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PROOF OF THE EVANS-VIGIER FIELD FROM THE DIRAC EQUATION OF THE FERMION IN THE CLASSICAL FIELD: REPLY TO RIKKEN

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The Dirac equation of the fermion in the classical electromagnetic field is used to prove the existence of the Evans-Vigier field from the first principles of relativistic quantum theory.

Key words: Dirac equation, fermion, Evans-Vigier field.

1. INTRODUCTION

Recently, an experiment [1] has been reported which did not detect the optical Faraday effect (OFE) in liquid benzene. This negative result was used to conclude that the Evans-Vigier field $\boldsymbol{B}^{(3)}$ [2–5] does not exist. The existence, however, of magneto-optic effects is well supported empirically [6–10] and the OFE had been reported experimentally [11], under more appropriate conditions, prior to Rikken's paper, which does not refer to this work [11]. In this reply to Rikken's paper [1] the Dirac equation of one fermion in the classical electromagnetic field is used to prove the existence of $\boldsymbol{B}^{(3)}$ and to define the experimental conditions under which its effects can be observed specifically. The OFE was first predicted by Kielich [12] using non-relativistic arguments.

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2. THE $\boldsymbol{B}^{(3)}$ FIELD FROM THE DIRAC EQUATION

Using the Dirac equation, it can be shown [2] that the interaction of a fermion with a circularly polarized electromagnetic field is described by the following equation, after averaging over many field cycles,

$$\begin{aligned}\big((En - mc^2 + ecA_0)(En + mc^2 + ecA_0) - c^2(\boldsymbol{p} + e\boldsymbol{A}^*)\\ \cdot(\boldsymbol{p} + e\boldsymbol{A}) - ie^2c^2\boldsymbol{\sigma}\cdot\boldsymbol{A}^*\times\boldsymbol{A}\big)\psi = 0.\end{aligned} \tag{1}$$

Here ψ is the four-spinor of Dirac [13], $A_\mu := (\boldsymbol{A}, A_0)$, is the field's potential four-vector; e, m, and $\boldsymbol{p}$ are the charge, mass and momentum, respectively, of the fermion; c is the speed of light in vacuo, En is the total energy of the fermion, and mc^2 its rest energy in the observer's frame of reference. The four-vector A_μ is electromagnetic in nature, so $\boldsymbol{A}$ is complex [14–16]. The term in $\boldsymbol{\sigma}\cdot\boldsymbol{A}^*\times\boldsymbol{A}$, where $\boldsymbol{\sigma}$ is a Pauli spinor [13], leads to the Evans-Vigier field as follows.

2.1 WEAK FIELD LIMIT ($mc^2 \gg ecA_0$)

This limit corresponds to

$$I \ll \left(\frac{cm^2}{\mu_0 e^2}\right)\omega^2, \tag{2}$$

where I is the field intensity (W m^{-2}); μ_0 the vacuum permeability in S.I. [17]; and ω the field angular frequency. For an electron,

$$I \ll 7.72 \times 10^{-9}\ \omega^2, \tag{3}$$

and for a proton,

$$I \ll 0.026\ \omega^2. \tag{4}$$

Using, additionally, the non-relativistic limits given by Dirac [13], i.e., $En \sim mc^2$, $\boldsymbol{p} \sim \boldsymbol{0}$, the limit for an initially slow moving fermion, Eq. (1), becomes

$$W\psi := (En - mc^2)\psi \sim \left(\frac{e^2}{2m}\left(\mathbf{A}^*\cdot\mathbf{A} + i\boldsymbol{\sigma}\cdot\mathbf{A}^*\times\mathbf{A} - ecA_0\right)\right)\psi . \tag{5}$$

The Evans-Vigier field [2–5] in this limit emerges from the Dirac equation (5) and is defined by

$$\mathbf{B}^{(3)*} := -i\frac{e}{\hbar}\mathbf{A}\times\mathbf{A}^* := -i\frac{e}{\hbar}\mathbf{A}^{(1)}\times\mathbf{A}^{(2)}, \tag{6}$$

where $\hbar$ is the Dirac constant. This definition makes the term in $\boldsymbol{\sigma}\cdot\mathbf{A}\times\mathbf{A}^*$ an ordinary Zeeman effect term [2–5], with fermion half-integral spin eigenvalue $\pm\hbar/2$ as usual.

2.2 STRONG FIELD LIMIT ($mc^2 < ecA_0$)

Using $En \sim mc^2$; $\mathbf{p} \sim \mathbf{0}$, the Dirac Eq. (1) in this limit becomes

$$W\psi = \left(ec\boldsymbol{\sigma}\cdot\frac{i}{A^{(0)}}\mathbf{A}^*\times\mathbf{A}\right)\psi, \tag{7}$$

where [2–5] $A^{(0)} = (\mathbf{A}\cdot\mathbf{A}^*)^{1/2} = A_0$ is the scalar amplitude of the vector $\mathbf{A}$. In this condition, A_μ is lightlike, i.e., $A_\mu A_\mu = 0$. Using [2–5] $A^{(0)} = cB^{(0)}/\omega$, the Evans-Vigier field emerges from the Dirac equation (7) as

$$\mathbf{B}^{(3)*} := -\frac{i}{B^{(0)}}\mathbf{B}\times\mathbf{B}^* := \frac{i}{B^{(0)}}\mathbf{B}^{(1)}\times\mathbf{B}^{(2)}. \tag{8}$$

Equation (6) is transformed into Eq. (8) using the equivalence principle [2–5],

$$\frac{e}{\hbar} = \frac{\kappa}{A^{(0)}}, \tag{9}$$

where $\kappa = \omega/c$ is the momentum magnitude of a free photon. Equation (9) applies when $I > 7.72\times10^{-9}\,\omega^2$ for the electron or when $I > 0.026\,\omega^2$ for the proton; and means that the field intensity is such as to accelerate the fermion infinitesimally close to c. The momentum magnitude, $eA^{(0)}$, transferred from field to fermion is for all practical purposes the momentum magnitude, $\hbar\kappa$, of the free photon itself. This means that the photon has given up all its energy and momentum to the fermion.

3. DISCUSSION

Equation (8) was the first to be proposed [2–5], using arguments based on rotation operators of the free field. The limits of application of Eq. (8) were first shown in Ref. 2, Vol. 1, Eq. (411), using classical arguments. Equation (8) shows that $\boldsymbol{B}^{(3)}$ is a fundamental vacuum field because $B^{(0)}$, $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ are fundamental quantities of the classical field. The manner in which $\boldsymbol{B}^{(3)}$ interacts with matter has been summarized in Sec. 1. Rikken [1] worked with a peak I of about 5.5×10^{12} W m^{-2} at a frequency of 10,640 cm^{-1}; ($\omega = 1.77 \times 10^{15}$ rad s^{-1}). Equation (3) and (4) both show that he worked well within the weak-field limit, a limit in which definition (6) must be used and in which definition (8) does not hold. *Unfortunately*, Rikken [1] appears to have used the latter definition and does not refer to our Ref. 2. His sample, benzene, has no free electron spin and so the only OFE expected is that due to a perturbation of the polarizability tensor by $\boldsymbol{A} \times \boldsymbol{A}^*$ [18]. This effect was not detected, so it must be very small, as expected theoretically. To assert on the basis of this type of experiment that $\boldsymbol{B}^{(3)}$ does not exist contradicts the fundamentals of relativity and quantum mechanics on which Dirac [13] based his original analysis. The emergence of $\boldsymbol{B}^{(3)}$ from the Dirac equation shows that vague criticisms of the $\boldsymbol{B}^{(3)}$ field based on symmetry [19] and assertion [20] are incorrect.

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