ADDITIONAL INFEROMETRY PAPERS

NON-ABELIAN ELECTRODYNAMICS AND MICHELSON INTERFEROMETRY

ABSTRACT

The phase difference in a Michelson interferometer is shown to be due to a line integral which is equal, through a non-Abelian Stokes theorem, to an area integral over a fundamental topological magnetic field, an area integral which has the units of volume multiplied by a topological magnetic monopole.

DISCUSSION

Contemporary non-Abelian gauge field theory {1-3} applied to electrodynamics gives rise to a topological magnetic monopole {4,5} and a topological charge {6} which are observable in electromagnetic phase effects {7,8} These are closely related to the Aharonov-Bohm effect {9} and exist for one photon {10}. The topological magnetic monopole (g_m) is defined as an area integral over a topological magnetic flux density, $\boldsymbol{B}^{(3)}$. In S.I. units:

$$g_m = \frac{1}{V}\iint B^{(3)} dAr \qquad (1)$$

where V is a volume and Ar is an area. The electromagnetic phase factor is:

$$\phi = g\oint A_\mu dx^\mu = -ig^2 \iint \left[A_\mu , A_\nu^*\right] d\sigma^{\mu\nu} \qquad (2)$$

where g is a dimensionality coefficient {11} and A_μ the non-Abelian four-potential {1-3}. Eqn. (2) originates in the demonstration {12} that all topological phases are due to parallel transport in Minkowski spacetime using covariant derivatives. Therefore the electromagnetic phase factor is directly proportional to the topological magnetic monopole:

$$\phi = g g_m V . \qquad (3)$$

These concepts do not exist in Maxwellian electrodynamics, which is by definition a linear and Abelian gauge field theory {1-3}. The topological phase is however observable in interferometry on the one photon level, and this implies that electrodynamics is a non-Abelian gauge field theory {11}.

In this paper, it is shown that the well known phase difference which gives rise to the standard Michelson interferogram {13-15} is due to the topological magnetic field $\boldsymbol{B}^{(3)}$ through the non-Abelian Stokes Theorem, eqn. (2).

In the Maxwellian theory of Michelson interferometry, {13-15} the electromagnetic phase factor is the well known Lorentz invariant, or retarded moving wave solution:

$$\phi = \kappa_\mu x^\mu = \omega t - \boldsymbol{\kappa} \cdot \boldsymbol{r} \qquad (4)$$

where ω is the angular frequency, and κ is the wave-number at point $\boldsymbol{r}$. Parity symmetry implies that, on reflection from a mirror in one arm of the interferometer, the phase factor becomes:

$$\phi = \omega t + \kappa \cdot \boldsymbol{r}. \tag{5}$$

For the complete optical path from beam splitter to mirror and back, the phase factor is therefore always ωt, and is independent of path length $\boldsymbol{r}$. The same is always true in the other path, so there is never a phase difference between the two beams recombining at the beam splitter, and in consequence, no interferogram. This is obviously contrary to experience {13-15} and so, in Maxwellian theory, a phase factor is added phenomenologically. Its origin is unknown.

In non-Abelian electrodynamics, on the other hand, the factor $\kappa_\mu x^\mu$ becomes a line integral of a non-Abelian Stokes theorem with g defined {11} to be $\kappa/A^{(0)}$, where $A^{(0)}$ is the scalar magnitude of A_μ:

$$\phi = \oint \kappa_\mu dx^\mu = \omega t - g \iint B^{(3)} dAr. \tag{6}$$

On reflection from a mirror, the path is reversed and the line integral changes sign together with the wave-vector κ, giving the complete phase factor:

$$\phi = \omega t - 2\oint \kappa \cdot d\boldsymbol{r} = \omega t - 2g \iint B^{(3)} dAr \tag{7}$$

where $\boldsymbol{r}$ is the distance from beamsplitter to mirror. If one mirror is moved with respect to the other, an interferogram appears as observed:

$$\Delta\phi = \cos(2\kappa \Delta r \pm 2\pi n) \tag{8}$$

where Δr is the path length difference between the two arms of the Michelson interferometer. For radiation consisting of many frequencies, the interferogram $\Delta\phi$ is a sum of cosines whose Fourier transform is a spectral function {13-15}.

Eqn. (7) means that there is a topological phase that gives rise to Michelson interferometry, and which is given by an area integral over the fundamental topological magnetic field $\boldsymbol{B}^{(3)}$ {11}. The existence of this topological phase has been well established experimentally {16, 17}. However, it is shown for the first time in this paper that Michelson interferometry is **fundamentally** a phenomenon of non-Abelian electrodynamics. The Michelson interferogram cannot be explained in terms of Maxwellian electrodynamics without additional phenomenology. Therefore electrodynamics in general is a non-Abelian gauge field theory because the linear, or Abelian theory is incomplete.

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