# ONE PHOTON AND MACROSCOPIC B CYCLIC EQUATIONS: REPLY TO VAN ENK

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The B cyclics of electrodynamics, which relate transverse and longitudinal fields in vacuo, are one photon relations which are also valid on a macroscopic scale. In the same way, the Maxwell equations in the received view were originally phenomenological relations between electric and magnetic fields, but, in the received view are also written down for one photon. Point by point replies to van Enk are given.

Key words: $\boldsymbol{B}^{(3)}$ field; classical theory, N photon theory.

## 1. INTRODUCTION

It has been established empirically that circularly polarized electromagnetic radiation magnetizes matter [1–5]. This effect can be understood through the conjugate product $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ of vector potentials of the electromagnetic wave in vacuo. The reply to the comments by van Enk [6] is based on the fact that such an effect exists theoretically for one photon interacting with one electron (Sec. 2). Section 3 extends the argument to a beam of many photons. Section 4 contains some detailed replies to the various points raised by van Enk

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[6].

## 2. INTERACTION OF ONE PHOTON WITH ONE ELECTRON

It has been established [7,8] that the inverse Faraday effect can be understood using the relativistic Hamilton-Jacobi equation and the conjugate product

$$\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} = \left(\frac{c}{\omega}\right)^2 \boldsymbol{B}^{(2)} \times \boldsymbol{B}^{(2)}, \qquad (1)$$

$\underset{\sim}{B}^{(1)}$

where $c$ is the speed of light in vacuo and $\omega$ the angular frequency. The quantum hypothesis [7,8],

$$eA^{(0)} = \hbar\kappa, \qquad (2)$$

reduces this to a one-photon theory. Here $\hbar\kappa$ is the linear momentum of one photon, where $\hbar$ is the Dirac constant and $\kappa$ the wavenumber $\kappa = \omega/c$. Here $e$ is the elementary charge and $A^{(0)}$ is the amplitude of $\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$. In the limit of complete transfer of energy from one photon to one electron, this theory shows consistently that the energy transferred is $\hbar\omega$, the energy of one photon. The Dirac equation has been used to confirm the result [8] in relativistic quantum field theory. Therefore for one photon, there exist the cyclic relations

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*}, \quad \text{et cyclicum}, \qquad (3)$$

between components of the vector

$$\boldsymbol{B} = \boldsymbol{B}^{(1)} + \boldsymbol{B}^{(2)} + \boldsymbol{B}^{(3)}, \qquad (4)$$

which is the total magnetic flux density of one photon expressed in the space basis ((1), (2), (3)). Equations (3) are established by: (a) the empirical existence of $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$; (b) the geometry of space; and in terms of flux $\boldsymbol{\Phi}$ (weber) are simply relations in space between rotation generators of O(3) for one fluxon, $\boldsymbol{\Phi}^{(0)} = \hbar/e$, the unit of magnetic flux carried by the photon. It is noted that Eqs. (3) are valid independently of any consideration of volume, they rely only on the existence of $\hbar/e$ and on the structure of the rotation group O(3).

The Dirac equation itself [8] establishes the existence of $\mathbf{A}^{(1)} \times \mathbf{A}^{(2)}$ in the interaction of one photon with one electron, and therefore establishes that of $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$. The geometry of space then establishes $\mathbf{B}^{(3)}$ through Eqs. (3), which are also spin angular momentum relations for one photon. Equations (3) form a local Poincaré invariant gauge theory which is also CPT invariant [9] and is therefore a self consistent field theory. Since $\mathbf{B}^{(3)}$ is phase free its divergence is zero as required and so is that of the complete vector $\mathbf{B}$. In the basis ((1), (2), (3)),

$$\mathbf{B}^{(1)} \cdot \mathbf{B}^{(2)} = \mathbf{B}^{(2)} \cdot \mathbf{B}^{(1)} = \mathbf{B}^{(3)} \cdot \mathbf{B}^{(3)}, \tag{5}$$

so that the square of $\mathbf{B}^{(3)}$ is the same as that of $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$. The energy density for one photon can then be defined through the energy density generated by $\mathbf{B}$ [10],

$$En = \hbar\omega = \frac{1}{\mu_0}\int \mathbf{B} \cdot \mathbf{B} dV, \tag{6}$$

where $\mu_0$ is the permeability in vacuo and $V$ the volume occupied by the photon. In general, since Eqs. (3) are spin angular momentum relations for one fluxon $\hbar/e$, the whole of spin angular quantum theory can be applied to them self-consistently, as for any particle with angular momentum.

## 3. MACROSCOPIC B CYCLICS

The empirical evidence for the existence of $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$ has been obtained with a beam of many photons. The Maxwell equations were used by Pershan [11] to predict the existence of the inverse Faraday effect. For many photons, as for one photon, Eqs. (3) follow from the empirical verification of $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$ [1–5] and from the geometry of space. In the same way the Maxwell equations in the received view are considered to be valid for a beam of $N$ photons for all $N$. It is assumed implicitly that photon-photon interaction effects do not affect the validity of the Maxwell equations when applied to a beam of many photons. Therefore for $N$ photons $\mathbf{B}^{(3)}$ is the macroscopic field defined by geometry and by the empirical existence of the macroscopic $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$. Thus $\mathbf{B}^{(3)}$ is simply the third space component of $\mathbf{B}$, the other two being $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$. Any theory that can be applied to $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$ can also be applied to $\mathbf{B}^{(3)}$. This is precisely in analogy to angular momentum theory in quantum mechanics for many particles. Any perceived conceptual problems caused by the fact that $\boldsymbol{B}$ is a flux density (tesla) can be removed by using the flux (weber) in which considerations of volume (or cross section of that volume, i.e., area) can be removed.

## 4. DETAILED REPLIES TO VAN ENK

In this section, the points raised by van Enk are answered individually.

(1) The $B$ cyclics are valid for one photon, and therefore, if properly applied, for a beam of $N$ photons, monochromatic or otherwise. They indicate non-Maxwellian electrodynamics in the vacuum, although $\boldsymbol{B}^{(3)}$ happens also to be a solution of Maxwell's linear approximation to the more general field equations [7]. If, however, one attempts to take a cross product of two modes with different electrodynamic phases, a phase dependent longitudinal term will result whose divergence is non-zero. Thus, in the Cartesian basis, if

$$\begin{aligned} \boldsymbol{B}_1^{(1)} &= \frac{B^{(0)}}{\sqrt{2}}(i\boldsymbol{i}+\boldsymbol{j})e^{i\phi_1}, \\ \boldsymbol{B}_2^{(2)} &= \frac{B^{(0)}}{\sqrt{2}}(-i\boldsymbol{i}+\boldsymbol{j})e^{-i\phi_2}, \end{aligned} \tag{7}$$

with phases $\phi_1 := \omega_1 t - \boldsymbol{\kappa}_1\cdot\boldsymbol{r}$ and $\phi_2 = \omega_2 t - \boldsymbol{\kappa}_2\cdot\boldsymbol{r}$ then,

$$\boldsymbol{B}_1^{(1)} \times \boldsymbol{B}_2^{(2)} = iB^{(0)2}e^{i(\phi_1-\phi_2)}\boldsymbol{e}^{(3)*}, \tag{8}$$

and the divergence of the quantity $\boldsymbol{B}_1^{(1)} \times \boldsymbol{B}_2^{(2)}$ is not zero. This means that we have carried out a cross product between transverse magnetic field components of two different modes, corresponding to two different photons of energies $\hbar\omega_1$ and ~~homegae~~$_2$. The result must therefore be a quantity constructed from more than one photon, i.e., from the interaction between two photons, and for this reason cannot be fundamental to one photon, unless the latter exists in two different places at the same time. The procedure is equivalent to taking an $\boldsymbol{i}$ unit vector through the origin of frame $O_1$, and cross multiplying it into a $\boldsymbol{j}$ unit vector through a different origin of a different frame $O_2$. The resultant $\boldsymbol{k}$ unit vector is a property neither of frame $O_1$

$\hbar\omega_2$

nor frame $O_2$.

(2) Our point (1) also shows why van Enk's point (2) is incorrect. Our $B$ cyclics are valid for one photon, and therefore for $N$ photons. If the beam is not monochromatic, there might be interaction effects, but these will not lead to divergence free magnetic fields in general. Terms from the interaction of different modes in our point (1) do not contribute to the fundamental $\boldsymbol{B}^{(3)}$ for one photon, i.e., interaction terms from different modes disappear as required in van Enk's point (2). The $B$ cyclics can also be written in terms of the fluxon, $\hbar/e$, the flux per photon (weber), with no reference to volume or area. This is the clearest way of showing that the $B$ cyclics are relations of spacetime itself. Linear superposition is always maintained, furthermore. If, for the sake of argument, we consider radiation with modes 1 to $n$ of different frequencies

$\underline{B}_1^{(2)}$ $\underline{B}_1^{(3)}$

$$\begin{aligned} \boldsymbol{B}_1 &= \boldsymbol{B}_1^{(1)} + \boldsymbol{B}_2^{(2)} + \boldsymbol{B}_3^{(3)}, \\ &\vdots \\ \boldsymbol{B}_n &= \boldsymbol{B}_n^{(1)} + \boldsymbol{B}_n^{(2)} + \boldsymbol{B}_n^{(3)}, \end{aligned} \tag{9}$$

and the total vector $\boldsymbol{B}$ is linear in (1), (2) and (3) at any frequency. If we allow cross terms to occur between different modes,

good

$$\boldsymbol{B}_1^{(1)} \times \boldsymbol{B}_2^{(2)} =?\ iB^{(0)} B_{12}^{(3)*},\ et\ cyclicum, \tag{10}$$

the complete quantity $\boldsymbol{B}_{12}$ is again linear in its components,

$$\boldsymbol{B}_{12} = \boldsymbol{B}_{12}^{(1)} + \boldsymbol{B}_{12}^{(2)} + \boldsymbol{B}_{12}^{(3)}, \tag{11}$$

but this is not in general a divergence free magnetic flux density. It is claimed by van Enk that

$$\int \boldsymbol{B}_{12} \cdot \boldsymbol{B}_{12} dV = 0. \tag{12}$$

This may or may not be true, but it does not affect the linear superposition represented by equations such as (9) and (11). The complete dot product,

$$\boldsymbol{B}_{tot} \cdot \boldsymbol{B}_{tot} = (\boldsymbol{B}_1 + \boldsymbol{B}_2 + \boldsymbol{B}_3 + \ldots) \cdot (\boldsymbol{B}_1 + \boldsymbol{B}_2 + \boldsymbol{B}_3 + \ldots), \tag{13}$$

is another linear superposition of auto and cross terms. In

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linear polarization we have relations such as

$$\left(\boldsymbol{B}_1^{(1)} \times \boldsymbol{B}_2^{(2)}\right)_R = -\left(\boldsymbol{B}_1^{(1)} \times \boldsymbol{B}_2^{(2)}\right)_L, \tag{14}$$

which imply $\boldsymbol{B}_{12}^{(3)*} = \mathbf{0}$ in linear polarization but not zero in circular polarization. Cross products of modes such as $\boldsymbol{B}_1^{(1)} \times \boldsymbol{B}_2^{(2)}$, however, produce in general a phase dependent quantity whose divergence is not zero. Only auto terms produce divergence and phase free fields such as $\boldsymbol{B}^{(3)}$. Therefore if a magnetic field is accepted as having to be free of divergence, only auto terms will contribute in the photons of an ensemble. If $\boldsymbol{B}^{(3)}$ is phase free, the divergence of $\boldsymbol{B}^{(1)} + \boldsymbol{B}^{(2)} + \boldsymbol{B}^{(3)}$ is zero. Equations such as (9), (11) and (13) represent linear superpositions of components in three dimensions, there is never any contradiction with linear superposition.

(3) For one photon of energy $\hbar\omega$, the spin angular momentum operator in the (3) axis, $\hat{J}^{(3)}$, is related to the photomagneton operator [7] $\hat{B}^{(3)}$ by $\hat{B}^{(3)} = B^{(0)}\hat{J}/\hbar$ and the classical limit of $\hat{J}^{(3)}/\hbar$ is the unit axial vector $\boldsymbol{k} = \boldsymbol{e}^{(3)}$ in the (3) axis. There is a similar relation for the one photon flux operator $\hat{\Phi}^{(3)} = \Phi^{(0)}\hat{J}^{(3)}/\hbar$, where $\Phi^{(0)} = \hbar/e$ is the fluxon. These relations are valid for one photon and therefore for $N$ photons if properly applied. van Enk claims that the definitions in his Eqs. (4) and (5) are mutually incompatible. However, $B_0 = |\boldsymbol{E} \times \boldsymbol{E}^*|/E_0 c$ (in his notation); and the eigenvalue of $\hat{J}/\hbar$ is $\boldsymbol{e}^{(3)}$. Therefore there is no inconsistency for one photon, and none for $N$ photons if the theory is properly applied.

$\hat{J}^{(3)}/\hbar$

$\hat{J}^{(3)}/\hbar$

(4) *Conventional theory* uses $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ and therefore, ineluctably, uses $iB^{(0)}\boldsymbol{B}^{(3)*}$. To assert otherwise is to assert a theory that is incompatible with the Lie algebra of the Poincaré group; and which is therefore unphysical in special relativity. *Conventional theory* cannot explain the inverse Faraday effect without the use of $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$, which implies the existence of $\boldsymbol{B}^{(3)}$ in vacuo. It is now known [12–32] that there are many types of vacuum longitudinal solutions of Maxwell's linear field equations themselves, and $\boldsymbol{B}^{(3)}$ is one of these, the fundamental phase free spin field [15–17]. The claim by van Enk that $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ is an operator, and that $\boldsymbol{B}^{(3)}$ is not a magnetic field, is just that, a *claim* of the received view. Equations (3) are $\hat{C}\hat{P}\hat{T}$ conserving equations of field theory which are therefore relativistically covariant. This is alone sufficient to show

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that $\boldsymbol{B}^{(3)}$ is a field component in vacuo, otherwise the $\hat{C}\hat{P}\hat{T}$ theorem [7] is rendered meaningless.

(5) This point is similar to van Enk's point (4): $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ always implies $\boldsymbol{B}^{(3)}$ for one photon if we accept the Lie algebra of the Poincaré group. It therefore implies $\boldsymbol{B}^{(3)}$ for $N$ photons, and this is shown empirically in magneto-optics [1].

(6) From our point (5), and from empirical data [1], it has, indeed, been demonstrated that $\boldsymbol{B}^{(3)}$ is a magnetic flux density, and as such, magnetizes material matter.

(7) Symmetry arguments, as embodied in group theory, are among the most powerful in physics, and a theory that allows for one photon the existence of $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$, but not of $iB^{(0)}\boldsymbol{B}^{(3)*}$, violates the symmetry of spacetime as described by the ten parameter Poincaré group. For example, such a theory produces the physically meaningless E(2) little group [7]. If $\boldsymbol{B}^{(3)} = ?\boldsymbol{0}$, Poincaré group symmetry is violated, and therefore Poincaré group symmetry does not allow this result.

(8) The trajectory of a fermion such as an electron in the $\boldsymbol{B}^{(3)}$ field has been evaluated self-consistently using the Dirac equation [8] and the relativistic Hamilton-Jacobi equation [7]. In both cases, $\boldsymbol{B}^{(3)}$ acts through its free space definition $-ie\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}/\hbar$ [8] and the interaction is relativistic. From this definition it is clear that $\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$ has been used in the calculation, using methods suggested by Landau and Lifshitz [33] and by Talin *et al*. [34] to calculate the trajectory of an electron in a circularly polarized electromagnetic field in a relativistic manner. In this context van Enk does not appear to have grasped fully the point that $\boldsymbol{B}^{(3)}$ is a phase-free, radiated electromagnetic component in vacuo; and is not a *static magnetic field*. The latter is a misnomer.

(9) If there is no photon-photon interaction, the energy of two photons is the sum of the energies of each individual photon,

$$En_T = \left(\frac{1}{\mu_0}\int \boldsymbol{B}\cdot\boldsymbol{B}dV\right)_1 + \left(\frac{1}{\mu_0}\int \boldsymbol{B}\cdot\boldsymbol{B}dV\right)_2 . \qquad (15)$$

The existence of $\boldsymbol{B}^{(3)}$ does not change this, because $\boldsymbol{B}^{(3)}$ is the third component of $\boldsymbol{B}$ in space. This is analogous with angular momentum theory for two non-interacting and spinning particles. If they interact then the total angular momentum is conserved and there may be a non-zero net interaction angular momentum. The difference between angular momentum and magnetic flux is that the latter must be free of divergence, as discussed already. Additionally photons propagate at $c$ in the received view, and the interaction of photons is a non-trivial, fully relativistic problem.

85(2)

The correct description of $\boldsymbol{B}^{(3)}$ in terms of photon number operators is Eq. (20), page 116, Vol. 85(1) of Ref. 1, a reference which van Enk selectively misquotes in favor of his erroneous equations (12) and (13). In Ref. 1,

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$$\hat{B}^{(3)} := \hat{B}_{\Pi} = \left(\frac{\epsilon_0}{8 I_0 c}\right)^{1/2} \left(\frac{2\Pi\omega}{n^2(\omega) V}\right) \hbar (\hat{n}_+ - \hat{n}_-), \tag{16}$$

e.c.

where $\epsilon_0$ is the S.I. vacuum permittivity, $I_0$ the beam intensity, $\omega$ the angular frequency, $n(\omega)$ the refractive index, $V$ the quantization volume, and $(\hat{n}_+ - \hat{n}_-)$ the dimensionless difference between left and right circularly polarized photon number operators. In our Eq. (16), the quantity $I_0$ is proportional to the inverse square root of photon number, and so $\boldsymbol{B}^{(3)} \cdot \boldsymbol{B}^{(3)*}$ is proportional to photon number as required classically and empirically. Therefore the product $\boldsymbol{B}^{(3)} \cdot \boldsymbol{B}^{(3)*}$ is linear in photon number. As in several other instances, van Enk misinterprets or misrepresents our published work. The energy of one photon is by definition $\hbar\omega$. If the quantization volume is $V$ (volume occupied by one photon) then the energy density for one photon is $\hbar\omega / V$, which is equal to $\boldsymbol{B} \cdot \boldsymbol{B} / \mu_0$ where $\boldsymbol{B} = \boldsymbol{B}^{(1)} + \boldsymbol{B}^{(2)} + \boldsymbol{B}^{(3)}$, and where $\mu_0$ is the permeability in vacuo. This definition self-consistently incorporates $\boldsymbol{B}^{(3)}$, and the energy of two photons is $2\hbar\omega$.

$I_0^{-1/2}$

my mistake

(10) The dual transform of fields [33],

$$\boldsymbol{B}^{(3)} \rightarrow \frac{-i\boldsymbol{E}^{(3)}}{c}, \qquad \boldsymbol{E}^{(3)} \rightarrow ic\boldsymbol{B}^{(3)}, \tag{17}$$

leaves Maxwell's equations of electrodynamics invariant in vacuo. This means that the quantity $-i\boldsymbol{E}^{(3)}$ is significant, and different from zero. Standard special relativity shows that the square of $i\boldsymbol{E}^{(3)} + c\boldsymbol{B}^{(3)}$ is a real, physical, Lorentz invariant in vacuo. In our self-consistent calculus, real quantities are physical, pure imaginary quantities are unphysical, but mathematically non-zero. The square modulus of $-i\boldsymbol{E}^{(3)}$ is a real, physical quantity proportional to the square of $\boldsymbol{B}^{(3)}$. This result is that of ordinary complex number algebra. The complex basis ((1), (2), (3)) is a valid basis of the O(3) rotation group; and as such is equivalent to the real basis (X, Y, Z). Chubykalo and Smirnov-Rueda [12–14] have shown that there is a real, physical $\boldsymbol{E}^{(3)}$ in vacuo, a novel solution of Maxwell's field equations. This is a polar vector, however, and as such, cannot be obtained from a cross product such as $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$, because the latter always produces an axial vector, i.e., $\boldsymbol{B}^{(3)}$. For this reason, it is not possible to incorporate a real $\boldsymbol{E}^{(3)}$ in cyclic constructs such as (3), but it is possible to include an imaginary $i\boldsymbol{E}^{(3)}$, which has magnetic symmetry in special relativity. Dr. van Enk has been made aware of the work in Ref. 12, but has apparently declined to refer to it.

(11) In the basis ((1), (2), (3)) the Poynting vector is a real vector formed from the cross product in this basis of $\boldsymbol{E}^{(1)}$ and $\boldsymbol{B}^{(2)}$. The Poynting vector is proportional to the real linear momentum of one photon in a one photon beam. It is real in any valid basis of O(3); as are the $\boldsymbol{B}^{(3)}$ and $\boldsymbol{E}^{(3)}$ fields. To assert otherwise is erroneous.

(12) The complex basis ((1), (2), (3)) and the real basis (X, Y, Z) are equivalent bases of the O(3) group. However, the equations of electrodynamics indicate the need for the complex, Minkowskian and pseudo Euclidean basis of spacetime. Accordingly, the Lie algebra of the Poincaré group is complex, and the Maxwell equations are the archetypical equations of special relativity, within which vector combinations such as $c\boldsymbol{B} + i\boldsymbol{E}$ occur routinely. We can therefore infer the existence of $i\boldsymbol{E}^{(3)}$.

(13) The real and physical fields $\boldsymbol{B}^{(3)}$ and $\boldsymbol{E}^{(3)}$ have by now been demonstrated independently [12,13] to be solutions in vacuo of Maxwell's field equations. Transverse components thereof have both real and imaginary parts in general, and the component $\boldsymbol{B}^{(1)}$, for instance, is generated by $i\boldsymbol{E}^{(1)}/c$. These points have been developed in van Enk's own Ref. 5, which he has evidently not grasped in full detail, and which is quoted out of context. The bases ((1), (2), (3)) and (X, Y, Z) are equivalent, and it is therefore ~~purposeless~~ meaningless to claim that one of them is *not necessary*. Which one? It is clear also that Maxwell's equations allow $c\boldsymbol{B}$ to be replaced in a dual transformation either by $-i\boldsymbol{E}$ or by $-\boldsymbol{E}$. Both replacements are equally valid mathematically because both regenerate Maxwell's electrodynamic equations intact. It is not possible to claim, however, that a real, phase free, (*static*) $\boldsymbol{B}^{(3)}$ must by dual transform generate a real $-\boldsymbol{E}^{(3)}$. In magnetostatics this is trivially clear because the only physical field present is a magnetic field. In electrostatics the only field present is an electric field. In neither case are we allowed to claim that one type must be transformed into another by dual transform. The correct deriva-

Vol. 3 of van Enk's ref (5) is quoted as if published, whereas it is actually still in press, (April 12th., 1996), evidently, but van Enk has not read probably not it and ~~probably does not~~ volumes 1 and 2 are quoted, but not actually read. ~~[illegible]~~

Also: The claim by van Enk [illegible] ref. 85(2) [illegible] ref. 1 is a collection of reprints of previous work is incorrect. ~~Several~~ The majority of the papers were original to ref. 85(2).

tion of the real $\boldsymbol{E}^{(3)}$ has been given in Ref. 12, a preprint of which was has been made available to Dr. van Enk. The $\boldsymbol{E}^{(3)}$ field is well defined as the Coulombic component advocated by several workers in this century [12–32]. As argued already, the real $\boldsymbol{E}^{(3)}$ is polar, and for this reason does not occur as a real $\boldsymbol{E}^{(3)}$ in cyclic constructs such as (3). It occurs as the imaginary $i\boldsymbol{E}^{(3)}$, which is magnetic and axial. The imaginary $i\boldsymbol{E}^{(3)}$ is unphysical (does not act as an electric field), the real $\boldsymbol{E}^{(3)}$ is physical (acts as an electric field). The symmetry of $\boldsymbol{E}^{(3)}$ is polar, the symmetry of $i\boldsymbol{E}^{(3)}$ is axial.

It is concluded that van Enk's critique contains several errors and misconceptions, and by now there is a large amount of evidence, reviewed in the following section, in favor of longitudinal fields in vacuo, of which $\boldsymbol{B}^{(3)}$ is the fundamental, phase free, spin field.

## 5. DISCUSSION

The *B* cyclics prove that in vacuum electrodynamics transverse fields imply longitudinal fields and vice versa. There have emerged recently a series of independent confirmations of the existence of longitudinal fields in vacuo; of subluminal and superluminal solutions of the vacuum Maxwell equations; and novel non-Maxwellian structures for the basic field equations. Early indications of non-Maxwellian behavior and of longitudinal, acausal fields, go back to Majorana, Dirac, Oppenheimer and Wigner; and have recently been confirmed independently and essentially re-discovered after fifty years or more. In this Section these indications, both theoretical and empirical, are reviewed briefly.

Chubykalo and Smirnov-Rueda [12–14] have demonstrated the existence in vacuo of $\boldsymbol{B}^{(3)}$ and $\boldsymbol{E}^{(3)}$ from the Maxwell equations. This important work clears up several paradoxes in fundamental electrodynamics using a combination of transverse and longitudinal field components in vacuo. Dvoeglazov [15,16] has independently demonstrated the inference [17] that $\boldsymbol{B}^{(3)}$ is the space part of the Pauli-Lyuban'ski four-vector that is the most fundamental description of the vacuum electromagnetic field. Dvoeglazov *et al.* [18] have discussed inconsistencies between the Maxwellian approach and the Joos-Weinberg equations. Dvoeglazov has also provided the following scholarly summary of the study of longitudinal vacuum components of electromagnetism in the twentieth century.

The acausal, longitudinal component appears to have been first inferred in the scientific manuscripts of Ettore Majorana, circa 1928 to 1932 [19]. It was inferred independently by Oppenheimer [20], and Dirac [21]. On page 733 of Oppenheimer [20] the $En = 0$ acausal solution is given. Wigner [22] later identified this as the *additional, discrete, phase* <u>*free*</u> *(my emphasis, MWE) variable*. Much later, it was inferred independently by Gianetto [23] and by Ahluwalia and Ernst [24].

Hunter and Wadlinger [25] have developed a relativistic three dimensional soliton theory of the photon which implies the existence of longitudinal solutions, and importantly, have provided empirical evidence for the theory. Several empirical indications have emerged of the existence of superluminal phenomena [26] accompanied by longitudinal fields. Theoretical methods have been developed by Recami *et al.* [27] for the interpretation of these data with classical and quantum tachyons. Rodrigues *et al.* [28] have demonstrated rigorously that there exist novel subluminal and superluminal solutions of Maxwell's equations in vacuo, solutions which are accompanied by longitudinal components. The existence of the latter has been inferred independently by
29 Muñera and Guzmán ~~[28]~~ and by Mészáros [30], using adiabatic thermodynamics. Pope [31] has developed the philosophical concepts of causal realism to deal with these new developments. Roy and Roy [32] have inferred the existence of a primordial $\boldsymbol{B}^{(3)}$ in cosmology.


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