

## On the Irrotational Nature of the $\mathbf{B}^{(3)}$ Field

Circularly polarized radiation produces phaseless magnetic effects in matter, an observation which can be explained through the fundamental  $\mathbf{B}^{(3)}$  field of the radiation. It is shown that the irrotational nature of this field is compatible with a multipole expansion of the radiation field.

Key words. Irrotational nature of  $\mathbf{B}^{(3)}$  ; multipole representation of  $\mathbf{B}^{(3)}$

### 9.1 Introduction

Several magneto-optic effects are known in nature, the earliest one to be observed is the inverse Faraday effect [1—3], in which circularly polarised radiation produces a phase free magnetization similar to that produced by a static magnetic field aligned in one axis ( $Z$ ). Magnetization by electromagnetic radiation has recently been interpreted [4—8] using a theorem which expresses the conjugate product of non-linear optics in terms of a phaseless magnetic field  $\mathbf{B}^{(3)}$  which is the space component of a Pauli-Lubanski four-vector ( $|\mathbf{B}^{(3)}|, \mathbf{B}^{(3)}$ ). If it is assumed that  $\mathbf{B}^{(3)}$  is zero there is no classical field helicity [5]. Therefore  $\mathbf{B}^{(3)}$  is a fundamental field

akin to the particle helicity introduced by Wigner [10]. Vector analysis leads to the conclusion that the empirically observed conjugate product [1—3] is uniaxial if  $\mathbf{B}^{(3)}$  is aligned with  $Z$ . In this simple case the field is therefore irrotational, its curl is zero because it is a simple axial vector in one axis: the empirical observation of the conjugate product in magneto-optics leads directly to this conclusion if we consider the conjugate product to be made up of plane waves propagating in the axis,  $Z$ , in which  $\mathbf{B}^{(3)}$  is aligned by definition. Therefore it can be expressed in terms of the gradient of a scalar function because the curl of such a function is always identically zero. If the plane waves are replaced by components of multipole radiation, then the  $\mathbf{B}^{(3)}$  vector in vacuo is still irrotational for all multipole components. This is shown as follows.

In Sec. 9.2 the  $\mathbf{B}^{(3)}$  field is worked out for multipole components of the radiated electromagnetic field. It is assumed for the sake of argument that the magnetic monopole does not exist in nature, although there are data which counter-indicate this assumption [11—13]. Therefore the scalar function of which  $\mathbf{B}^{(3)}$  is a gradient obeys Laplace's equation, whose solutions are well known in electrostatics. It is therefore straightforward to show that  $\mathbf{B}^{(3)}$  in general can be expressed in terms of multipole components in the spherical harmonic expansion, involving, as usual, the well known Legendre polynomials.

It is emphasized that every individual component in the multipole expansion of  $\mathbf{B}^{(3)}$  is irrotational, because every component is a particular solution of the Laplace equation for the scalar function of which  $\mathbf{B}^{(3)}$  is a gradient both by empirical observation [1—3] and by definition. This is a direct and clear consequence of two basic premises: that  $\mathbf{B}^{(3)}$  is phaseless (produces observable phase free magnetic effects [1—3]) and that there exist no observed magnetic monopoles in nature. If data show that magnetic monopoles exist on the contrary, the Laplace equation is replaced by a Poisson equation, with physical consequences which can be worked out with the well known solutions of Poisson's equation.

This simple line of argument has been developed in this paper in order to show that  $\mathbf{B}^{(3)}$  in multipole radiation is irrotational for all

multipole components if there are no magnetic monopoles. Recent arguments in the literature which claim that  $\mathbf{B}^{(3)}$  is somehow not irrotational [14,15] are counter-indicated by the arguments developed here and discussed in Sec. 9.3, in which the scalar function of which  $\mathbf{B}^{(3)}$  is a gradient is identified as a Stratton scalar potential [16—18]. The Laplacian of this scalar potential is zero if there are no magnetic monopoles in nature, and the solution of the Laplace equation allows  $\mathbf{B}^{(3)}$  to be expanded in terms of multipoles, in precisely the same way as angular momentum. The relation between the longitudinal  $\mathbf{B}^{(3)}$  and the transverse  $\mathbf{B}^{(1)} = \mathbf{B}^{(2)*}$  in vacuo (the B cyclic theorem) is a theorem of  $\hat{C}$  negative angular momentum components. As pointed out by Atkins [19] this allows the development of a large fraction of all quantum theory. The B cyclic theorem can therefore be used straightforwardly to quantize the electromagnetic field in vacuo.

## 9.2 Laplace Equation for the Gradient Function of $\mathbf{B}^{(3)}$

Since  $\mathbf{B}^{(3)}$  is empirically phaseless (*i.e.*, observed in nature to be phaseless and independent of the electromagnetic frequency and wavevector) and if it is assumed that there are no magnetic monopoles (magnetic charges or sources present) then it can be expressed in terms of the gradient of a scalar function:

$$\mathbf{B}^{(3)} = -\nabla \Phi_B, \quad (2.9.1)$$

which is determined by the well known Laplace equation [18,20],

$$\nabla^2 \Phi_B = 0. \quad (2.9.2)$$

If  $\mathbf{B}^{(3)}$  depended on the electromagnetic phase, it would oscillate at high frequencies and no phaseless magneto optic effects would have been observed [1—3]. Therefore the time dependent part of  $\mathbf{B}^{(3)}$  is zero, leading to Eq. (2.9.1). (Analogously a Coulomb field can be expressed as the

gradient of a scalar potential which obeys the Laplace equation in a source free region such as the vacuum in conventional electrostatics.)

To find the general form of  $\mathbf{B}^{(3)}$  in a multipole expansion, we therefore solve the Laplace equation for  $\Phi_B$ , and evaluate the gradient of this solution, which is [19,20],

$$\Phi_B = \frac{U(r)}{r} \rho(\theta) Q(\phi), \quad (2.9.3)$$

in spherical polar coordinates  $(r, \theta, \phi)$ . The general solution (2.9.3) can be written as [19,20],

$$\phi_B = (Ar^l + Br^{-2})Y_{lm}(\theta, \phi), \quad (2.9.4)$$

where  $Y_{lm}(\theta, \phi)$  are the spherical harmonics and  $A$  and  $B$  are constants. Here  $m$  and  $l$  are integers, with  $l$  running from  $-m$  to  $m$ . The solution of Laplace's equation is therefore obtained [19,20] as a product of radial and angular functions. The latter are orthonormal functions, the spherical or tesseral harmonics, which form a complete set on the surface of the unit sphere for the two indices  $l$  and  $m$ . Integer  $l$  defines the order of the multipole component,  $l = 1$  is a dipole;  $l = 2$  is a quadrupole;  $l = 3$  is an octopole;  $l = 4$  is a hexadecapole and so forth. The properties of the spherical harmonics are very well known.

The most general form of  $\mathbf{B}^{(3)}$  from Laplace's equation is therefore,

$$\left. \begin{aligned} \mathbf{B}^{(3)} &= -\nabla \Phi_m, \\ \Phi_m &= (Ar^l + Br^{-2})Y_{lm}(\theta, \phi). \end{aligned} \right\} \quad (2.9.5)$$

This is the phaseless magnetic field of multipole radiation. The solution (2.9.5) reduces to the simple [4—8],

$$\mathbf{B}^{(3)} = B^{(0)}\mathbf{e}^{(3)} = B^{(0)}\mathbf{k},$$

when  $l = 1, m = 0, r = Z, \theta = 0, A = -B^{(0)}, B = 0$  and  $\nabla = (\partial/\partial Z)\mathbf{k}$ . More generally, there exist other irrotational forms of  $\mathbf{B}^{(3)}$ ,

- a) The  $\mathbf{B}^{(3)}$  for dipolar radiation,  $l = 1, m = -1, 0, 1$ .
- b) The  $\mathbf{B}^{(3)}$  for quadrupole radiation,  $l = 2, m = -2, -1, 0, 1, 2$ .
- c) The  $\mathbf{B}^{(3)}$  for octopole radiation,  $l = 3, m = -3, -2, -1, 0, 1, 2, 3$ .
- d) The  $\mathbf{B}^{(3)}$  for hexadecapole radiation,  $l = 4, m = -4, -3, -2, -1, 0, 1, 2, 3, 4$ .
- e) The  $\mathbf{B}^{(3)}$  for  $n$  pole radiation,  $l = n, m = -n, \dots, n$ .

The  $\mathbf{B}^{(3)}$  for  $n$ -pole fields are irrotational for all  $n$  and are all solutions of Maxwell's equations and generalizations such as those due to Majorana [21] and Weinberg [22], Ahluwalia *et al.* [23] and Dvoeglazov *et al.* [24]. They are all phaseless and all contribute to magneto optical effects. In every case the longitudinal and transverse components are angular momentum components expressible in the language of spherical harmonics.

### 9.3 Discussion

In Sec. 9.2, we have firstly used empirical evidence from magneto optics to argue that the fundamental  $\mathbf{B}^{(3)}$  field is irrotational because for plane waves it is a simple vector defined in one axis ( $Z$ ). The possible forms of  $\mathbf{B}^{(3)}$  for  $n$  pole radiation were then worked out using the Laplace equation, *i.e.*, by expressing  $\mathbf{B}^{(3)}$  as the negative of the gradient of a scalar function. This procedure is equivalent to using the static part of a Stratton potential for the magnetic field [25]. The complete form of the Stratton potential is [26],

$$\mathbf{B} = -\nabla \phi_m - \frac{1}{c} \frac{\partial \mathbf{A}_m}{\partial t}, \quad (2.9.7)$$

and as shown recently by Afanasiev and Stepanofsky [27], the Stratton potential is needed for a complete description of the classical electromagnetic helicity in terms of a conservation equation and Noether's Theorem. This finding is consistent with the fact that the helicity is zero if  $\mathbf{B}^{(3)}$  is zero, a basic inconsistency in conventional electrodynamics [20], which uses a  $U(1)$  gauge and asserts that  $\mathbf{B}^{(3)}$  is zero.

It is also well known (for example problem 6.6 of Jackson's first edition [20]) that *any* vector field ( $\mathbf{B}$ ) can be expressed as the sum of irrotational and divergentless components under well defined conditions. This is consistent with the fact that the transverse plane wave  $\mathbf{B}^{(1)} = \mathbf{B}^{(2)*}$  is divergentless while the longitudinal  $\mathbf{B}^{(3)}$  is irrotational. Therefore we can write:

$$\mathbf{B} = \mathbf{B}^{(1)} + \mathbf{B}^{(2)} + \mathbf{B}^{(3)}, \quad (2.9.8a)$$

$$\nabla \cdot \mathbf{B}^{(1)} = \nabla \cdot \mathbf{B}^{(2)} = \nabla \cdot \mathbf{B}^{(3)} = 0, \quad (2.9.8b)$$

$$\nabla \times \mathbf{B}^{(3)} = \mathbf{0}, \quad (2.9.8c)$$

$$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = i\mathbf{B}^{(0)} \mathbf{B}^{(3)*}, \quad \text{et cyclicum}, \quad (2.9.8d)$$

and the B cyclic equation (2.9.8) is a condition under which  $\mathbf{B}^{(1)} = \mathbf{B}^{(2)*}$  is divergentless and  $\mathbf{B}^{(3)}$  is both irrotational and divergentless. This is self-consistent and consistent with empirical data from magneto optics [1—3]. This result will not be found in conventional electrodynamics because the former introduces an  $O(3)$  gauge through Eq. (2.9.8). It is, however, consistent with Laplace's equation as shown in Sec. 9.2.

In conventional electrostatics, the Coulomb field is expressed as the negative of the gradient of a scalar function in the presence of charges (electric monopoles). The Poisson equation is then solved to show that the scalar potential is proportional to the charge density in the universe. If there is no charge density, the scalar potential is zero and there is no Coulomb field. In the Coulomb gauge therefore there is no longitudinal, irrotational

electric field present in the conventional treatment of electrostatics and electrodynamics. The introduction of Maxwell's displacement current (a vacuum current) allows in electrodynamics the existence of transverse waves which are conventionally unaccompanied by the Coulomb field. Therefore the development of conventional (Maxwellian) electrodynamics is based on the existence without charges of a current, Maxwell's displacement current, made up of the time derivative of a transverse electric field which exists in the absence of sources (electric monopoles). This is self-inconsistent in several ways, as discussed recently by Chubykalo *et al.* [28] and by Lehnert *et al.* [29]. The most fundamental inconsistency is that the charge (or monopole) and the field take on a separate identity, the field, according to Maxwell, can exist without the charge, because Maxwell's displacement current can exist in the absence of sources. If so, it is equally valid, following Lehnert [29] to assume that the divergence of the electric field is non-zero in the absence of sources, or to introduce the vacuum convection current, following Chubykalo *et al.* [28]. Both procedures lead directly to  $\mathbf{B}^{(3)}$  in the vacuum. Furthermore, the B cyclic theorem (2.9.8d) is a relation between field components in the absence of magnetic monopoles, *i.e. in vacuo*, or *in the vacuum*.

A point of major importance, and a turning point in the development of electrodynamics, is that magneto-optical data have now been identified as giving direct and unequivocal empirical support for the existence of  $\mathbf{B}^{(3)}$  and an  $O(3)$  gauge. This is also logical support for Lehnert *et al.* [29] and for Chubykalo *et al.* [28], who have developed recently a self-consistent form of electrodynamics. Ultimately, it seems logical to develop electrodynamics and unified field theory on the basis that the primordial field exists in the vacuum, following Maxwell, and to assume that charge is a manifestation of the field as originally supposed by Faraday [28]. It also seems possible [28] to develop a fully covariant theory in electrodynamics which allows velocities greater than  $c$  and which allows the interrelation of field theory with action at a distance theory [28]. Proceeding on this basis, the B cyclic theorem becomes the archetypical theorem of the primordial vacuum field. It is simply a relation between components of spin angular momentum multiplied by a  $\hat{C}$  negative coefficient. Thus, the electromagnetic field is a

physical entity which has angular and linear momentum, as observed empirically in nature.

Finally, if we assume that magnetic monopoles exist in the universe, the fundamental magnetic field  $\mathbf{B}^{(3)}$  can be expressed as the negative of the gradient of a scalar function which is the solution of a Poisson equation,

$$\Phi_B(\mathbf{r}) = \Phi_B(0) + \frac{1}{4\pi} \int \frac{\nabla' \cdot \mathbf{B}(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} d^3r'. \quad (2.9.9)$$

Here  $\Phi_B(0)$  is a constant of integration, as discussed by Jackson [20] on his page 8 of the first edition, and where the divergence of the complete magnetic field  $\mathbf{B} = \mathbf{B}^{(1)} + \mathbf{B}^{(2)} + \mathbf{B}^{(3)}$  is non-zero because magnetic multipoles are assumed to be present in the universe. The  $\mathbf{B}^{(3)}$  field from this solution is axial, conservative and irrotational, in precise analogy to the central Coulomb field in the presence of electric monopoles in the universe.

It is concluded that the fundamental  $\mathbf{B}^{(3)}$  field responsible for magneto optical effects is irrotational both in the absence and in the presence of magnetic monopoles. As described on Jackson's page nine of the first edition [20], the line integral of Stokes' Theorem  $\oint \mathbf{B}^{(3)} \cdot d\mathbf{l}$  is zero over any closed path. This is a counter argument to Comay's recent assertion [14] that  $\mathbf{B}^{(3)}$  is not irrotational. Clearly, if this were true,  $\mathbf{B}^{(3)}$  would not be proportional to the empirically observable conjugate product appearing in the B cyclic theorem (2.9.8d), and  $\mathbf{B}^{(3)}$  would not be a fundamental C negative angular momentum of the electromagnetic field in vacuo: the primordial and fundamental electromagnetic spin angular momentum. As such,  $\mathbf{B}^{(3)}$  remains irrotational for n pole radiation and for plane waves in the presence and absence of electric and magnetic monopoles.

## Acknowledgments

The Alpha Foundation is warmly thanked for an affiliation and many colleagues for interesting Internet discussions.

## References

- [1] J. P. van der Ziel, P. S. Pershan, and L. D. Malmstrom, *Phys. Rev. Lett.* **15**, 190 (1965); *Phys. Rev.* **143**, 574 (1966).
- [2] J. Deschamps, M. Fitaire, and M. Lagoutte, *Phys. Rev. Lett.* **25** 1330 (1970); *Rev. Appl. Phys.* **7**, 155 (1972).
- [3] R. Zawodny in M. W. Evans and S. Kielich, eds., *Modern Nonlinear Optics* Vol. 85(1) of *Advances in Chemical Physics*, I Prigogine and S. Rice, eds. (Wiley Interscience, New York, 1997, paperback, 3rd printing), a review of magneto optics with ca. 150 references.
- [4] M. W. Evans, *Physica B* **182**, 227, 237 (1992).
- [5] M. W. Evans and J.-P. Vigiier, *The Enigmatic Photon, Volume 1: The Field  $\mathbf{B}^{(3)}$*  (Kluwer Academic, Dordrecht, 1994).
- [6] M. W. Evans and J.-P. Vigiier, *The Enigmatic Photon, Volume 2: Non-Abelian Electrodynamics* (Kluwer Academic, Dordrecht, 1995).
- [7] M. W. Evans, J.-P. Vigiier, S. Roy and S. Jeffers, *The Enigmatic Photon, Volume 3: Theory and Practice of the  $\mathbf{B}^{(3)}$  Field* (Kluwer Academic, Dordrecht, 1996).
- [8] M. W. Evans, J.-P. Vigiier, and S. Roy, eds., *The Enigmatic Photon, Volume 4: New Directions* (Kluwer Academic, Dordrecht, 1998), with review articles from leading specialists.
- [9] M. W. Evans and A. A. Hasanein, *The Photomagnetron in Quantum Field Theory* (World Scientific, Singapore, 1994).
- [10] E. P. Wigner, *Ann. Math.* **40**, 149 (1939).
- [11] M. Israelit, *Magnetic Monopoles and Massive Photons in a Weyl Type Electrodynamics* LANL Preprint 9611060 (1996), *Found. Phys.*, in press.
- [12] B Barish, G. Lin, and C. Lane, *Phys. Rev. D* **36**, 2641 (1987).
- [13] N. Rosen, *Found. Phys.* **12**, 213 (1982).

- [14] E. Comay, *Chem. Phys. Lett.* **261**, 601 (1996).
- [15] M. W. Evans and S. Jeffers, *Found. Phys. Lett.* **9**, 587 (1996), reply to Ref. 15.
- [16] A. F. Ranada, *Eur. J. Phys.* **13**, 70 (1992).
- [17] A. F. Ranada, *J. Phys. A* **25**, 1621 (1992).
- [18] H. Bacry, *Helv. Phys. Acta* **67**, 632 (1994).
- [19] P. W. Atkins, "*Molecular Quantum Mechanics*, 2<sup>nd</sup> edn., (Oxford Univ. Press, Oxford, 1983).
- [20] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962).
- [21] E. Majorana, *Nuovo Cim.* **14**, 171 (1937); papers in the Domus Galileiana, Pisa; *E. Gianetto, Lett. Nuovo Cim.* **44**, 140 (1985), theory developed in ref. (8) by E. Recami and M. W. Evans.
- [22] S. Weinberg, *Phys. Rev.* **133B**, 1318 (1964); 134B, 882 (1964).
- [23] D. V. Ahluwalia and D. J. Ernst, *Mod. Phys. Lett* **7A**, 1967 (1992).
- [24] V. V. Dvoeglazov, *Int. J. Theor. Phys.* **35**, 115 (1996); V. V. Dvoeglazov, Yu. N. Tyukhtyaev and S. V. Khudyakov, *Russ. J. Phys* **37**, 898 (1994), *Rev. Mex. Fis. Suppl.* **40**, 352 (1994); "Majorana Like Models in the Physics of Neutral Particles.", Int. Conf. Theory Electron, Cuautitlan, Mexico, 1995; in Ref.(8, "The Weinberg Formalism and New Looks at Electromagnetic Theory.", a review with ca. 100 references.
- [25] H. K. Moffat, *Nature* **347**, 367 (1990).
- [26] H. Pfister and W. Gekelman, *Am. J. Phys.* **59**, 497 (1994).
- [27] G. N. Afanasiev and Yu. P. Stepanovsky, "The Helicity of the Free Electromagnetic Field and its Physical Meaning." Preprint E2-95-413, J.I.N.R., Dubna, 1995; *Nuovo Cim.*, in press. Also M. W. Evans, *Physica A* **214**, 605 (1995); V. V. Dvoeglazov, "On the Claim that the Antisymmetric Field Tensor is Longitudinal after Quantization." in Ref. 8.
- [28] A. E. Chubykalo and R. Smirnov-Rueda, *Phys. Rev. E* **53**, 5373 (1996); *ibid.*, Ref. 8, "Action at a Distance and Self-Consistency of Classical Electrodynamics.", a review; *ibid.*, "The Convective Displacement Current." (World Scientific, planned); A. E. Chubykalo, M. W. Evans, and R. Smirnov-Rueda, *Found. Phys. Lett.* **10**, 93 (1997).

- [29] B. Lehnert, *Phys. Scripta* **53**, 204 (1996); *Optik* **99**, 113 (1995); in Ref. 8, "Electromagnetic Space-Charge Waves in Vacuo.", a review; B. Lehnert and S. Roy, "Electromagnetic Theory with Space Charges in Vacuo and Non-Zero Rest Mass of Photon." (World Scientific, planned)