Hyam Wyn Evans, Form IV c, Geometry Test — 12-3-65

1. ~~The~~ A parallelogram is a quadrilateral with opposite sides parallel. ✓

Given. AD // BC, AB // DC., diagonal AC, //gm. ABCD.
To prove. AC bisects parallgm. ABCD. i.e $\Delta ACD = \Delta ABC$
Proof. In Δs ABC and ACD.
AC = AC (common).
$\angle DAC = \angle ACB$ (alternate angles)
$\angle DCA = \angle BAC$ (alternate angles)
State which lines are parallel.
$\therefore \Delta ABC \equiv \Delta ADC$ (A.A.S). ✓
$\therefore$ area of $\Delta ABC = \frac{1}{2}$ area of parallgm. ABCD.
$\therefore$ ~~The~~ diagonal AC bisects parallgm. ABCD. ✓

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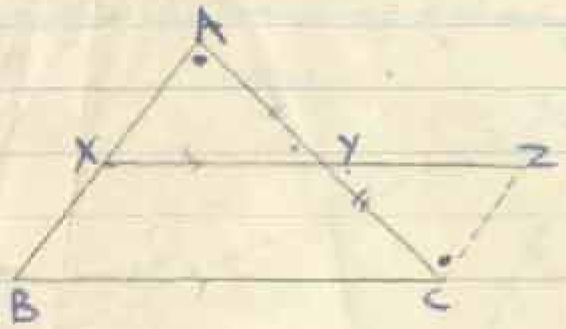
3. The line joining the mid-points of 2 sides of a triangle is parallel to the 3rd side and equal to ~~the~~ one half of it. ✓



<u>Given.</u> ΔABC, $AX = XB$, $AY = YC$.

<u>To Prove.</u> $XY \parallel BC$, $XY = \frac{1}{2} BC$.

<u>Construction</u> Draw $ZC \parallel$ to AB to meet XY produced at Z.

<u>Proof.</u> In Δs AXY and YZC.

$AY = YC$ (given).

$\angle AYX = \angle ZYC$ (~~alternate~~ vertically opposite angles)

$\angle ZCA = \angle CAX$ (alternate angles, by construction).

$\therefore \Delta AXY \equiv \Delta YZC$ (A.A.S.).

$\therefore ZC =$ ~~XB~~. AX but $AX = XB$ (given) $\therefore ZC = XB$.

but $ZC \parallel XB$ (construction).

$\therefore XZCB$ is a $\parallel$gm. (opposite sides $\parallel$ and equal).

$\therefore XZ \parallel BC$. (~~also~~ opp. sides of $\parallel$gm.)

$\therefore XY \parallel BC$

but $XY = YZ$ (proved)

and $XZ = BC$ (opp. sides of $\parallel$gm).

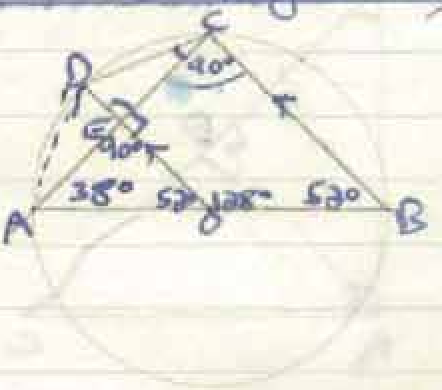
$\therefore XY = \frac{1}{2} BC$

$\therefore XY \parallel BC$, $XY = \frac{1}{2} BC$.

20/20

Myn Wyn Evans Form VI, Geometry 2-12-65

3(i)



Given

AB diameter of circle DCBA, DO ∥ CB.

~~To Prove~~. To find ∠ACD.

∠ACB = 90° (∠ in a semicircle)

∴ ∠CBA = 180° − 38° − 90° (∠s of a △)

= 52°.

∴ ∠CEO = 90° (int opp ∠s = 180°, DO ∥ CB)

∴ ∠EOB = 180° − 52° (∠s of a quadrilateral)

= 128°

∠AEO = 90° (∠s on a st line = 180°)

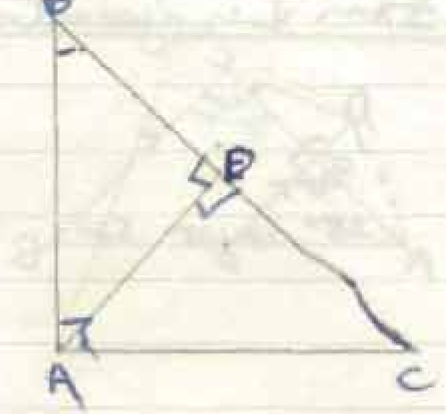
∴ ∠DEC = 90° (vert opp ∠).

∠EDC + ∠DCE = 90°

~~Cyclic~~ ∠ODC + 90° + ∠DCE = 180°

180
128
52

3(ii)



$\cos \angle ABC = \frac{BA}{BC} = \frac{BD}{BA}$

$\therefore AB^2 = BD . BC.$

$\cos \angle C = \frac{AC}{BC} = \frac{DC}{AC}.$

$\therefore AC^2 = BC . DC$

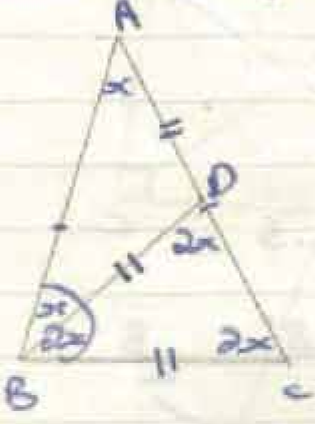
$\therefore AC^2 + AB^2 = \underbrace{BD . BC + BC . DC}_{= BC(BD+DC)}$

$\therefore AC^2 + AB^2 = BC^2$

1/10

Mugwa Wya Vau, Form II, Geometry Test.

2(i)



$\angle BAD = x^{\circ}$ (given)

$\therefore \angle ABD = x^{\circ}$ (base $\angle$s of an isos. Δ)

$\therefore \angle ADB = 180^{\circ} - 2x$ ($\angle$s of a Δ).

$\angle BDC = 180^{\circ} - (180^{\circ} - 2x)$ ($\angle$s on st. line)

$= 2x$ (ext $\angle$ of an isos Δ ADB)

$\therefore \angle DCB = 2x$ (base $\angle$s of an isos Δ)

$\therefore \angle ABC = 2x$ (base $\angle$s of an isos Δ)

$\therefore \angle DBC = 180 - (2x + 2x)$ ($\angle$s of a Δ).

$= 180 - 4x$

$\therefore$ ~~[illegible]~~

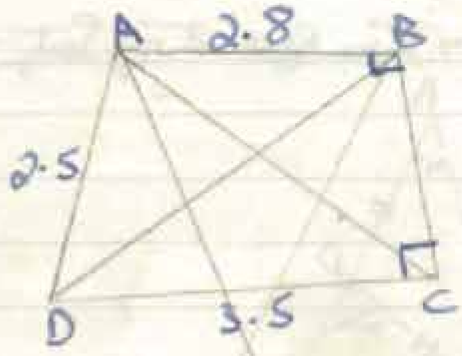
$\angle A + \angle ABC + \angle ACB = 180^{\circ}$ ($\angle$s of a Δ)

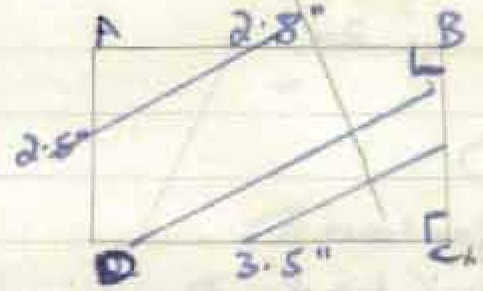
$\therefore x + 2x + 2x = 180.$

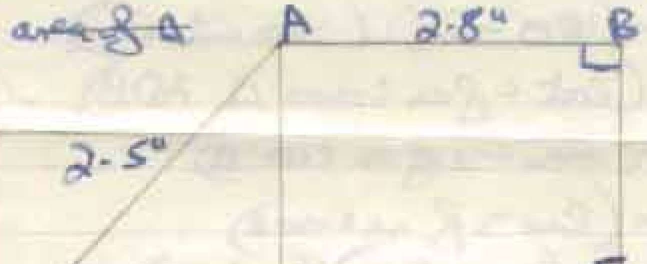
$5x = 180$

$x = \frac{180}{5} = 36^{\circ}.$

(ii)







area of quad $= (2.8 + 3.5) \times \frac{1}{2} \times BC$

$= 6.3 \times \frac{1}{2} \times BC$

area of Δ ADX $= \frac{1}{2} DX \times BC$.

<u>By Pythagoras' Theorem</u>

$AX^2 + DX^2 = AD^2$.

$AX^2 + 0.7^2 = 2.5^2$.

$AX^2 + 4.9 = 6.25$.

$AX^2 = 6.25 - ~~4.9~~\ 0.49$.

$= 1.35$

$AX = 2.4$

2.5
2.5
500
1125
6.25

2.5
2.8
.7

6.25
4.9
1.35

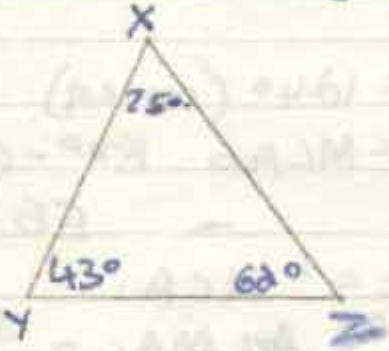
6.25
0.49
5.76

$\frac{8\frac{1}{2}}{10}$

$\frac{13\frac{1}{2}}{15}$

[illegible], Form II, Geometry Test. 3-12-65

2(i)



(i) $\angle X = 180° - 43° - 62°$
$= 75°$

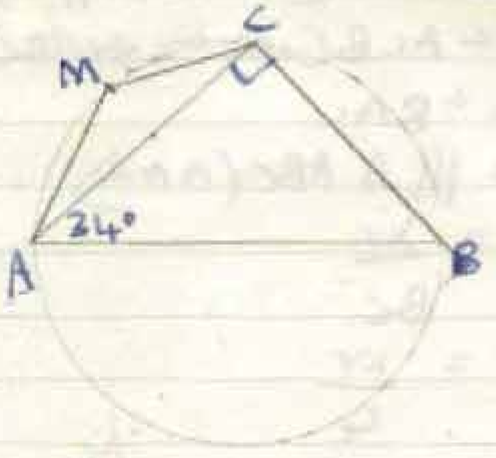
$\therefore$ The longest side is YZ because the longest side of a triangle is opposite the largest angle.

(ext ∠s of a polygon = 360°)

(ii) One exterior ∠ of the regular polygon $= \frac{360°}{16} = 22\frac{1}{2}°$.

$\therefore$ One interior ∠ of the " " $= 180° - 22\frac{1}{2}°$ (s.t. line)
$= 157\frac{1}{2}°$ ans.

(iii)



$\angle ACB = 90°$ (∠ in a semicircle = 90°).
$\therefore \angle CBA = 180° - 90° - 34$
$= 56°$.
$\therefore \angle AMC = 180° - 56°$ (opp ∠s of cyclic quad = 180°).
$= 124$

In $\triangle$ MAC $\therefore \angle MAC = \angle MCA$ (equal arcs related of ... 's at the arc).

but $\angle AMC = 124°$ (proved)

$\therefore \angle MAC + \angle MCA = 180° - 124°$

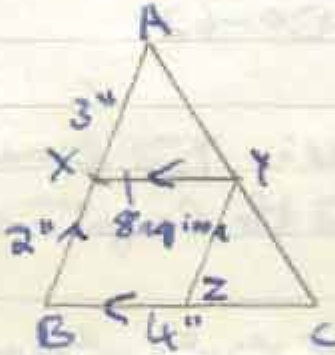
$= 56°$

4

but $\angle MAC = \angle MCA$.

$\therefore \angle MAC = \frac{1}{2} \times 56° = 28°$.

3i).



In $\triangle$s ABC and AXY

$\angle AXY = \angle ABC$ (alt $\angle$s XY // BC).

$\angle AYX = \angle ACB$ (alt $\angle$s XY // BC).

$\angle XAY = \angle BAC$

$\therefore \triangle AXY ||| \triangle ABC$ (AAA).

$\therefore \frac{AX}{AB} = \frac{XY}{BC}$.

$\therefore \frac{3}{5} = \frac{XY}{4}$

$\therefore 5XY = 12$

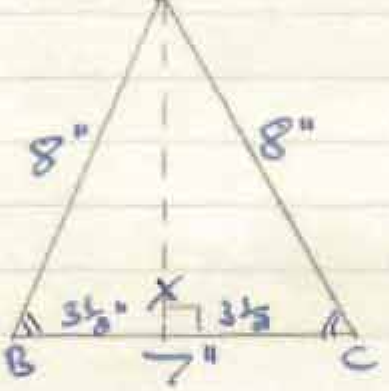
$XY = 2\frac{2}{5}$ unit

In $\triangle$s ACB and YZC

$\angle ZYC = \angle BAC$ (alt $\angle$s ZY // AB).

Myron Wyn Evans, Form IV, Geometry Test 3-12-65

Use Trigonometrical tables to find the angles and the area of an isosceles triangle with sides, 8", 8" and 7"



Given: isos Δ ABC with AB = AC = 8". BC = 7".

In Δ ABC BX = XC (perpendicular from height to base of isos Δ divides the base equally, in two rats).

$\therefore$ In Δ ABX ∠ AXB = 90°, BX = ½ × 7 = 3½", AB = 8"

$\cos \angle ABX = \dfrac{BX}{AB} = \dfrac{3.5}{8} = 0.4375$

$\therefore \angle ABX = 64° 3'$

$\therefore \angle ACB = 64° 3'$ (base ∠s of an isos Δ).

$\therefore \angle BAC = 180° - 64° 3' - 64° 3'$ (∠s of a Δ).

$= 51° 54'$

In Δ ABX $\sin \angle ABX = \dfrac{AX}{AB}$

$\therefore \sin 64° 3' = \dfrac{AX}{8}$

5/5

$\therefore AX = 8 \times \sin 64° 3'$

$= 8 \times 0.8992$

$= 7.1936"$

$\therefore$ area Δ ABC = ½ 7 × 7.1936 sq" = 25.1776.

= ~~3.5 ×~~ ½ × 50.3552 sq inches