A GENERAL THEORY OF NON-ABELIAN ELECTRODYNAMICS

ABSTRACT

The general theory of gauge fields is used to develop a theory of electrodynamics in which the fundamental structure is non-Abelian and in which the internal gauge field symmetry is O(3), based on the existence of circular polarization and the third Stokes parameter. The theory is used to provide an explanation for the Sagnac effect with platform at rest and in motion. The Sagnac formula is obtained by considering the platform in motion to be a gauge transformation. The topological phases can be described straightforwardly with non-Abelian electrodynamics, which produces a novel magnetic field component for all types of radiation, a component which is proportional to the third Stokes parameter. The theory provides a natural explanation for the inverse Faraday effect without phenomenology.

INTRODUCTION

Radiation always contains the third Stokes parameter, which is proportional to the conjugate produce {1-5} of complex vector potentials $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$. The third Stokes parameter characterizes circular polarization, and all other polarizations can be thought of as combinations of circular polarization. For example, linear polarization is a mixture of half right and half left circular polarization. Therefore a self consistent theory of radiation, or electrodynamics, must consider the fact that $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ is always non-zero. In this paper, we develop a theory of electrodynamics in which this basic requirement is fulfilled self-consistently within the framework of general gauge field theory {6, 7} and demonstrate its advantages through a novel explanation of the Sagnac effect with platform at rest and in motion. The additional phase shift observed in the moving platform is derived from a gauge transformation of the non-Abelian gauge field theory. In Section 2, the structure of the novel non-Abelian electrodynamics is derived from a closed loop in Minkowski spacetime, leading to a satisfactory description of the third Stokes parameter in terms of a novel magnetic field $\boldsymbol{B}^{(3)}$ which is self-dual and does not give rise to Faraday induction. In Section 3, considerations of gauge transformation in the O(3) internal gauge symmetry of the electrodynamic theory are given and in Section. 4, the electrodynamical theory is applied to an explanation of the Sagnac effect. In Section 5, the theory is applied to the topological phase using a novel non-Abelian Stokes theorem, and in Section 6, to a novel explanation of interferometric effect in optics.

DERIVATION OF THE ELECTRODYNAMIC THEORY

Electrodynamics can be derived from gauge theory by using a round trip with covariant derivatives {8}. Such a procedure is valid for any internal gauge group symmetry and electrodynamics can be derived in consequence for any internal gauge group. A general multi-component field vector ψ is acted upon by an operator which transports the vector around a closed loop using the theory of infinitesimal generators. The result of the trip around the closed loop is expressed as:

$$\psi'' = e^{ix}e^{iy}e^{-ix}e^{-iy}\psi \tag{1}$$

The four exponentials in this expression are operators which can be expanded in a Taylor series. To second order, the series takes the form:

$$e^{ix}e^{iy}e^{-ix}e^{-iy} = 1-(xy-yz)+x^2+y^2+\ \ldots. \tag{2}$$

The object $zy - yz$ is a commutator of operators. For example, if x and y denote Lorentz boosts, {8} it can be shown that the commutator leads to the Thomas precession. In general, the commutators are determined by group theory {8} and the method is valid for all groups.

In order to recover a theory of electrodynamics from this powerful general approach, the symbols x and y are defined as follows:

$$x \equiv D_\mu \Delta x^\mu ,$$
$$y \equiv D_\upsilon \Delta x^\upsilon , \qquad (3)$$

where D_μ and D_υ are covariant derivatives and where x^μ and x^υ are four-vectors in Minkowski spacetime {8}. The covariant derivatives can be defined in any gauge group symmetry and can be expressed in the shorthand notation {8}

$$D_\mu \equiv \partial_\mu - igA_\mu , \qquad (4)$$

using Eqs. (2) and (3):

$$\psi' = \left(1 - \left[D_\mu , D_\upsilon\right]\Delta x^\mu \Delta x^\upsilon + D_\mu D^\upsilon \Delta x_\mu \Delta x^\upsilon + D_\mu D^\upsilon \Delta x_\mu \Delta x^\upsilon + \ . \ . \ .\right)\psi . \qquad (5)$$

The effect of the journey around the closed spacetime loop represented by eqn. (5) is defined by the way in which the covariant derivatives enter into the expression {8}. To second order, the closed loop journey ends in a commutator containing covariant derivatives ($xy - yx$) and quadratic products containing derivatives (x^2 and y^2).

The commutator gives the field tensor {8} in any gauge group, and products such as x^2 contain field components to higher order, and this process gives us an ever more accurate theory of nonlinear electrodynamics and nonlinear optics systematically from first principles. The latter are very fundamental to modern physics.

If A_μ is the electromagnetic potential, then the electromagnetic field is

$$G_{\mu\upsilon} \equiv \partial_\mu A_\upsilon - \partial_\upsilon A_\mu - ig\left[A_\mu , A_\upsilon\right], \qquad (6)$$

where g is a universal constant, the ratio of e to $\hbar$. The field (6) is part of the commutator of covariant derivatives $[D_\mu, D_\upsilon]$. The Jacobi identity

$$\sum\left[D_\rho , \left[D_\mu , D_\upsilon\right]\right] \equiv 0 \qquad (7)$$

follows and is obeyed identically by the commutator $[D_\mu, D_\upsilon]$ for any gauge group. This Jacobi identity is the homogeneous field equation for any gauge group. In general relativity, it is the Bianchi identity {8}. These are well known results of modern gauge field theory. Our purpose in this opening section is to show that the gauge group of electromagnetism has to be a non-Abelian group such as O(3) in order to obtain self-consistently the homogeneous field equation and the first and third Stokes parameters from this general gauge field theory, which we have truncated at second order in the Taylor series (5). A theory to higher order in the Taylor series gives non-linear field tensors and equations, currently unexplored. More generally, these equations must be solved numerically and this process can be carried out in any gauge group. The current approach to electrodynamics is to assert that it is a linear theory based on a U(1) gauge group, and this loses a substantial amount of information. In particular, the third Stokes parameter is undefined in this procedure. In general, gauge theory shows that the equations of electrodynamics are non-linear to all orders in all internal gauge group symmetries and the process is always Lorentz and gauge covariant. The gauge theory conserves the fundamental symmetries of nature.

Taking the O(3), rotation group, symmetry for the internal gauge symmetry of the general theory applied to an electrodynamic vector potential, the $B^{(3)}$ component {9-14} emerges in the vacuum as

$$B^{(3)*} \equiv -i\frac{e}{\hbar}A^{(1)} \times A^{(2)}, \tag{8}$$

so $B^{(3)}$ is proportional to the third Stokes parameter. The latter is contained within the definition of the field tensor and is therefore contained within the commutator of covariant derivatives. The quadratic product of covariant derivatives contains the zero order Stokes parameter:

$$S_0 \equiv E^{(1)} \bullet E^{(2)} = \omega^2 A^{(1)} \bullet A^{(2)}, \tag{9}$$

and so an O(3) internal gauge group symmetry produces the result

$$S_0 = \pm S_3, \tag{10}$$

as required by the existence of circular polarization and the third Stokes parameter S_3.

If this method is applied with a U(1) internal symmetry group, the $B^{(3)}$ field vanishes along with the third Stokes parameter S_3, which is obviously counter-indicated by data (the existence of circular polarization). The field tensor becomes the familiar scalar in the internal, Abelian, gauge space:

$$G_{\mu\upsilon}(\mathrm{U}(1)) = \partial_\mu A_\upsilon - \partial_\upsilon A_\mu, \tag{11}$$

and the familiar homogeneous field equation is obtained:

$$\partial_\mu \tilde{G}^{\mu\upsilon}(\mathrm{U}(1)) \equiv 0. \tag{12}$$

Although these equations are widely known and accepted, they are self-inconsistent because they imply a zero S_3 in gauge field theory. In order to obtain a non-zero S_3 self-consistently, the existence of the $B^{(3)}$ field must be postulated. Inter alia, $B^{(3)}$ is a fundamental field observed in the third Stokes parameter, and in circular polarization. In field matter interaction $B^{(3)}$ is observed in magneto-optical effects {9-14}.

GAUGE TRANSFORMATION

The rotation of the general multi-dimensional field ψ takes place as follows {8}:

$$\psi' = S\psi \tag{13}$$

and is a special case of the closed loop or round trip represented in eqn. (1). In special relativity, both S and ψ are functions of x^μ, and the derivative

$$\partial_\mu \psi' = \partial_\mu (S\psi) = S\partial_\mu \psi + \psi \partial_\mu S, \tag{14}$$

in consequence is not covariant, because it does not transform under S in the same way as the field itself. There is an extra term on the right-hand side of eqn. (14). The concept of gauge transformation enters into field theory through the use of the covariant derivative as follows:

$$D'_\mu \psi' = SD_\mu \psi, \tag{15}$$

Where

$$D'_\mu = \partial_\mu - igA'_\mu. \tag{16}$$

Equation (15) is covariant and has the same algebraic form as the original eqn. (13). In consequence of the introduction of A'_μ as defined in eqn. (16), gauge transformation is a frame transformation in which A_μ changes its value to A'_μ. It can be shown that

$$A'_\mu = SA_\mu S^{-1} - \frac{i}{g}\partial_\mu SS^{-1}. \tag{17}$$

If A_μ did not change to A'_μ, we would have:

$$\begin{aligned} A'_\mu &= A_\mu = SA_\mu S^{-1}, \\ \partial_\mu SS^{-1} &= 0, \end{aligned} \tag{18}$$

and so the second term on the right-hand side of eqn. (17), known as the inhomogeneous term, would vanish.

If we adopt an O(3) internal gauge symmetry for electrodynamics and consider a rotation about the Z axis, then

$$S = e^{iJ_Z\alpha}, \tag{19}$$

where J_Z is the O(3) rotation generator {15} and α is an angle of rotation, an Euler angle. In other words, gauge transformation resulting from a rotation of the field ψ, is a physical rotation through a finite angle α. The potential function in the O(3) gauge theory is defined {8} as

$$A_\mu \equiv J^a A^a_\mu, \tag{20}$$

where repeated indices are summed over as usual. For a rotation about the Z axis, it can be shown straightforwardly that this O(3) gauge transformation results in:

$$A^Z_\mu \rightarrow A^Z_\mu + \frac{1}{G}\partial_\mu \alpha. \tag{21}$$

In the next section, we use this result to explain the Sagnac effect.

EXPLANATION OF THE SAGNAC EFFECT

When the well-known Sagnac interferometer is set up on a static platform, a phase shift, or interferogram, is observed, even though there is no path difference between radiation propagating clockwise (C) or anticlockwise (A). The phase shift cannot therefore be described with U(1) gauge field theory, but as shown in this section, it can be described with O(3) symmetry gauge field theory in which there occurs {16} a real-valued phase factor

$$\phi_C = -\phi_A = g\oint_C A_\mu dx^\mu = -ig^2 \iint \left[A_\mu, A^*_\upsilon\right] d\sigma^{\mu\upsilon} \tag{22}$$

from a non-Abelian Stokes theorem {17}. The latter arises in turn from parallel transport around a loop in Minkowski spacetime with O(3) covariant derivatives {18}. The four-potential A_μ is a rotation generator in the internal gauge space as defined by eqn. (20). The factor g is by dimensionality $\kappa/A^{(0)}$, where κ is the wave-vector magnitude and $A^{(0)}$ the scalar magnitude of A_μ. Therefore, the phase shift observed in the Sagnac interferometer with platform at rest is, from the area integral in eqn. (22):

$$\Delta\phi = 2\kappa^2 Ar = 2\frac{\omega^2}{c^2}Ar, \tag{23}$$

where Ar is the area enclosed by the loop. The observable phase change, or interferogram, is the single valued function

$$\Delta\phi = \cos(\Delta\phi \pm 2\pi n); \tag{24}$$

where ω is the angular frequency of the light and c is the vacuum speed of light.

The rotation of the Sagnac platform is well known to induce an extra phase shift which U(1) gauge field theory(Maxwell-Heaviside theory) cannot explain {19}. In O(3) electrodynamics, the explanation of the extra phase shift is straightforward if we consider the rotation of the Sagnac platform to be a gauge transformation described by eqn. (21).

Consideration of the $\mu = 0$ component of this equation gives

$$\omega \to \omega \pm \Omega, \tag{25}$$

where $\omega = \frac{\partial \alpha}{\partial t}$ is the angular frequency of rotation of the platform. From eqns. (23) and (25), we obtain, with platform in motion,

$$\Delta\phi = \frac{2}{c^2}(\omega \pm \Omega)^2 Ar \tag{26}$$

and

$$\Delta\Delta\phi = \pm 4\frac{\omega\Omega Ar}{c^2} \tag{27}$$

if

$$\Omega^2 \ll \omega^2. \tag{28}$$

Equation (27) is the well-observed phase shift when the platform is set in motion in the Sagnac effect {19}. Since $\omega \sim 10^{15}$ rads^{-1} and $\Omega \sim 10$ rads^{-1}, condition (28) is true to an excellent approximation. The result (27) is frame and gauge invariant and the Sagnac effect is observed empirically to be independent of whether the observer is on or off the platform, i.e., a frame invariant {20}.

THE TOPOLOGICAL PHASES IN O(3) ELECTRODYNAMICS

The electromagnetic phase in O(3) electrodynamics is given by {20}

$$\begin{aligned}\phi &= \exp\left(\oint D_\mu dx^\mu\right) \\ &= \exp\left(\iint \left[D_\mu, D_\nu\right] d\sigma^{\mu\nu}\right),\end{aligned} \tag{29}$$

a closed loop with covariant derivatives in Minkowski spacetime. The line and surface integrals of this non-Abelian Stokes theorem involve O(3) covariant derivatives with potentials defined as in eqn. (20), where

$$\left[J_1, J_2\right] = iJ_3. \tag{30}$$

Therefore dx^μ is a line element in Minkowski spacetime and $d\sigma^{\mu\nu}$ is an element of hyper-surface on the Poincaré sphere. The closed loop (or round trip) in spacetime generates the free electromagnetic field:

$$G_{\mu\upsilon} = i\frac{\hbar}{e}\left[D_\mu, D_\upsilon\right]. \tag{31}$$

Theorem (29) incorporates in one novel equation the Maxwell-Heaviside and Wu-Yang phases {20}. The former appears from the fundamental de Broglie (wave-particle) dualism of quantum mechanics:

$$\begin{aligned} \partial_\mu &= -i\kappa_\mu, \\ D_\mu &= -i\left(\kappa_\mu + \frac{e}{\hbar}A_\mu\right), \end{aligned} \tag{32}$$

and the Wu-Yang phase is part of the covariant derivative. The complete 0(3) electromagnetic phase is therefore summarized by the integral of the electromagnetic field tensor over the hyper-surface on the Poincaré sphere:

$$\oint = \exp\left(-i\frac{\hbar}{e}\iint G_{\mu\upsilon}\, d\sigma^{\mu\upsilon}\right), \tag{33}$$

where the field tensor contains a non-zero commutator of potentials:

$$G_{\mu\upsilon} = \partial_\mu A_\upsilon - \partial_\upsilon A_\mu - i\frac{e}{\hbar}\left[A_\mu, A_\upsilon\right] \tag{34}$$

The only non-vanishing (non-oscillatory) part of this integral is the topological phase

$$\phi = \exp\left(-i\frac{e}{\hbar}\iint -i\frac{e}{\hbar}\left[A_\mu, A_\upsilon\right] d\upsilon^{\mu\upsilon}\right). \tag{35}$$

This does not exist in the Maxwell-Heaviside theory, because the latter is an Abelian gauge theory of U(1) symmetry, unable therefore, to describe the empirically observable topological phase. These concepts are well established in non-Abelian gauge field theory in general {20} but do not occur in the Maxwell-Heaviside theory, whose linear Abelian nature prohibits the description of the non-linear, non-Abelian, topological phase.

The general theorem (29) can be developed for one photon using the wave particle momentum dualism:

$$\hbar\kappa = eA^{(0)}, \tag{36}$$

where $A^{(0)}$ is the scalar magnitude of a longitudinal, phase free, vector potential peculiar to O(3) electrodynamics {20}. It is again undefined in the Maxwell-Heaviside theory and in consequence, this theory violates Newton's third law in radiation/matter interaction such as the Compton effect. In O(3) electrodynamics, the Compton effect is understood straightforwardly {20} from the correspondence of the quantized momentum for one photon, $\hbar k$, and its classical equivalent $eA^{(0)}$. The latter does not exist in the Maxwell-Heaviside theory. If we restrict consideration to a plane wave propagating in the Z axis with a beam cross section defined in the XY plane, we obtain from eqn. (36):

$$\oint A_3\, dx^3 = -i\iint \frac{\kappa}{A^{(0)}}\left[A_1, A_2\right] d\sigma^{12}, \tag{37}$$

whence it is easily shown that the magnetic flux in vacua is

$$\Phi = B^{(3)} Ar = -i\iint \frac{\kappa}{A^{(0)}}\left[A_1, A_2\right] d\sigma^{12} \tag{38}$$

and originates in the σ^{12} component of the hypersurface on the Poincaré sphere. The physical origin of the magnetic flux is the interference between the (1) and (2) (complex conjugate) components of the circularly polarized beam. These exist on the single photon level, and so the topological phase for one photon originates in the latter's $\boldsymbol{B}^{(3)}$ field. We have therefore demonstrated the physical origin of the topological phase {21}, which is physically distinct from
the dynamical phase.

The magnetic flux carried by one photon is

$$\Phi = \frac{\hbar}{e}, \tag{39}$$

and so h becomes the helicity {22}

$$h = \pm 1 \tag{40}$$

for a plane wave in circular polarization propagating in Z. This result is self-consistently regained from Eqs. (37) and (38) as follows:

$$\begin{aligned} r &= \kappa\, Ar = \frac{1}{\kappa}, \\ \kappa \bullet \boldsymbol{r} &= \pm 1. \end{aligned} \tag{41}$$

Therefore the topological phase is governed by the $\boldsymbol{B}^{(3)}$ field in various forms of interferometry {22}. For example the Pancharatnam phase {22} is distinguished experimentally by the fact that it is achromatic and depends only on polarization; is non-additive (has no r dependence), is unbounded (depends on cyclic and therefore periodic changes in the state of polarization) and be observed in unseparated beams. The dynamical phase in contrast is chromatic, additive, bounded and needs separate beams in interferometry in order to become effective. The P phase and concomitant $\boldsymbol{B}^{(3)}$ field arise from cycling in the polarization state of light while keeping the direction of the beam fixed, and are equivalent to a gauge potential in the parameter or momentum space. They are due to parallel transport in the presence of a gauge field of O(3) internal symmetry and are equivalent to an optical Aharanov-Bohm effect. The Berry phase is related to the P phase and can be observed in a Faraday effect due to $\boldsymbol{B}^{(3)}$, the rotation of a plane of polarization of light transmitted through a fiber wound helically on a cylinder {23}. This phenomenon is due to cycling in the direction of a beam of light so that the tip of the spin vector of a photon in this beam traces out a closed curve on the sphere of spin directions {24}.

The fundamentally non-Abelian nature of these phenomena means that the Maxwell-Heaviside equations become:

$$\begin{aligned} D_\mu \tilde{\boldsymbol{G}}^{\mu\nu} &\equiv \boldsymbol{0}, \\ D_\mu \boldsymbol{H}^{\mu\nu} &= \boldsymbol{J}^\nu, \end{aligned} \tag{42}$$

where the field tensors $\tilde{\boldsymbol{G}}^{\mu\nu}$ and $\boldsymbol{H}^{\mu\nu}$ and four-current $\boldsymbol{J}^\mu$ are vectors in the O(3) symmetry internal gauge space. A systematic development of these equations of 0 (3) electrodynamics is given elsewhere {25}. The key importance of the topological phase is that it indicates conclusively the empirical existence of the Evans-Vigier field $\boldsymbol{B}^{(3)}$ of the photon. The latter does not exist in the Maxwell-Heaviside theory, whose field equations are written in a scalar (U(1)) internal gauge space. In this space, the commutator defining the P phase in eqn. (35) is zero by definition. The O(3) gauge group is preferred to the SU(2) group because the latter is simply connected and does not support an Aharonov-Bohm effect or topological phase {23-25} because the mappings are deformable to a constant map. The O(3) group is doubly connected and supports the Aharonov-Bohm and topological phase effects. The O(3) group is homomorphic with SU(2) but is its covering group.

The adoption of the O(3) gauge symmetry for the whole of electrodynamics is therefore indicated by the empirical existence of $B^{(3)}$ in the topological phase on the one photon-level. This course of action brings with it several fundamental conceptual advances in electrodynamics and unified field theory (26} . The most important of these is that electrodynamics is a non-Abelian gauge theory and this is indicated fundamentally by the fact that a classical polarized wave is always constituted of two vectorial components, for example, (1) and (2) in circular polarization, which gives rise to $B^{(3)}$ and the topological phase through a non-zero commutator of potentials. Without this commutator, there is no topological phase. As soon as a non-Abelian structure is adopted for electrodynamics, the field tensor becomes gauge covariant in the vacuum and a gauge transformation becomes a physical rotation. In the Maxwell-Heaviside theory, the potentials are not gauge invariant on the classical level and are not regarded as physical for this reason {27}. In the O(3) electrodynamics, the gauge transformation of the potentials is a well-defined physical process giving rise to a characteristic inhomogeneous term responsible for physical effects on the classical level. An example is the Sagnac effect described in Section 4.

Another major conceptual advance is the realization in O(3) electrodynamics that the constant e, like $\hbar$, is a fundamental constant of physics that exists under all conditions, in the field as well as on the electron. This realization straightforwardly unifies the Maxwellian concept of material charge being a result of the field {28}, and the Lorentzian concept of the field being the result of charged matter. The fundamental de Broglie duality {23} follows from O(3) electrodynamics and allows a simple classical explanation of the Compton effect as well as of the topological phase in experiments such as the Sagnac effect and interferometry in general.

INTERFEROMETRIC EFFECTS IN OPTICS

Contemporary non-Abelian gauge field theory applied to electrodynamics gives rise to a topological magnetic monopole and a topological charge {28}, which is observable in electromagnetic phase effects such as the Sagnac effect. These are closely related to the Aharonov-Bohm effect and exist for one photon. The topological magnetic monopole can be defined simply as the area integral over the topological magnetic flux density $B^{(3)}$:

$$g_m = \frac{1}{V}\iint B^{(3)} dAr, \tag{43}$$

where V is a volume and Ar a cross-sectional area of radiation. The electromagnetic phase factor is as defined in (35). Therefore the electromagnetic phase factor is directly proportional to the topological magnetic monopole

$$\phi = g g_m V. \tag{44}$$

The topological phase is observable in interferometry {29} on the one-photon level, and this implies again that electrodynamics is a non-Abelian gauge field theory {30}.

The well-known phase difference that gives rise to the standard Michelson interferogram {31} is due to $B^{(3)}$ through the non-Abelian Stokes theorem, eqn. (35). In the Maxwellian theory of Michelson interferometry, the electromagnetic phase factor is the well-known Lorentz invariant, or retarded moving wave solution:

$$\phi = \kappa_\mu x^\mu = \omega t - \boldsymbol{\kappa} \cdot \boldsymbol{r} \tag{45}$$

where ω is the angular frequency, $\boldsymbol{\kappa}$ the wave-number at point $\boldsymbol{r}$. Parity symmetry implies that on reflection from a mirror in one arm of the interferometer, the phase factor becomes

For the complete optical path from beam splitter to mirror and back, the phase factor is therefore always zero, and independent of path length $\boldsymbol{r}$. The same is always true in the other path, so there is never a phase difference between the two beams recombining at the beam splitter and in consequence no interferogram for perfect reflection from the mirrors. This is obviously contrary to experience, and so in Maxwellian theory, a phase factor is added phenomenologically, and attributed to imperfections in the mirrors and so forth.

In non-Abelian electrodynamics, on the other hand, the factor becomes a linear integral of a non-Abelian Stokes theorem with g defined {28} to be κ/A^0, where $A^{(0)}$ is the scalar magnitude of A_μ:

$$\phi = \oint \kappa_\mu dx^\mu = \omega t - g \iint B^{(3)} dAr. \tag{47}$$

On reflection from a mirror, the path is reversed and the line integral changes sign together with the wave vector $\boldsymbol{\kappa}$, giving the complete phase factor

$$\phi = \omega t - 2\oint \kappa \cdot dr = \omega t - 2g \iint B^{(3)} dAr, \tag{48}$$

where $\boldsymbol{r}$ is the distance from the beamsplitter to mirror. If one mirror is moved with respect to the other, an interferogram appears as observed:

$$\Delta\phi = \cos(2\kappa\Delta r \pm 2\pi n), \tag{49}$$

where Δr is the path length difference between the two arms of the Michelson interferometer. For radiation consisting of many frequencies, the interferogram $\Delta\phi$ is a sum of cosines whose Fourier transform is a spectral function {31}.

Equation (48) means that there is a topological phase that gives rise to Michelson interferometry and which is given by an area integral over the fundamental topological magnetic field $\boldsymbol{B}^{(3)}$ {28}. The existence of this topological phase has been well established experimentally {32}. Michelson interferometry is a fundamental phenomenon of non-Abelian electrodynamics. The Michelson interferogram cannot be explained in terms of Maxwellian electrodynamics without additional phenomenology. Therefore electrodynamics in general is a non-Abelian gauge field theory because the linear, or Abelian, theory is incomplete. This manifests itself clearly in the inverse Faraday effect {33}, due to $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$, which does not exist in U(1) electrodynamics and again is phenomenological.

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