac{1-\tan^2\frac{1}{2}\theta}{2\tan\frac{1}{2}\theta}.$$

$$= \frac{\cancel{2}\cos^2\frac{1}{2}\theta}{\cancel{2}\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta}$$

$$L.H.S. = \frac{1}{2\cos\theta\sin\frac{1}{2}\theta} + \frac{2\tan\frac{1}{2}\theta}{1-\tan^2\frac{1}{2}\theta}. \quad = \cot\frac{1}{2}\theta.$$

=

iii) $2 + \sin\theta = 3\cos\theta.$

$\therefore (2 + \sin\theta)^2 = 9\cos^2\theta.$
$= 9 - 9\sin^2\theta.$

$\therefore 4 + 4\sin\theta + \sin^2\theta = 9 - 9\sin^2\theta.$

$10\sin^2\theta + 4\sin\theta - 5 = 0.$

$\therefore 10\sin^2\theta + 4\sin\theta - 5 = 0$

$$\therefore \sin\theta = \frac{-4 \pm \sqrt{16 + 200}}{20}.$$

$$= \frac{4 \pm 14.7}{20} = \frac{18.7}{20} = \frac{1.87}{2} = 0.935.$$

or $\sin\theta = \frac{4 - 14.7}{20} = -\frac{10.7}{20} = -0.535$

$\therefore \theta = 69°14'$ or $180° - 69°14' = 110°46'.$

This method sometimes gives extraneous roots.

2. By the Cosine Rule.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$

But $\cos A = 2\cos^2\tfrac{1}{2}A - 1.$

$$\therefore 2\cos^2\tfrac{1}{2}A - 1 = \frac{b^2 + c^2 - a^2}{2bc} + 1$$

$$\therefore 2\cos^2\tfrac{1}{2}A = \frac{b^2 + c^2 - a^2 + 2bc}{2bc}$$

$$\therefore \cos^2\tfrac{1}{2}A = \frac{b^2 + c^2 - a^2 + 2bc}{4bc}$$

$$= \frac{(b+c)^2 - a^2}{4bc}$$

$$= \frac{(b+c+a)(b+c-a)}{4bc} \qquad \text{If } 2s = b+c+a$$

$$\text{Then } \cos^2 \tfrac{1}{2}A = \frac{\cancel{2}s(2s-2a)}{4bc}$$

$$= \frac{s(s-a)}{bc}$$

$$\therefore \cos \tfrac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$$

$$\text{Also } \frac{b^2+c^2-a^2}{2bc} = 1 - 2\sin^2 \tfrac{1}{2}A.$$

$$\therefore 2\sin^2 \tfrac{1}{2}A = 1 - \left(\frac{b^2+c^2-a^2}{2bc}\right)$$

$$= \frac{2bc - (b^2+c^2-a^2)}{2bc}$$

$$= \frac{2bc - b^2 - c^2 + a^2}{2bc} = \frac{a^2 - (b-c)^2}{2bc}$$

$$= \cancel{\frac{a^2 - (b+c)^2}{2bc}}$$

$$= \frac{(a+b-c)(a-b+c)}{2bc} \qquad \text{If } 2s = a+b+c$$

$-1+0.97$
$-1+0.980$

$$\sin^2 \tfrac{1}{2}A = \frac{(2s-2b)(2s-2c)}{4bc} = \frac{(s-b)(s-c)}{bc} =$$

$$\therefore \tan \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

No	Log
1014·1	3·0060
1058·2	3·0244
	6·0304
1430	3·1553
787·7	2·8964
	6·0517
	$\bar{1}$·9787

ii) $a = 642.3$, $b = 415.9$, $c = 371.8$

$~~(a+b+c) = 1429.0~~$ $(a+b+c) = 1430$

$~~\therefore \tan \tfrac{1}{2}A = \sqrt{(1013.1)(1057.2)}~~$

$$\tan \tfrac{1}{2}A = \sqrt{\frac{(1014.1)(1058.2)}{(1430)(787.7)}}$$

$~~\therefore \tfrac{1}{2}A = 43°36'~~$
$~~\therefore A =~~$ $\therefore \tan^2 \tfrac{1}{2}A = 0.9521$

$\therefore \tan \tfrac{1}{2}A = 0.9757$

$\therefore \tfrac{1}{2}A = 44° 18'$
$\therefore A = 88° 36'.$

~~By the Sine Rule~~
Use same rule for $\tan \tfrac{1}{2}B$

3) i). $x+3y=2$

$\therefore 3y = -x+2$

$\therefore$ gradient of line $= -\frac{1}{3}$

For new line

By $y-k = m(x-h)$

$y+4 = -\frac{1}{3}(x-2)$

$\therefore y+4 = -\frac{1}{3}x + \frac{2}{3}$

$3y+12 = -x+2$

$3y+x+10 = 0.$

$\therefore$ eqn of line is $3y+x+10=0$ —(1)

ii) gradient of $2y=x+5$ is $\frac{1}{2}$.

$\therefore$ gradient of new line is -2

$\therefore$ by $y-k = m(x-h)$

~~equation~~ eqn of new line is

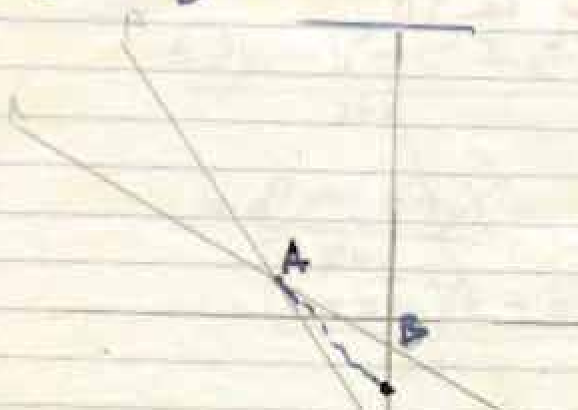
$y+8 = -2(x+6)$

$\therefore y+8 = -2x-12$

$y+2x+20=0.$

$\therefore$ eqn of line is $y+2x+20=0.$ —(2)

iii)



eq^n ① cuts y axis when $x = 0$.
i.e. $3y = -10$. $y = -\frac{10}{3}$
eq^n ② cuts y axis when $\frac{x}{3} = 0$
i.e. $y = -20$.

$\therefore$ dist $BC = 20 - \frac{10}{3} = \frac{50}{3}$.

$\therefore \frac{1}{2}BC = \frac{50}{6} = \frac{25}{3}$.

$\therefore$ co ordinates of mid point are $\left(0, \frac{35}{3}\right) = \left(0, -\frac{35}{3}\right)$

When $y + 2x + 20 = 0$ and $3y + x + 10 = 0$ intersect

$$-2x - 20 = \frac{-x - 10}{3}$$

$\therefore x + 10 = 6x + 60$
$5x = -50$
$x = -10$.

$y_2 = 0$
$y_1 = -\frac{35}{3}$
$x_2 = -10$
$x_1 = 0$.

$x = -10$, $y - 20 + 20 = 0$
$y = 0$.

$\therefore$ lines intersect at $(-10, 0)$

$\therefore$ by $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$

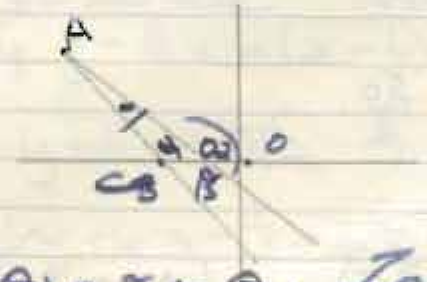
$-\frac{35}{30} = \frac{y - \frac{35}{3}}{x - 0}$

$\therefore 35x + 30y + 350$ $\therefore$ eq is $-7x = 6y - 70$.

$6y + 7x = 70$.

Let [illegible] be α.

$\therefore \tan\alpha = \dfrac{m_1 \sim m_2}{1 - m_1 m_2}$ where m_1, m_2 are gradients of AB and [illegible] respectively



Of $\angle$ ~~BB~~A$= \alpha$
$\angle$ ACO $= \theta_2$ $\angle$ ABO $= \theta_1$
Of AC is line $y = m_2x + c_1$
Of AB is line $y = m_1x + c_2$

$\theta_2 = \alpha + \theta_1$ (ext $\angle$ of Δ)

$\therefore \alpha = \theta_2 - \theta_1$

$\tan\alpha = \tan(\theta_2 - \theta_1) = \dfrac{\tan\theta_2 \mp \tan\theta_1}{1 + \tan\theta_2 \tan\theta_1}$

$= \dfrac{m_2 \mp m_1}{1 + m_2 m_1}$.

~~6.) $2^{2x+4} - 33(2^x) + 8 = 0$.~~

~~[illegible]~~

7.) Sum of the first n natural numbers $= \frac{1}{2}n(n+1)$

Sum of the squares of the first n natural no $= \frac{1}{6}n(n+1)(2n+1)$

$1.5. + 3.7. + 5.9. + \ldots\ldots$ r th term

common diff. of [illegible] 1. 3. 5. 7. ... etc

1st term $= 1$.

$= 2$.

$\therefore$ nth term $= (n-1)d$ $= 2(n-1) + 1$ $= 2n - 2 + 1$ $= 2n - 1$

common diff. of terms 5. 7. 9. 11. .. etc $= 2$

first term $= 5$ $= 2n + 3$

$\therefore$ nth term $= 5 + (n-1)2$ $= [illegible]$ $5 + 2n - 2$

$\therefore$ nth term of series $= 20(n-1)^2$.

$= \cancel{3}(2n+3)(2n-1) = (2n+3)(2n-1)$

$= 4n^2 + \cancel{6n} + 4n - 3$

$\therefore$ Sum to n terms $= \sum 4n^2 + \sum 4n - \sum 3$

$= 4\sum n^2 + 4\sum n - 3\sum 1$

$= \frac{2}{3}n(n+1)(2n+1) + 2n(n+1) - 3n.$

$= n\left(\frac{2}{3}(n^2 - 3n + 2) + 2n^2 + 2n - 3n\right)$

$= n\left(\frac{2}{3}n^2 \cancel{- 2n} + \frac{4}{3} + 2n^2 \cancel{+ 2n} - 3n\right)$

$= n\left(\frac{8}{3}n^2 - 3n + \frac{4}{3}\right)$

$= \frac{1}{3}n(8n^2 - 9n + 4)$

When $n = 30$,

$S_n = 10(7200 - 270 + 4)$

$= 10 \times 6934 = 69340.$

7200
270
6934

$\frac{69340}{107.}$ Sum to n terms $=$

$n\left(\frac{2}{3}(n+1)(n+2) + 2(n+1) - 3\right)$

$= n\left(\frac{2}{3}n^2 + 2n + \frac{4}{3} + 2n + 2 - 3\right)$

$= n\left(\frac{2}{3}n^2 + 4n + \frac{1}{3}\right).$

7200
~~7200~~
270
6934
6934

$+4n$

$= n\left(\frac{2}{3}n^2 + \cancel{2n} + \frac{4}{3} + 6n + \frac{20}{\cancel{12}}n + \frac{2}{\cancel{3}} - 3\right)$

$= n\left(\frac{2}{3}n^2 + \frac{42}{6}n + \frac{1}{3}\right)$

$= 3n\frac{}{6}$

$-\frac{5}{2}+\frac{1}{2}$ when $n = 30$

$-\frac{5}{6}+\frac{2}{6}$ $S_{30} = 30\left(600 \cancel{+ 210} + \frac{1}{3}\right)$

$= 18000 \cancel{+ 900} + 10$

$= 18010$

$= \cancel{6}\frac{178902}{107} = \frac{27000}{107}$

$= n\left(\frac{2}{3}n^2 + 4n + \frac{1}{3}\right)$

$= 30\left(600 + 120 + \frac{1}{3}\right) = \cancel{56} = \frac{18010}{1078} =$

$= 18000 + 3600 + 10$

$= 21610$

18000 / 101

18000 / 91 / 1700

18000 / 3600 / 21610 / 107

iii) $\left(\frac{1}{2}x^2 - \frac{1}{x}\right)^{12}$

$(r+1)^{th}$ Term $= \frac{12.11.10.\ldots(12-r+1)}{r!}\left(\frac{1}{2}x^2\right)^{12-r}\left(\frac{-1}{x}\right)^{12}$

~~coeff~~ x^0.

i.e. $24 - 2r - 12 = 0$

$2r = 12$

$r = 6. \; 3d$

$\therefore (r+1)^{th}$ Term $= \frac{12.11.\cancel{10}.\cancel{9}.\cancel{8}.7}{1.\cancel{2}.\cancel{3}.\cancel{4}.\cancel{5}.\cancel{6}}\left(\frac{1}{2}\right)^6 \cancel{(-1)}$ ✓

84 / 11 / 924

$= \cancel{84}$ $= \frac{924}{64} = \frac{261.}{16.}$

8 i) $y = x^3$

let x increase by small amount δx

let y " by a small amount.

$\therefore y + \delta y = (x + \delta x)^3 = x^3 + 3x^2\delta x + 3x(\delta x)^2 + \delta x^3$

$\therefore \delta y = \cancel{x^3} + 3x^2\delta x + 3x(\delta x)^2 + (\delta x)^3 - \cancel{x^3}$

$\therefore \frac{\delta y}{\delta x} = 3x^2 + 3x\,\delta x + (\delta x)^2$

$\cancel{x \to 0}.\ \frac{dy}{dx} = \underset{x \to 0}{\int} \frac{\delta y}{\delta x} = 3x^2.$

ii) $y = \frac{x(x+1)}{x^3 - 2} = \frac{x^2 + x}{x^3 - 2}$

$\cancel{\text{let } x^3 - 2 = u}$

$\cancel{\therefore \frac{du}{dx} = 3x^2}$

$\cancel{\therefore \delta = \frac{x(x+1)}{x^3 - 2}}$

$\therefore \frac{dy}{dx} = \frac{(2x+1)(x^3 - 2) - 3x^2(x^2 + x)}{(x^3 - 2)^2}$

$= \frac{2x^4 + x^3 - 4x - 2 - 3x^4 - 3x^3}{(x^3 - 2)^2}$

$= \frac{-x^4 - 2x^3 - 4x - 2}{(x^3 - 2)^2} = -\frac{(x^4 + 2x^3 + 4x + 2)}{(x^3 - 2)^2}$

2) $y = \log(e^{2x} + 2x)$

~~$\frac{dy}{dx} = \frac{1}{2}\log$~~

Let $e^{2x} + 2x = u$.

$\therefore \frac{du}{dx} = 2e^{2x} + 2$.

$y = \log u$

$\therefore \frac{dy}{du} = \frac{1}{u}$

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{2e^{2x} + 2}{e^{2x} + 2x}$.

iii) $y = \cos 2x + 2\cos x$

$\therefore \frac{dy}{dx} = -2\sin 2x - 2\sin x$.

For turning values

$\frac{dy}{dx} = 0$

$\therefore 2\sin 2x - 2\sin x = 0$

$\cancel{2}\sin 2x = \cancel{2}\sin x$

$\sin 2x = \sin x$

$\therefore x = 0.$ or $x = \pi$.

$\therefore 2\sin x \cos x = \sin x$

$\therefore \sin x(2\cos x - 1) = 0$

$\frac{d^2y}{dx^2} = \cancel{2\cos 2x} = -4\cos 2x - 2\cos x$

$\sin x = 0$ or $\cos x = \frac{1}{2}$

When $x = 0$. $\frac{d^2y}{dx^2} < 0$ $\therefore x = 0$ is a maximum value

$\sin x = \pi$.

$\frac{d^2y}{dx^2} > 0$. $\therefore$ $x = \pi$ is a maximum value

9i). $$\int_4^5 \frac{dx}{(x-4)(x-3)}.$$ P.F.

Let $(x-3) = u$

9ii) $$\int_0^{\frac{\pi}{2}} \cos^3 x \,.\, dx$$

$\sin x = 0$ $u = 0$.

$\sin x = \frac{\pi}{2}$. $u = 1$.

Let $u = \sin x$

$\therefore \frac{du}{dx} = \cos x$

$$\therefore I = \int_0^{\frac{\pi}{2}} \cos x \cos^2 x \; dx$$

$$= \int_0^{\frac{\pi}{2}} \cos^2 x \; du.$$

$\cos x = u$

$\frac{du}{dx} = -\sin x$.

6 i) $2^{2x+2} - 33(2^x) + 8 = 0$. Put $2^x = u$

Taking logs to base 10
$(2x+2)\log 2 - (\log 33 + x\log 2) + \log 8 = 0$.

$0.6020x + 0.6020 - 1.5185 - 0.3010x + 0.9031$

$\therefore 0.3010x = 0.0134$. Can't do this

No	log
0.0134	$\bar{2}.1271$
0.3010	$\bar{1}.4786$
	$\bar{2}.6485$

$\therefore x = \frac{0.0134}{0.3010}$

$= 0.04451$. ans

ii) The logarithm of a no. M to the base a is the no. obtained when ~~the~~ ~~base~~ multiplied by the logarithm.

$\log_a M . \log_M a = 1$. Let $\log_a M = y$.

~~Let $\log_a M = x$~~

~~$\therefore x = \sqrt[a]{M}$~~

~~$\therefore \log_a M = \sqrt[a]{M}$~~

~~Similarly $\log_M a = \sqrt[M]{a}$~~

$\therefore M = a^y$.
Taking logs to base b.
$\log_b M = y \log_b a$

$\therefore y = \frac{\log_b M}{\log_b a}$ $\therefore \log_a M = \frac{\log_b M}{\log_b a}$

if $M = b$. $y = \log_a M = \frac{\log_M M}{\log_M a}$

$\therefore \log_a M . \log_M a = 1$

1.5185
1.5051
0.8134

Let $\log_b a = y$. y.
$\therefore \log a = b$
Let $\log_c b = x$ x
$\therefore b = c$
Let $\log_a c = z$ z.
$\therefore c = a$

$\therefore \log_b a \cdot \log_c b \cdot \log_a c = \cancel{xzy} \cdot z = b^y \cdot c^x \cdot a^z$

$= \log_b a \times \cancel{\frac{1}{\log_b c}} \times \cancel{\frac{\log_b c}{\log_b a}} = 1.$

iii) $x^2 + 5x + 10 = 0.$
if roots are $\alpha, \beta.$
$(\alpha + \beta) = -\frac{5}{1} = -5$ — ①.

$\alpha\beta = \frac{10}{1} = 10.$ — ②

from ② $\alpha = \frac{10}{\beta}$

$\therefore$ in ① $\frac{10}{\beta} + \beta = -5$

$\therefore 10 + \beta^2 = -5\beta$
$\therefore \beta^2 + 5\beta + 10 = 0$
$\therefore \beta = -\frac{5 \pm \sqrt{25 - 40}}{2}$

$\therefore$ roots of the equation are imaginary.

Let required eq^n be
$x^2 + px + q = 0.$

then sum of roots
$(2\alpha + 3\beta) + (2\beta - 3\alpha)$
$= -p$

$\therefore -\alpha - \beta = -p$
$\alpha + \beta = p$

10. $3y = (x-2)^2$

$\therefore 3y = x^2 - 4x + 4.$

when $x = 2$, $y = 0$.

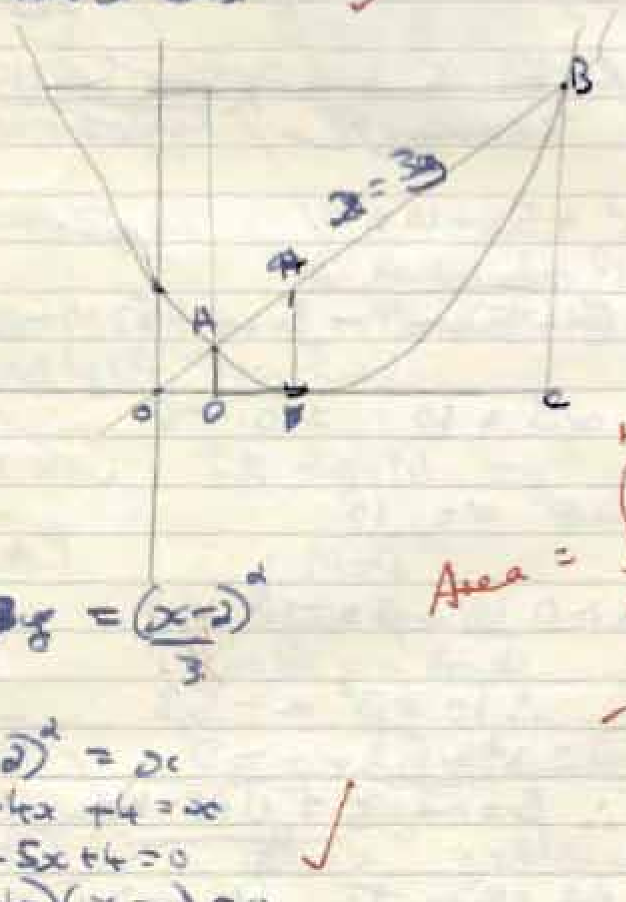
~~[illegible]~~

as $y \to \infty$, $x \to \infty$

as $y \to -\infty$ $x \to -\infty$

graph cuts x axis at points 2 and 2

$\therefore$ curve touches x axis ✓



$$\text{Area} = \int_1^4 \frac{x}{3}\,dx$$

$$- \int_1^4 \frac{(x-2)^2}{3}$$

$y = \frac{3x}{3}$ cuts $3y = \frac{(x-2)^2}{3}$

when $(x-2)^2 = x$

$x^2 - 4x + 4 = x$

$x^2 - 5x + 4 = 0$ ✓

$(x-4)(x-1) = 0$

$x = 1$ or $x = 4$.

required area in diagram $= - \left[\int_1^2 \frac{(x-2)^2}{3}\,dx + \int_2^4 \frac{(x-2)^2}{3} \right.$

+ area quad. ABCD.

$$= \text{area } \Delta ABO - \text{area } \Delta AOD - \int_0^4 \left(\frac{x-2}{3}\right)^2 dx + \int_1^2 \left(\frac{x-2}{3}\right)^2 dx.$$

$$= \text{area } \Delta ABO - \text{area } \Delta AOD - \left[\int_0^4 \frac{x^2-4x+4}{3} dx + \int_1^2 \frac{(x^2-4x+4)}{3} dx\right]$$

$$= -\left[\frac{x^3}{9} - \frac{4x^2}{6} + \frac{4x}{3}\right]_0^4 + \left[\frac{x^3}{9} - \frac{4x^2}{6} + \frac{4x}{3}\right]_1^2$$

$$= \left\{\left[\frac{64}{9} - \frac{64}{6} + \frac{16}{3}\right] - \cancel{\left[\frac{8}{9} - \frac{16}{6} + \frac{8}{3}\right]}\right\} + \left\{\cancel{\left[\frac{8}{9} - \frac{16}{6} + \frac{8}{3}\right]} - \left[\frac{1}{9} - \frac{4}{6} + \frac{4}{3}\right]\right\}$$

$$= \frac{64}{9} - \frac{64}{6} + \frac{16}{3} - \frac{1}{9} + \frac{4}{6} - \frac{4}{3}$$

$$= 7 - 10 + 4 = 1 \text{ sq units}$$

area Δ ABO ~~area Δ AOD~~ $= \frac{1}{2} \times \frac{4}{3} \times 4 = \frac{8}{3}$ sq units

area Δ ADO $= \frac{1}{2} \times 1 \times \frac{1}{3} = \frac{1}{6}$ sq units.

$\therefore$ area inclosed by curve $= \frac{8}{3} - \frac{1}{6} - 1 = \frac{7}{6}$ sq units.

vol-went out = vol of cone OBC − vol cone AOD

$$- \pi \int_1^4 y^2 dx$$

$$= \frac{1}{3}\pi \left(\frac{4}{3}\right)^2 4 - \frac{1}{3}\pi \left(\frac{1}{3}\right)^2 1. - \pi \int_1^4 y^2 dx$$

$$\pi \int_1^4 y^2\,dx = \pi \int_1^4 \frac{(x-2)^4}{9}$$

~~By the Binomial Theorem~~ $\cancel{(x-2)^4} = (-2+x)^4$

$$\cancel{= 16 + 4(-2)^3 x + \frac{4.3}{1.2}(-2)^2 x^2}$$

$$\cancel{+ \frac{4.3.2}{1.2.3}(-2)^3 x^3 + x^4}$$

$$\cancel{= 16 - 32x + 24x^2 - 32x^3 + x^4}$$

$$\cancel{\therefore I = \int_1^4}$$

Let $(x-2) = u$.

When $x = 4$ $u = 2$
when $x = 1$ $u = -1$.

$\frac{du}{dx} = 1.$

$$\therefore I = \pi \int_{-1}^{2} \frac{u^4}{9}\,du.$$

$$= \pi \left[\frac{u^5}{45}\right]_{-1}^{2} = \pi\left\{\left[\frac{32}{45}\right] - \left[\frac{-1}{45}\right]\right\}$$

$$= \pi\left[\frac{33}{45}\right] = \pi\,\frac{11}{15}.$$

$$\therefore \text{Vol} = \frac{64\pi}{9} - \frac{\pi}{9} - \frac{11\pi}{15}$$

$$= \pi\left[7 - \frac{11}{15}\right] = \pi\left[\frac{194}{15}\right]$$

$\therefore$ new vol = $\frac{194\pi}{15}$ cubic units. ans.