ON WHITTAKER'S *F* AND *G* FLUXES, PART III: THE EXISTENCE OF PHYSICAL LONGITUDINAL AND TIME-LIKE PHOTONS

ABSTRACT

In this third of a series of papers developing Whittaker's F and G fluxes, it is shown by canonical quantization that there exist physical time-like and longitudinal photons in vacuo. Canonical quantization proceeds straightforwardly from the classical Klein-Gordon field proportional to Whittaker's F potential. The properties of the time-like and longitudinal photons are summarized briefly, and imply the existence of the Evans-Vigier field in quantized form: the photomagneton.

INTRODUCTION

In two recent papers {1, 2}, Whittaker's development {3, 4} of the electromagnetic entity in terms of two magnetic fluxes F and G has been used to disprove the basic ansatz of contemporary field theory, its gauge freedom, and to show that classical potentials and physical. Whittaker's development leads directly to the existence of longitudinal magnetic fluxes f and g, whose density in O(3) electrodynamics is the Evans-Vigier field $\boldsymbol{B}^{(3)}$ {5, 12}. The latter has been defined directly in terms of the physical F and G. It has been shown on the classical level that both the transverse and longitudinal parts of the electromagnetic potential are physical, and that under well defined conditions, there exist potentials in vacuo without any Maxwell-Heaviside field being present. The only field present is $\boldsymbol{B}^{(3)}$, which is derived from a gauge theory applied to electrodynamics with a physical internal O(3) symmetry gauge space based on circular polarization {5, 12}. The notion of a gauge invariant electrodynamics has been disproved and replaced by a theory based on gauge covariance, and example being O(3) electrodynamics, another example being the SU(2) electrodynamics of Barrett {13}.

In this third part of the series, it is demonstrated that there exists physical time-like and longitudinal photons which have an independent physical existence. The Gupta-Bleuler method is disproved and canonical quantization is shown to proceed straightforwardly from a classical Klein-Gordon field using standard methods {14}. The properties of the time-like and longitudinal photons are summarized briefly. Canonical quantization proceeds from potentials, not fields, and under certain circumstances, there may exist photons without fields being present at all. An example of such condition was given in part two {2} of this series. Whittaker's F and G have a more fundamental meaning than the received four potential, which is usually quantized to give photons which are asserted to be transverse space-like. Longitudinal and time-like photons are removed in the received view {14} using the Gupta-Bleuler method, which in this paper is disproved.

THE LONGITUDINAL FOUR POTENTIAL

It has been shown {1, 2} that the Whittaker magnetic flux on vacuo, F, can be represented by:

$$F = i\frac{A^{(0)}}{\sqrt{2}}(X - iY)e^{i(\omega t-\kappa Z)}; \qquad f = F\boldsymbol{k} \tag{1}$$

in the laboratory frame (X, Y, Z). The radius of the beam of light is defined by:

$$R^2 = X^2 + Y^2. \tag{2}$$

Here the exponential is the usual Maxwell-Heaviside electromagnetic phase ω is the angular frequency at time t and κ the wavevector at point Z in the propagation axis. There exists then {1,2 } a four potential whose space-like part is longitudinally directed along the propagation axis Z of the beam, and whose time-like part is its modulus:

$$A_L^\mu = i\omega F(1, \boldsymbol{k}) \tag{3}$$

$$A_L = \phi_L \boldsymbol{k}. \tag{4}$$

The time-like part of this novel four-potential is therefore the classical potential

$$\phi_L = -\omega \frac{A^{(0)}}{\sqrt{2}} (X - iY) e^{i(\omega t - \kappa Z)} \tag{5}$$

which obeys the d'Alembert equation in vacuo:

$$\Box \phi_L = 0. \tag{6}$$

This can be regarded as a Klein-Gordon equation for a classical field {14} for a particle without mass. Canonical quantization of this type of Klein-Gordon equation is well known {14} and is free of any complications. In the next section, the potential ϕ_L is quantized to give a physical time-like photon, whose existence implies that of a concomitant physical longitudinal photon. The Gupta-Bleuler method, which asserts that there can be neither, is therefore disproved straightforwardly using Whittaker's results {3, 4}.

CANONICAL QUANTIZATION OF A_L^μ

This is a very well known procedure - the quantization of the real Klein-Gordon field, the physical part of ϕ_L being real. The complex representation of ϕ_L used in eqn. (5) is for convenience only. Therefore, in this section, there is need only to summarize the main results of such a procedure {14}, full details being available in any good textbook.

The Klein-Gordon equation (6) for the time-like potential is regarded as an equation for a classical; field, ϕ_L. The field ϕ_L is next treated as an operator subject to the commutation relation of quantum mechanics. The **field** is therefore being quantized. The procedure gives a positive definite Hamiltonian, which was shown in the second part of this series {2} to be defined entirely in terms of the Evans-Vigier field $\boldsymbol{B}^{(3)}$ for a beam of light of definite radius R. There is no problem of negative energy therefore, as in the single particle Klein-Gordon equation. Field quantization proceeds by expressing the field as a Hermitian operator, whose Fourier expansion is:

$$\phi_L = \int \frac{d^3\kappa}{(2\pi)^3 2\omega_\kappa} \left[a(\kappa) e^{-i\kappa Z} + a^+(\kappa) e^{i\kappa Z} \right] \tag{7}$$

with the frequencies $\omega_\kappa = \kappa c$. Therefore many different frequencies emerge from the quantization of the time-like photon. The coefficients a and a^+ are operators. As described on pp. 131 ff. of Ryder, for example {14}, commutation relations are constructed between a and a^+ of eqn. (7). These commutator relations are

$$[a(\kappa), a(\kappa')] = [a^+(\kappa), a^+(\kappa')] = 0 \tag{8}$$

$$[a(\kappa), a^+(\kappa')] = (2\pi)^3 2\omega_\kappa \delta^3(\kappa - \kappa'). \tag{9}$$

The operator

$$N(\kappa) = a^+(\kappa)a(\kappa) \tag{10}$$

is the operator for the number of particles with momentum $\hbar\kappa$ and with energy $\hbar\omega$. The Hamiltonian after quantization is of the harmonic oscillator form:

$$H = \frac{1}{2}P^2(\kappa) + \frac{\omega_\kappa^2}{2}Q^2(\kappa) \tag{11}$$

where

$$P(\kappa) = \left(\frac{\omega_\kappa}{2}\right)^{1/2}\left[a(\kappa) + a^+(\kappa)\right] \tag{12}$$

$$Q(\kappa) = \frac{1}{(2\omega_\kappa)^{1/2}}\left[a(\kappa) - a^+(\kappa)\right] \tag{13}$$

and the time-like potential ϕ_L becomes after canonical quantization an infinite sum of oscillators.

The operators a and a^+ are the annihilation and creation operators for the field quanta. The energy of the quantized field is non-negative. The particles which are quanta of the longitudinal field obey Bose-Einstein statistics {14}, any number of particles may exist in the same quantization state, and the particles are bosons., These are of course massless photons with spin zero.

The time-like and space-like parts of the four potential A_L^μ therefore quantize directly to photons propagating along the Z axis. There is a direct relation between the classical A_L^μ and the quantum mechanical four wavevector:

$$\kappa^\mu = \hbar\left(\frac{\omega}{c}, \kappa\right) \tag{14}$$

and the classical energy momentum of one particle:

$$p^\mu = \left(\frac{En}{c}, \boldsymbol{p}\right). \tag{15}$$

DIFFICULTIES WITH REGULAR CANONICAL QUANTIZATION

It is well known {14} that canonical quantization in the radiation gauge, where the four potential has transverse components only, results in photons with spin 1 and -1 which are transverse photons. There are no physical time-like or longitudinal photons. This is in direct conflict with the fact that particulate photons are longitudinal in nature from, for example, the Compton and photoelectric effects. In the Lorenz gauge {14}, the end result, using the Gupta-Bleuler method, is that transverse photons are physical, but not time-like or longitudinal photons. This again leaves us in the same quandary as radiation gauge canonical quantization, as just described for radiation gauge quantization.

Whittaker's method leads to photons with spin -1, 0 and +1 and this is precisely the conclusion obtained from O(3) electrodynamics, where the flux density of F and G, the $\boldsymbol{B}^{(3)}$ field, is longitudinally directed. The angular momentum properties of the O(3) electrodynamics lead to the result that the photon is a boson with spin -1, 0 and 1. The little group of the Poincaré group becomes the O(3) group {5-12}, which is physical.

CONCLUSION

Canonical quantization of the time-like field obtained from Whittaker's analysis {3, 4} gives rise to photons which are longitudinally directed and which are observed in the Compton and photoelectric effect. In the received view, canonical quantization leads to the self-conflicting result that photons are transverse only. This makes no physical sense, yet is the contemporary opinion. Whittaker's fluxes F and G lead directly to O(3) electrodynamics {1, 2}.

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