U(1) REFUTATION

NON-ABELIAN ELECTRODYNAMICS AND THE INVERSE FARADAY EFFECT

ABSTRACT

The inverse Faraday effect is explained from first principles using novel electrodynamical field equations which are derived from contemporary non-Abelian gauge field theory. The field equations linearize into two Maxwellian type equations together with a novel equation of motion for a topological magnetic field, defined in terms of a standard topological magnetic monopole. The inverse Faraday effect is thus explained as magnetization of one electron due to this topological magnetic field. No phenomenology is required in this explanation. The topological magnetic field does not give Faraday induction, as verified empirically, and is a fundamental property of the electromagnetic field in the vacuum.

1. INTRODUCTION

The theory of electromagnetism is based on the Maxwell-Heaviside equations which appear in every textbook {1-3}. In terms of gauge field theory, {4-7} it is a linear and Abelian theory of the electromagnetic sector that does not allow for the existence of a topological magnetic monopole. In this paper, it is argued that the well known inverse Faraday effect (magnetization by circularly polarized electromagnetic radiation {8-10}) is describable by gauge field theory applied to electromagnetism in such a way as to include a topological magnetic monopole {11, 12}, defined as the area integral over a topological magnetic field divided by a volume of radiation. The inverse Faraday effect is the magnetization caused in matter (one electron) by this topological magnetic field. The latter is governed by a non-Maxwellian field equation which shows that there is no Faraday induction due to it. It manifests itself as the magnetization of one electron through the optical conjugate product {8-10}, which is usually introduced phenomenologically into a linear Maxwellian structure.

Section 2 introduces the field equations necessary for the first principles explanation of the inverse Faraday effect in gauge field theory. These equations are linearized using a self-consistent procedure {13, 14} which produces three equations from the original gauge field equations. Two of these are Maxwellian and the third is the equation of motion for the topological magnetic field, a constant of motion which does not produce Faraday induction. This topological magnetic field (labeled $B^{(3)}$) is defined from a standard topological magnetic monopole g_m{11, 12}through the equation:

$$g_m = \frac{1}{V}\iint B^{(3)} Ar \tag{1}$$

which is the area integral of a non-Abelian Stokes theorem divided by a radiation volume. In terms of the well known optical conjugate product, the topological magnetic field in the vacuum is defined through:

$$B^{(3)*} \equiv -igA^{(1)} \times A^{(2)} \tag{2}$$

where $A^{(1)} = A^{(2)*}$ is a complex vector potential in the complex spherical basis ((1), (2), (3)) and where g is a proportionality factor. For one photon, it is the inverse {13, 14} of the magnetic fluxon, $\hbar/e$, where $\hbar$ is Planck's constant and e the elementary charge. In the vacuum, the de Broglie duality implies that {13, 14}:

$$p = \hbar\kappa = eA^{(0)} \tag{3}$$

where κ is the scalar magnitude of the wave-vector and where $A^{(0)}$ is the scalar magnitude of the vector potential $\boldsymbol{A}^{(1)}$, for example a plane wave. In Section 3, the magnetization of matter by $\boldsymbol{B}^{(3)}$ in the inverse Faraday effect is considered in detail through non-Abelian field equations with the factor g in the vacuum modified self-consistently for field matter interaction, including the interaction of the topological magnetic field with matter. Therefore the inverse Faraday effect becomes empirical evidence for the well known topological magnetic field of contemporary non-Abelian gauge field theory {15, 16}.

2. THE NON-ABELIAN FIELD EQUATIONS

In order to describe magnetization by a topological magnetic field, the equations of the electromagnetic field are written in an O(3) internal gauge symmetry as follows:

$$D_\mu \tilde{\boldsymbol{G}}^{\mu\nu} \equiv \boldsymbol{0} \tag{4}$$

$$D_\mu \boldsymbol{H}^{\mu\nu} \equiv \boldsymbol{J}^\nu . \tag{5}$$

These are similar to the Maxwell-Heaviside equations but the bold-faced symbols are vectors in the internal gauge space. Covariant derivatives D_μ are used in this gauge space. Thus $\tilde{\boldsymbol{G}}^{\mu\nu}$ is a field strength tensor in Minkowski space-time and a vector in the internal gauge space; and $\boldsymbol{H}^{\mu\nu}$ is a similarly structured magnetization tensor. The four-current $\boldsymbol{J}^\nu$ is a four-vector in Minkowski space-time and a vector in the O(3) gauge space. The homogeneous field equation (4) is the Feynman Jacobi identity

$$\left(\partial_\mu + g\boldsymbol{A}_\mu \times\right)\tilde{\boldsymbol{G}}^{\mu\nu} \equiv \boldsymbol{0} \tag{6}$$

where g is the inverse of the fluxon $\hbar/e$ and where $\boldsymbol{A}_\mu$ is a potential four-vector in Minkowski space-time and a vector in the O(3) internal gauge space {15, 16}. A particular solution of this equations is:

$$\partial_\mu \tilde{\boldsymbol{G}}^{\mu\nu} = \boldsymbol{A}_\mu \times \tilde{\boldsymbol{G}}^{\mu\nu} \equiv \boldsymbol{0}. \tag{7}$$

Eqn. (7) gives a complex conjugate pair of homogeneous Maxwellian equations:

$$\partial_\mu \tilde{\boldsymbol{G}}^{\mu\nu(1)} = 0 \tag{8}$$

$$\partial_\mu \tilde{\boldsymbol{G}}^{\mu\nu(2)} = 0 \tag{9}$$

and the field equation in the vacuum for the topological magnetic field $\boldsymbol{B}^{(3)}$ defined in eqn. (2):

$$\partial_\mu \tilde{\boldsymbol{G}}^{\mu\nu(3)} = 0. \tag{10}$$

In the vacuum, the topological magnetic field is a constant of motion governed by the field equation:

$$\frac{\partial \boldsymbol{B}^{(3)}}{\partial t} = \boldsymbol{0} \tag{11}$$

a result which is consistent with its definition in non-Abelian gauge field theory {15, 16} with an O(3) internal gauge space:

$$\boldsymbol{B}^{(3)*} = -i\frac{e}{\hbar}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}. \tag{12}$$

Similarly the inhomogeneous field equation (5) can be shown to reduce in the vacuum to two Maxwellian equations in indices (1) and (2) and to a second equation of motion for the topological magnetic field:

$$\partial_\mu G^{\mu\nu(3)} = 0 \tag{13}$$

whose vector form is:

$$\nabla \times \boldsymbol{B}^{(3)} = \boldsymbol{0}. \tag{14}$$

The vector equations (11) and (14) for the topological magnetic field in the vacuum show that there is no Faraday induction due to it, as observed empirically {17}.

3. THE INVERSE FARADAY EFFECT

The inverse Faraday effect {8-10} cannot be described self-consistently in the linearized gauge field theory that gives rise to the Maxwell-Heaviside equations because the linearization process means that the conjugate product $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$, needed to describe the inverse Faraday effect, **vanishes**. For a consistent first principles explanation of the inverse Faraday effect, we need the non-Abelian, inhomogeneous field equation (5), which is written out in full as follows:

$$\partial_\mu \boldsymbol{H}^{\mu\nu(1)*} = \boldsymbol{J}^{\nu(1)*} + ig\boldsymbol{A}_\mu^{(2)} \times \boldsymbol{H}^{\mu\nu(3)} \tag{15}$$

$$\partial_\mu \boldsymbol{H}^{\mu\nu(2)*} = \boldsymbol{J}^{\nu(2)*} + ig\boldsymbol{A}_\mu^{(3)} \times \boldsymbol{H}^{\mu\nu(1)} \tag{16}$$

$$\partial_\mu \boldsymbol{H}^{\mu\nu(3)*} = \boldsymbol{J}^{\nu(3)*} + ig\boldsymbol{A}_\mu^{(1)} \times \boldsymbol{H}^{\mu\nu(2)} \tag{17}$$

in which the magnetization observable as the inverse Faraday effect is given by:

$$\boldsymbol{H}^{(3)*} = -i\frac{g'}{\mu_0}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} \tag{18}$$

where μ_0 is the vacuum permeability, and where g' is related to g by:

$$g' = \frac{\mu_0}{\mu}g. \tag{19}$$

We can write eqn. (15) as:

$$\partial_\mu \boldsymbol{H}^{\mu\nu(1)*} = \boldsymbol{J}^{\nu(1)*} + \Delta\boldsymbol{J}^{\nu(1)*} \tag{20}$$

where the transverse current

$$\Delta\boldsymbol{J}^{\nu(1)*} = ig\varepsilon\,\boldsymbol{A}_\mu^{(2)} \times \boldsymbol{G}^{\mu\nu(3)} \tag{21}$$

causes a signal in an induction coil due to the topological magnetic field $\boldsymbol{B}^{(3)}$ appearing in $G^{\mu\nu}$. This is the inverse Faraday effect as observed empirically {8-10}. The non-Abelian gauge field theory leading to eqn.(21) describes the effect without phenomenology.

In contrast, the explanation of the effect in the linear gauge field theory leading to the Maxwell-Heaviside equations relies on the phenomenological introduction of $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$. This is diametrically self-inconsistent because the linear and Abelian gauge field structure of the Maxwell-Heaviside equations means that $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ is everywhere zero under all conditions.

Eqn. (15) can be developed to show that:

$$\Delta J^{\nu(2)} = -\frac{\varepsilon}{c}\omega^2 A^{\nu(2)} \tag{22}$$

and if we write the four-current for one electron as:

$$\Delta J^{\nu(2)} = \frac{e}{mcV} p^{\nu(2)} \tag{23}$$

where m is the mass of the electron, V the sample volume, and $p^{\mu(2)}$ the electronic energy/momentum vector:

$$p^{\nu(2)} \equiv eA^{\nu(2)} \tag{24}$$

eqns. (22) and (23) give the permittivity:

$$\varepsilon = -\frac{e^2}{m\omega^2 V}. \tag{25}$$

Define the one electron susceptibility as:

$$x \equiv -\frac{e^2 c^2}{m\omega^2} \tag{26}$$

then:

$$\varepsilon = \frac{x}{c^2 V}. \tag{27}$$

This result is self-consistent with the fact that the inverse Faraday effect can be described through the same susceptibility x by using the relativistic Hamilton Jacobi equation {13, 14} for one electron in the classical electromagnetic field. The current $\Delta J^{\nu(2)}$ is due to a field induced transverse electronic momentum and the field equation (5) give a self-consistent and complete description of this process without the phenomenological {8-10} introduction of the conjugate product.

DISCUSSION

The inverse Faraday effect is magnetization of an electron by the topological magnetic field $B^{(3)}$, which does not occur in the Maxwell-Heaviside theory of electrodynamics. This conclusion is inevitable once we accept the fact that the phenomenological introduction of $A^{(1)} \times A^{(2)}$ into the U(1) gauge field theory is self-inconsistent. The reason for this inconsistency is that the field tensor in U(1) electrodynamics is linear in the potential four-vector, being its four-curl {18}. In the O(3) symmetry gauge field theory used in this paper, the field tensor contains a non-zero commutator of potentials, a commutator which defines the topological magnetic field missing from U(1) electrodynamics. This topological field also appears in the well observed topological phases {19, 20}, which are achromatic and which depend on an integral over $B^{(3)}$. The topological phases result from a closed loop with non-Abelian covariant derivatives in Minkowski space-time {21}. If the commutator of potentials is zero, as in the Maxwell-Heaviside theory, there is no topological phase. This is counter-indicated by data from interferometry {5}, data which reveal beyond doubt the existence of the topological phase. Recently, the O(3) symmetry electrodynamics developed here has been used to give a straightforward explanation {22} of the Sagnac effect, so there is considerable evidence for the conclusion that electrodynamics is a non-Abelian gauge field theory, as first conjectured by Yang and Mills {23}. A systematic development of this theory is available {24}.

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