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THE PHOTOMAGNETON $\hat{B}^{(3)}$ AND LONGITUDINAL GHOST FIELD $B^{(3)}$ OF ELECTROMAGNETISM

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The concepts are introduced of the longitudinal ghost field $B^{(3)}$ and photomagneton $\hat{B}^{(3)}$ of electromagnetism: $B^{(3)} = \langle\hat{B}^{(3)}\rangle = B^{(0)}\langle\hat{J}\rangle/\hbar$, where $B^{(0)}$ is the magnetic flux density amplitude and $\hat{J}$ the angular momentum operator of a photon beam. The major implication is that the individual photon has *three* degrees of polarization, the longitudinal one being accompanied by the ghost field $B^{(3)}$ which has no energy or linear momentum, and is generated from the angular momentum of the photon.

Key words: ghost field, photomagneton, electromagnetism.

1. INTRODUCTION

In order to derive without classical electrodynamics Planck's radiation law [1], S. N. Bose [2] used the notion that light has only two degrees of polarization. In this context, Pais [3] clearly explains that Bose used this notion to derive the correct value of the premultiplier in Planck's law, i.e., $8\pi\nu^2/c^2$. Here ν is the oscillator frequency in the old quantum theory of light, and c is the speed of light in vacuo. At that time (1924) the idea that a particle (named the photon in 1926 by Lewis) can have only two polarizations in three dimensional space was unprecedented. In Euclidean and Minkowski spaces it is intuitively expected that a particle have three degrees of space-like freedom. Pais further recounts [3] that there exists no rest frame for photon spin *if the photon is massless*; and contemporary gauge invariance means that the separation between orbital and intrinsic (spin) angular momentum is ambiguous. The restriction to two polarizations also means that the little group of the Poincaré group of electromagnetism [4] is E(2), which is well known [4] to have no physical meaning. Most seriously, the customary approach is deeply flawed in that it leads to a loss of manifest covariance in special relativity, in that the four-potential A_μ must give a field with only two, transverse components. The Bohm-Aharonov effect [5] shows that A_μ is physically meaningful, and so must have four components in free space.

In this Letter, we demonstrate the fact that in the classical and quantum theories of electromagnetism, there exist the longitudinal *ghost field (Gespensterstrahlung)* $\boldsymbol{B}^{(3)}$, which is the expectation value of the longitudinal *photomagneton* $\hat{B}^{(3)}$:

$$\boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{k} = \langle\hat{B}^{(3)}\rangle = B^{(0)}\frac{\langle\hat{J}\rangle}{\hbar}. \tag{1}$$

Thus $\boldsymbol{B}^{(3)}$ and $\langle\hat{B}^{(3)}\rangle$ are frequency-independent magnetic flux densities in free space. Here $B^{(0)}$ is the scalar amplitude of the electromagnetic flux density, $\boldsymbol{k}$ an axial unit vector in the propagation axis, $\hat{J}$ the beam angular momentum, and $\hbar$ the reduced Planck constant, $h/2\pi$. It is shown that $\hat{B}^{(3)}$ has no energy, no linear momentum, and does not affect the Planck law, because its associated oscillator frequency ν is zero. It owes its existence purely to the intrinsic spin of the photon, whose scalar magnitude in the quantum theory is $\hbar$. The photomagneton and its classical equivalent, the ghost field, can nevertheless be detected experimentally through its ability to magnetize, and phenomena such as the inverse Faraday effect [6–13], and optical Faraday effect [14, 15] can be described in terms of $\hat{B}^{(3)}$ at first and higher orders. This provides unequivocal experimental support for the existence of $\hat{B}^{(3)}$. The latter is consistent with the idea that the photon may have a tiny mass [16–18]. However small, finite photon mass means immediately that the photon must have three well-defined polarizations, and consequently, the existence of $\boldsymbol{B}^{(3)}$ in Maxwellian theory and $\hat{B}^{(3)}$ in the quantum theory means that finite photon mass is a consistent and natural idea.

2. CLASSICAL (MAXWELLIAN) ELECTRODYNAMICS

Within Maxwellian theory in free space, the ghost field $\boldsymbol{B}^{(3)}$ is related to the usual wave fields $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ by a cyclical Lie algebra [19]:

$$\begin{aligned} \boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} &= iB^{(0)}\boldsymbol{B}^{(3)*} = iB^{(0)}\boldsymbol{B}^{(3)}, \\ \boldsymbol{B}^{(2)} \times \boldsymbol{B}^{(3)} &= iB^{(0)}\boldsymbol{B}^{(1)*} = iB^{(0)}\boldsymbol{B}^{(2)}, \\ \boldsymbol{B}^{(3)} \times \boldsymbol{B}^{(1)} &= iB^{(0)}\boldsymbol{B}^{(2)*} = iB^{(0)}\boldsymbol{B}^{(1)}. \end{aligned} \tag{2}$$

Here $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ are complex conjugate wave fields (the usual magnetic components in circular polarization of the electromagnetic plane wave), and represent two transverse modes with orthogonal (circular) polarizations. In the standard Maxwellian theory of electrodynamics [20, 21] these are the only two polarizations considered, and describe left- and right- circularly polarized plane waves. However, Eqs. (2) show clearly that this picture is incomplete, *because if the ghost field* $\boldsymbol{B}^{(3)}$ *were zero,* $\boldsymbol{B}^{(1)}$ *and* $\boldsymbol{B}^{(2)}$ *would vanish, and all electromagnetism would be lost.*

We assert therefore that in classical electrodynamics there are three magnetic components $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ *and* $\boldsymbol{B}^{(3)}$ *of a travelling plane wave in vacuo. These are interrelated in the circular basis by Eq. (2). The third component, the ghost field*

$$\boldsymbol{B}^{(3)} = \frac{\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}}{(iB^{(0)})} = B^{(0)}\boldsymbol{k}, \tag{3}$$

is real and independent of phase [19].

By considerations of the Planck law of radiation, which is known [1, 3] to be valid experimentally, and which is based on only two polarizations (1) and (2), it follows that $\boldsymbol{B}^{(3)}$ cannot contribute to free space electromagnetic energy density. It can be shown [19, 23] that this is true if $\boldsymbol{B}^{(3)}$ is accompanied by the imaginary $i\boldsymbol{E}^{(3)}$, so that

$$U^{(3)} = \frac{1}{2}\left(\epsilon_0 i\boldsymbol{E}^{(3)} \cdot i\boldsymbol{E}^{(3)} + \frac{1}{\mu_0}\boldsymbol{B}^{(3)} \cdot \boldsymbol{B}^{(3)}\right) = 0. \tag{4}$$

Since $\boldsymbol{B}^{(3)}$ is a real magnetic flux density, it causes magnetization, as observed in the inverse and optical Faraday effects [6–15]. It can be shown [23] that the inverse

Faraday effect *vanishes*, contrary to observation [6–13], if $\boldsymbol{B}^{(3)} = 0$. In contrast, since $i\boldsymbol{E}^{(3)}$ is imaginary, it is considered not to be a physical electric field strength and to produce no spectral effects. Significantly, no spectral effects due to a longitudinal electric field of this type have been recorded, in contrast to the magnetic inverse and optical Faraday effects. The expression (4) is also consistent with the fact that $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ do not contribute to the Poynting vector and therefore do not contribute to the intensity of radiation $I(\nu)$, the density of states, or the Planck law. This again is as observed experimentally.

3. THE FUNDAMENTAL PHOTOMAGNETON $\hat{B}^{(3)}$

The transmutation of these classical ideas to the quantum theory takes place through the usual concepts of photon and photon spin. Since [24]

$$I(\nu) = c\rho(\nu), \tag{5}$$

where $\rho(\nu)$ is the density of radiation oscillator states, it is seen that $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ do not affect the Planck law, because they add nothing to $I(\nu)$ or to $\rho(\nu)$. In Planck's law,

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3}\left(\frac{e^{-h\nu/kT}}{1-e^{-h\nu/kT}}\right), \tag{6}$$

so that the Planck frequency ν associated with $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ is zero. This is consistent with the fact that these fields do not contribute to the classical equivalent of Eq. (6), the Rayleigh-Einstein-Jeans law. Nevertheless, in quantum theory it can be shown [19] that $\boldsymbol{B}^{(3)}$ becomes the *photomagneton* operator

$$\hat{B}^{(3)} = B^{(0)}\frac{\hat{J}}{\hbar}, \tag{7}$$

where $\hat{J}$ is the angular momentum of the photon beam. For one photon, the magnitude of $\hat{J}$ is $\hbar$, so that the expectation value $\langle\hat{J}\rangle/\hbar = \boldsymbol{k}$, the axial unit vector that defines the classical ghost field $\boldsymbol{B}^{(3)}$,

$$B^{(3)} = \langle \hat{B}^{(3)} \rangle = B^{(0)}\mathbf{k}. \tag{8}$$

We are led to the conclusion that the photon has three degrees of polarization.

In quantum theory the photomagneton $\hat{B}^{(3)}$ is phase-free, and as we have seen is associated with $\nu = 0$, i.e., with zero energy ($h\nu$) and zero *linear* momentum ($h\nu/c$). It is directly proportional, however, to the frequency-independent photon *angular* momentum, which is 0, ±ħ, not ±ħ as in the original theory of S. N. Bose [2]. Because of the existence of $\hat{B}^{(3)}$, the photon becomes a boson with three degrees of polarization; and this does not affect the validity of the Planck law, Eq. (6).

4. THE EXISTENCE OF PHOTON REST MASS

A boson with three degrees of polarization is a particle with mass. A boson with two degrees of polarization is a massless particle with only two helicities, the latter being components of particle spin along the direction of particle translation. In the standard Poincaré group [14], the helicity is the ratio of the Pauli-Lubansky pseudo-four-vector to the generator of space-time translation. In contemporary gauge theory, however, the existence of photon rest mass is usually not considered, because the gauge invariance condition,

$$m_0 A_\mu A_\mu = 0, \tag{9}$$

is solved by $m_0 = 0$. It has been shown recently, however [25], that the alternative solution

$$m_0 \neq 0, \quad A_\mu A_\mu = 0 \tag{10}$$

is consistent with the Dirac condition, and importantly, with the *experimental* fact that A_μ is physically meaningful through the Bohm-Aharonov effect [5]. The solution (10) allows m_0 to be non-zero, and this is consistent with our finding that the photon has three degrees of polarization (left and right circular and longitudinal).

The extensive and detailed work of de Broglie [16] and Vigier [18] and others on photon mass is therefore supported *strongly* by the existence of $\hat{B}^{(3)}$. We note that there is irrefutable experimental evidence for the magnetizing effects of $\hat{B}^{(3)}$ in the inverse Faraday effect [6–13] and optical Faraday effect [14, 15]. Recently the optical Cotton-Mouton effect has been reported experimentally [26] and interpreted [27] with $B^{(3)2}$. Note also that if $\hat{B}^{(3)}$ were zero, as required by the customary two polarization model, then the antisymmetric part of light intensity would vanish from Eq. (3), in conflict with experimental evidence from light scattering, for example. The Stokes S_3 parameter would vanish [19, 25], an incorrect result. In other words, if $\boldsymbol{B}^{(3)} =?\ 0$, then $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} =?\ 0$; but it is known experimentally that $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ is not zero, so $\boldsymbol{B}^{(3)}$ is not zero, and the photon has three polarizations.

Finally, it appears that the Einstein-de Broglie theory of light [28], in which waves and particles in light are both real, and co-exist, can accommodate the photomagneton $\hat{B}^{(3)}$ through the existence of photon spin, which, if the photon has mass, is now well defined in its rest frame as the angular momenta 0, ±ħ. The field $\hat{B}^{(3)}$ becomes the Einstein-de Broglie guiding (or pilot) field of the frequency-independent photon (particle) spin. This makes a profound difference to the basic theory of electromagnetism; for example A_μ becomes manifestly covariant, with four components, as needed, and the obscure E(2) little group [4] is replaced by a well-defined and physically transparent rotation group in three dimensions. These are two of the consequences of the existence of the photomagneton $\hat{B}^{(3)}$ of light; and the ghost field $\boldsymbol{B}^{(3)}$, its expectation value. There is no algebra akin to Eq. (2) for $i\boldsymbol{E}^{(3)}$, and it appears that this field is unphysical. The photon spin cannot generate a longitudinal electric field by symmetry [19].

Further experimental work on the inverse and optical Faraday effects is urgently required to elucidate the intensity dependence of the influence of $\hat{B}^{(3)}$. The original experiment of van der Ziel *et al.* [6] showed a dominating second-order effect, but effects at first order in $\hat{B}^{(3)}$ are also expected.

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