RADIATION-INDUCED FERMION RESONANCE

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The Dirac equation is solved for two novel terms which describe the interaction energy between the half integral spin of a fermion and the classical, circularly polarized, electromagnetic field. A simple experiment is suggested to test the new terms and the existence of radiation-induced fermion resonance.

Key words: Dirac equation, $\mathbf{B}^{(3)}$ field, radiation-induced ESR.

Recently, Warren *et al.* [1,2] have made the first attempt to detect radiation-induced fermion resonance due to irradiation by a circularly polarized electromagnetic field [3-6]. In this Letter the Dirac equation is solved for one fermion in a classical electromagnetic field. Two new terms are inferred which show the theoretical existence of radiation-induced fermion resonance, and the experimental conditions under which this phenomenon can be detected are defined for an electron beam. The demonstration is based on the standard Dirac

Hamiltonian operator (Gaussian units)

$$\hat{H} = c\boldsymbol{\alpha} \cdot \left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right) + \beta mc^2 + eV, \tag{1}$$

describing a fermion (e.g. an electron) of mass m and charge e interacting with a classical electromagnetic field, with vector potential

$$A_\mu = \left(V, \mathbf{A}\right), \tag{2}$$

through the eigenvalue equation

$$\hat{H}\psi = E\psi. \tag{3}$$

Here $\boldsymbol{\alpha}$ and β are the usual Dirac matrices and ψ the four-component Dirac spinor. The rest energy of the fermion is mc^2 and its three momentum is $\mathbf{p}$, as usual.

In the usual non-relativistic approximation the calculation proceeds by setting up Eq. (3) for the proper dominant wavefunction ϕ:

$$\hat{H}'\phi = E\phi. \tag{4}$$

Writing the 4-component Dirac spinor

$$\psi = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} \tag{5}$$

where ψ_A and ψ_B are respectively the *large* and *small* component satisfying the condition (at second order in v/c),

$$\int \left(\psi_A^\dagger \psi_A + \psi_B^\dagger \psi_B\right) d^3x \;\simeq\; \int \psi_A^\dagger \left(1 + \frac{\pi^2}{8m^2c^2}\right) \psi_A d^3x \;\simeq\; 1; \tag{6}$$

(here $\boldsymbol{\pi} = \mathbf{p} - e/c\mathbf{A}$) the dominant two-component wavefunction ϕ in the non relativistic limit is given by (again at second order in v/c)

$$\phi \;\simeq\; \left(1 + \frac{\pi^2}{8m^2c^2}\right) \psi_A. \tag{7}$$

With standard methods of solution, we find that the Hamiltonian in Eq. (4) is made up of six terms as follows:

$$H = \frac{\pi^2}{2m} + eV - \frac{eh}{2mc}\boldsymbol{\sigma} \cdot \mathbf{B}, \tag{H1}$$

$$-\frac{\pi^4}{8m^2c^2}, \tag{H2}$$

$$-\frac{e\hbar^2}{8m^2c^2}\left(\nabla\cdot\mathbf{E}+\frac{1}{c}=\frac{\partial}{\partial t}\left(\nabla\cdot\mathbf{A}\right)\right), \tag{H3}$$

$$-\frac{e\hbar}{4m^2c^2}\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\times\mathbf{p}, \tag{H4}$$

$$+\frac{e^2\hbar}{4m^2c^3}\boldsymbol{\sigma}\cdot\mathbf{E}\times\mathbf{A}, \tag{H5}$$

$$-\frac{e^2\hbar}{4m^2c^4}\boldsymbol{\sigma}\cdot\left(\mathbf{A}\times\frac{\partial\mathbf{A}}{\partial t}\right). \tag{H6}$$

Here $\mathbf{B}=\boldsymbol{\nabla}\times\mathbf{A}$ is the magnetic field, and $\boldsymbol{\epsilon}:=\mathbf{E}+(1/c)(\partial\mathbf{A}/\partial t)$, where $\mathbf{E}=-\boldsymbol{\nabla}V-(1/c)(\partial\mathbf{A}/\partial t)$ is the electric field.

These terms can be interpreted as follows: (H1) is the well-known Schrödinger-Pauli-Hamiltonian; (H2) is the relativistic correction to the kinetic energy; (H3) is the Darwin term; (H4) gives the spin-orbit coupling term. These four terms are well known corrections to order $(v/c)^2$ of the Schrödinger equation (in the non relativistic limit). Terms (H5) and (H6) are novel, and represent the coupling between the half integral fermion spin and the circularly polarized classical electromagnetic field. They give rise to radiation induced fermion resonance, and we will focus on them.

In S.I. units, terms (H5) and (H6) both give rise to the interaction eigenenergy

$$H=\frac{\mu_0\hbar}{4c}\left(\frac{e}{m}\right)^2\frac{I}{\omega}\boldsymbol{\sigma}\cdot\mathbf{k}, \tag{8}$$

where I is the beam power density (Wm^{-2}) and ω its angular frequency. Here μ_0 is the vacuum permeability. In deriving Eq. (6) it has been assumed that $\mathbf{E}$ and $\mathbf{A}$ are plane waves in vacuum and that

$$I=\frac{c}{\mu_0}B^{(0)2}, \tag{9}$$

where $B^{(0)}$ is the scalar amplitude of $\mathbf{B}=\nabla\times\mathbf{A}$. The resonance frequency from Eq. (8) is calculated from spinor states as usual and is

$$\omega_{res}=\frac{\mu_0}{2c}\left(\frac{e}{m}\right)^2\frac{I}{\omega}=6\cdot4832\times10^7\frac{I}{\omega} \tag{10}$$

for the electron.

For example, if I is 100 watts per square centimeter ($10^6 Wm^{-2}$), and ω is tuned to 1.2815 MHz, ω_{res} occurs at the same frequency. This means that the resonance absorption can be detected as a decrease in the r.f. output applied to an electron beam. This simple experiment tests the existence of terms (H5)and (H6) provided that the r.f. output is accurately circularly polarized. There is no effect expected in linear polarization.

The clear importance of a positive result to this experiment would be that site specific shifts [1,2] of easily a megahertz or more could be induced by irradiation in an ESR spectrum, giving a new analytical technique. The prediction in Eq. (8) can be tested to probably one part in 10^9 if the resonance frequency can be resolved to a hertz or less. This gives a very severe test in fundamental physics of the semi-classical Dirac equation, and also of its presumably more accurate counterparts in quantum electrodynamics and unified field theory, 2E. Similarly, site specific shifts in the range of perhaps one to a thousand Hertz can be induced in an NMR spectrum [1,2] by pulsing with circularly polarized r.f. radiation. An NMR spectrometer has sub-hertzian resolution, so these site specific shifts should, in theory, lead to a new analytical technique as was the purpose of Refs. [1] and [2].

In deriving the result (7), we have used the proportionality of $\mathbf{A} \times \mathbf{A}^*$ to the conjugate product, $\mathbf{B} \times \mathbf{B}^*$ of magnetic flux densities in vacuo. For transverse plane waves, this is easily demonstrated through the relation $\mathbf{B} = \mathbf{\nabla} \times \mathbf{A}$. Both $\mathbf{A} \times \mathbf{A}^*$ and $\mathbf{B} \times \mathbf{B}^*$ are well known to be proportion al to the fundamental magnetic flux density $\mathbf{B}^{(3)}$ [5] of $O(3) = SU(2)$ electrodynamics. Therefore radiation induced fermion resonance is due to the interaction of electron spin with the $\mathbf{B}^{(3)}$ field.

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