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THE OPTICAL AHARONOV-BOHM EFFECT

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It is shown that the vector potential of a circularly polarized laser causes the optical equivalent of the Aharonov-Bohm effect. An estimate is made of the expected fringe shift due to a circularly polarized laser directed through an optical fiber in an electron diffraction experiment, and it is shown that the effect is equivalent to that of a magnetic field.

Key words: optical Aharonov-Bohm, field $\mathbf{B}^{(3)}$, electromagnetic

1. INTRODUCTION

The Aharonov-Bohm (AB) effect [1-3] is well known to produce a fringe shift in an electron diffraction pattern by means of a gauge transformation into the non-trivial topology [3] of the vacuum of the potential four-vector, A_μ, which is thus shown to have a physical significance in the quantum field theory. The gauge transformation ensures that there is a fringe shift due to the extra electron momentum $-e\mathbf{A}^{(t)}$ even in regions where there is no magnetic flux density, and therefore no non-zero curl of the vector potential. Here $\mathbf{A}^{(t)}$ is the gauge transformed vector potential with zero curl. The AB effect is therefore based on the fact that a magnetic flux density of any kind is invariant under a gauge transformation. In the thirty years since its prediction [1] and verification [2] AB theory has been developed extensively to include fundamental concepts such as the topology of the vacuum [3], and the experimental development has resulted in new methods for mapping magnetic patterns in materials such as superconductors [4]. There is no reasonable doubt therefore that A_μ has physical significance. In this Letter we propose the optical Aharonov-Bohm (OAB) effect, due to a circularly polarized laser beam, which replaces the iron whisker, or equivalent solenoid device [2] of the original AB experiment. An estimate shows that the OAB effect is feasible with a laser beam of moderate power directed between the incoming electron beams, and contained within an optical fiber of about a micron in radius. The basic theory of the OAB effect is shown to be straightforward, and is based on the text-book result [5] that a circularly polarized laser drives an electron in a circular trajectory, i.e., generates a non-zero transverse momentum, a momentum which can be worked out from the real part of the vector potential of the circularly polarized laser. It is, furthermore, well known [5] that a static magnetic effect generates the same type of electron trajectory, a circle, so that under circumstances defined in this Letter, the effect on an electron of a phase free magnetic field (denoted $\boldsymbol{B}^{(3)}$) and a circularly polarized laser is identical.

Experimental conditions are suggested under which the existence of the OAB can be proven unequivocally, by isolating from the electrons the magnetic field of the fiber guided laser with a shield made, for example, of a ferroelectric material. This procedure attempts to ensure that the gauge transformed four-potential $A_\mu^{(t)}$ with zero curl is the only laser property present in regions occupied by electron wavefunctions, and is a procedure analogous to isolating the magnetic field in a solenoid in the traditional AB experiment [2]. The OAB phenomenon can in principle be developed into a useful analytical technique by embedding the laser carrying fiber in a material to be analyzed, and looking at the resulting electron interference patterns.

1. FUNDAMENTAL THEORY OF THE OAB EFFECT

In its simplest form [3] the theory of the conventional AB effect considers the electron wavefunction

$$\psi = \psi_0 \exp\left(\frac{i}{\hbar}\boldsymbol{p}\cdot\boldsymbol{r}\right), \tag{1}$$

where ψ_0 is its scalar amplitude, $\boldsymbol{p}$ the electron momentum, $\boldsymbol{r}$ a space coordinate and $\hbar$ the reduced Planck constant as usual. The electron momentum $\boldsymbol{p}$ is augmented in simple AB theory [3] by

$$\boldsymbol{p} \rightarrow \boldsymbol{p} - e\boldsymbol{A}^{(t)}, \tag{2}$$

where $\boldsymbol{A}^{(t)}$ has no curl, and is generated by a gauge transformation into the vacuum of the vector potential whose curl defines $\boldsymbol{B}$ inside a solenoid (or iron whisker) placed in the shadow of the interfering electron beams.

The existence of an *optically generated* AB effect is based on the inference that under certain circumstances, the *direct* effect on a single electron of a circularly polarized laser is the same precisely as that of a real, phase free, magnetic flux density, $\boldsymbol{B}^{(3)}$. The extra transverse momentum imparted *directly* to the electron by a field $\boldsymbol{B}^{(3)}$ is

$$\boldsymbol{p} = -e\boldsymbol{A}_\perp = -e\frac{B^{(0)}}{2}(-Y\boldsymbol{i} + X\boldsymbol{j}), \tag{3}$$

where $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are unit vectors in X, Y, and Z of the Cartesian frame, and where $\boldsymbol{B}^{(3)} := B^{(0)}\boldsymbol{k}$. This generates directly a non-zero electronic angular momentum

$$\boldsymbol{J} = e\boldsymbol{r}\times\boldsymbol{A}_\perp. \tag{4}$$

The transverse electronic momentum imparted by a circularly polarized laser is

$$\boldsymbol{p} = -e\frac{B^{(0)}c}{\sqrt{2}\omega}(-\boldsymbol{i}\sin\phi + \boldsymbol{j}\cos\phi), \tag{5}$$

where ω is the laser's angular frequency and ϕ its phase. The magnetic flux density amplitude of the laser is $B^{(0)}/\sqrt{2}$. The linear transverse momenta imparted directly to the electron by $\boldsymbol{B}^{(3)}$ and by the circularly polarized laser are *identical* under the condition

$$R := (X^2 + Y^2)^{\frac{1}{2}} = \sqrt{2}\,\frac{c}{\omega}; \tag{6}$$

and so, under this condition, the effect of the laser can be understood precisely in terms of the equivalent phase free magnetic field $\boldsymbol{B}^{(3)}$. The equivalence condition (6) is a constraint on the vector potential of $\boldsymbol{B}^{(3)}$,

$$\boldsymbol{A}_{\perp} = \frac{B^{(0)}}{2}(-Y\boldsymbol{i} + X\boldsymbol{j}), \qquad X^2 + Y^2 = 2\kappa^2, \tag{7}$$

where $\boldsymbol{\kappa}$ is the wave vector of the laser. This means that a circularly polarized laser generates a field $\boldsymbol{B}^{(3)}$, and it can be shown [6–10] that this is related to the usual transverse, oscillating, phase-dependent fields of the laser by a cyclically symmetric set of equations

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*}, \ \textit{et cyclicum}, \tag{8}$$

in a circular basis (1), (2), (3) defined by the unit vectors [11],

$$\boldsymbol{e}^{(1)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j}), \qquad \boldsymbol{e}^{(2)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} + i\boldsymbol{j}), \qquad \boldsymbol{e}^{(3)} = \boldsymbol{k}. \tag{9}$$

Thus $\boldsymbol{B}^{(2)}$ is the complex conjugate of $\boldsymbol{B}^{(1)}$, and $\boldsymbol{B}^{(3)}$ is pure real and equal to its complex conjugate [6–10]. The conjugate product $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ is observed in the inverse Faraday effect [12], and is the antisymmetric part of light intensity [6]. Therefore $\boldsymbol{B}^{(3)}$ is real and observable in the inverse Faraday effect and related magnetic effects of light and electromagnetic radiation in general.

Having realized the existence of $\boldsymbol{B}^{(3)}$, it is a simple matter to adopt the conventional AB theory [3] for the fringe shift expected from a circularly polarized laser in a fiber as described in the introduction. We use the standard [5] vacuum electrodynamical relation

$$B^{(0)} = \left(\frac{W}{\epsilon_0 c^3 A r}\right)^{\frac{1}{2}} = \left(\frac{I_0}{\epsilon_0 c^3}\right)^{\frac{1}{2}}, \tag{10}$$

where W is the power of the laser in watts, Ar the area of the optical fiber, and ϵ_0 the permittivity in vacuo in S.I. units. In Eq. (10) I_0 is the laser intensity in watts per square meter. The flux (in weber, i.e., tesla m^{-2}) through the fiber is therefore

$$\Phi = ArB^{(0)} = \left(\frac{ArW}{\epsilon_0 c^3}\right)^{\frac{1}{2}}, \tag{11}$$

and standard AB theory gives the fringe shift (in meters)

$$\Delta x = \frac{L}{d}\frac{e}{m_0 c}\Phi . \tag{12}$$

In Eq. (12), d is the inter-slit distance, L the interplate distance, $\lambda = h/(m_0 c)$ the electron wavelength, e the charge and m_0 the mass of the electron respectively. Assuming $L/d = 10$, then for a one micron radius fiber, Eq. (12) reduces to

$$\Delta x \sim 10^{-3} W^{\frac{1}{2}}\ \mu m. \tag{13}$$

For a laser of moderate power, the fringe shift of the OAB is easily observable using contemporary pulsing methods, which can routinely generate a power of up to 10^{12} watts.

Equation (12) is the simplest description of the optical Aharonov-Bohm effect – the shift in the electron interference fringes of a two slit interference device caused by a circularly polarized laser contained within an optical fiber.

By using the standard theorem [3],

$$\oint \boldsymbol{A}\cdot d\boldsymbol{r} = \int \nabla\times\boldsymbol{A}\cdot d\boldsymbol{s}, \tag{14}$$

it becomes clear that the OAB effect can be described in terms of a magnetic field

$$\boldsymbol{B}^{(3)} = \nabla\times\boldsymbol{A}_{\perp} \tag{15}$$

and that the non-zero $\boldsymbol{A}_{\perp}$ is a vector function whose curl *inside the fiber* is also non-zero,

$$\boldsymbol{A}_{\perp} = \frac{B^{(0)}}{2}(-X\boldsymbol{i} + Y\boldsymbol{j}), \qquad X^2 + Y^2 = 2\left(\frac{c}{\omega}\right)^2. \tag{16}$$

Therefore the unequivocal observation of the OAB would demonstrate experimentally the existence of the magnetic field $\boldsymbol{B}^{(3)}$ which is the curl of $\boldsymbol{A}_{\perp}$.

DISCUSSION

Under the equivalence condition (6), the non-zero angular momentum imparted directly to a single electron by a circularly polarized laser is the angular momentum due to the magnetic field $\boldsymbol{B}^{(3)}$, and can be expressed in terms of the vector potential $\boldsymbol{A}_{\perp}$. The field $\boldsymbol{B}^{(3)}$ of the laser therefore induces a magnetic dipole moment

$$\boldsymbol{m}^{(3)} = \chi' \boldsymbol{B}^{(3)} = -\frac{e^2 c^2}{m_0 \omega^2} \boldsymbol{B}^{(3)}, \tag{17}$$

where χ' is the real part of the single electron's susceptibility. It is noteworthy that this classical susceptibility is exactly the same in form as that routinely calculated in atoms and molecules [13]. The OAB is not of course due to the *direct* action of $\boldsymbol{B}^{(3)}$ on the electron, but is due to the gauge transformation into the vacuum of the vector potential $\boldsymbol{A}_{\perp}$ associated with $\boldsymbol{B}^{(3)}$. Nevertheless, the OAB would show the presence of $\boldsymbol{B}^{(3)}$ in the fiber in precisely the same way as the AB shows the presence of a field $\boldsymbol{B}$ in the traditional iron whisker [2]. If the shielding around the fiber is removed, or if a free laser beam is allowed to interact directly with the interfering electron beams, there is, in principle, a shift due to the field $\boldsymbol{B}^{(3)}$ interacting directly with the electrons of the beam as in Eqs. (3) and (5).

Extensive theoretical work on $\boldsymbol{B}^{(3)}$ [6–10] has shown that it is consistent with the principles of classical and quantum electrodynamics, and with the conservation theorems [5] of electromagnetism and matter. In quantum electrodynamics it is directly proportional to the angular momentum of the photon, a quantity ($\hbar$) which is also phase free. Equation (17) shows that $\boldsymbol{B}^{(3)}$ can act directly at first order on a single electron, producing a magnetic dipole moment.

The OAB shows the effect of a gauge transformed vector potential due to $\mathbf{B}^{(3)}$. From the standard relativistic Hamilton-Jacobi equation of motion of e in A_μ [5],

$$\mathbf{m}^{(3)} = \xi \chi' \mathbf{B}^{(3)}, \tag{18}$$

where

$$\xi = \left(1 + \left(\frac{m_0 \omega}{e B^{(0)}}\right)^2\right)^{-\frac{1}{2}} \tag{19}$$

is a relativistic correction factor which renders Eq. (17) correctly covariant. Equation (18) can be expressed for $\left(m_0\omega / eB^{(0)}\right)^2 \leq 1$, , using an ordinary Maclaurin series, as a sum of terms

$$\mathbf{m}^{(3)} = \chi' \mathbf{B}^{(3)} \left(1 + \left(\frac{m_0 \omega}{e B^{(0)}}\right)^2 + \frac{1}{2}\left(\frac{m_0 \omega}{e B^{(0)}}\right)^4 + \ldots\right), \tag{20}$$

the first of which is Eq. (17). Equations (17) and (18) show conclusively that the electron behaves in A_μ as if it were under the influence of the novel field $\mathbf{B}^{(3)}$, which is a fundamental field of electromagnetism, defined by the cyclic equations (8), and whose magnitude is $B^{(0)}$.

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REFERENCES

[1] Y. Aharonov and D. Bohm, *Phys. Rev.* **115**, 484 (1959).
[2] R. G. Chambers, *Phys. Rev. Lett.* **5**, 3 (1960).
[3] L. H. Ryder, *Quantum Field Theory* (Cambridge University Press, Cambridge, 1987).

[4] A. Tonomura, *Nature* **360**, 51 (1992); R. G. Chambers, *Rev. Mod. Phys.* **57**, 339 (1985).
[5] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon, Oxford, 1975).
[6] M. W. Evans, in Vol 85(2) of Ref. 14.
[7] M. W. Evans, *Physica B* **182**, 227, 237 (1992); **183**, 103 (1993); *Mod. Phys. Lett.* **7**, 1247 (1993); M. W. Evans and F. Farahi, *J. Mol. Struct.* **300**, 435 (1993); J.-P. Vigier and M. W. Evans, *Found. Phys.*, in press; M. W. Evans, *Found. Phys. Lett.* and *Found Phys.*, in press.
[8] M. W. Evans, *The Photon's Magnetic Field* (World Scientific, Singapore, 1992); M. W. Evans and A. A. Hasanein, *The Photomagneton in Quantum Field Theory*, Vol. 1 (World Scientific, Singapore, 1994).
[9] M. W. Evans and J.-P. Vigier, *The Enigmatic Photon, Vol. 1 The Field* $\mathbf{B}^{(3)}$ (Kluwer, Dordrecht, 1994).
[10] M. W. Evans, in *Waves and Particles in Light and Matter*, A. Garuccio and A. van der Merwe, eds. (Plenum, New York, 1994).
[11] R. Zawodny, in Vol. 85(1) of Ref. 14.
[12] A. Piekara and S. Kielich, *Arch. Sci.* **11**, 304 (1958); P. S. Pershan, *Phys. Rev.* **130**, 919 (1963); J. P. van der Ziel, P. S. Persham, and L. D. Malmstrom, *Phys. Rev. Lett.* **15**, 190 (1965) and *Phys. Rev.* **143**, 574 (1966); P. W. Atkins and M. H. Miller, *Mol. Phys.* **75**, 503 (1968); Y. R. Shen, *Nonlinear Optics* (Wiley, New York, 1984); J. Deschamps, M. Fitaire, and M. Lagoutte, *Phys. Rev. Lett.* **25**, 1330 (1970) and *Rev. Appl. Phys.* **7**, 155 (1972); T. W. Barrett, H. Wohltjen, and A. Snow, *Nature* **301**, 694 (1983); S. Woźniak, M. W. Evans, and G. Wagnière, *Mol. Phys.* **75**, 81 (1992); R. Zawodny, Vol. 85(1) of Ref. 14, (a review).
[13] P. W. Atkins, *Molecular Quantum Mechanics* (Oxford University Press, Oxford, 1983).
[14] M. W. Evans, and S. Kielich, eds., *Modern Nonlinear Optics*, Vols. 85(1), 85(2),and 85(3) of *Advances in Chemical Physics*, I. Prigogine and S. A. Rice, eds. (Wiley Interscience, New York, 1993/1994).