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UNIFICATION OF ELECTROMAGNETISM AND GRAVITATION: THE EQUATIONS OF ELECTRODYNAMICS DERIVED FROM THE RIEMANN TENSOR IN GENERAL RELATIVITY

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The equations of free space electrodynamics are derived directly from the Riemann curvature tensor and the Bianchi identity of general relativity by contracting on two indices to give a novel antisymmetric Ricci tensor. Within a factor $e/\hbar$, this is the field strength tensor $G_{\mu\nu}$ of free space electrodynamics. The Bianchi identity for $G_{\mu\nu}$ describes free space electrodynamics in a manner analogous to, but more general than, Maxwell's equations for electrodynamics, the critical difference being the existence in general and special relativity of the Evans-Vigier field, $\boldsymbol{B}^{(3)}$.

Key Words: electrodynamics, gravitation, unification, $\boldsymbol{B}^{(3)}$ field, Ricci tensor.

1. INTRODUCTION

The experimentally verifiable existence of the vacuum $\boldsymbol{B}^{(3)}$ field shows that the theory and equations of vacuum electrodynamics become structured as for those of gravitation [1-10]. For example, the partial derivatives used in Maxwell's equations are replaced by covariant derivatives as used in general

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relativity. In this Letter, the unification of gravitation and electrodynamics in curved spacetime is demonstrated directly and simply from the Bianchi identity of gravitation theory [11,12]. In Section Two, the Bianchi identity is shown to reduce straightforwardly to the Evans-Vigier field equations [2] recently developed to account self-consistently for the existence of $\boldsymbol{B}^{(3)}$ in the vacuum. Section Three develops electromagnetism and gravitation as respectively the antisymmetric and symmetric parts of the same Ricci tensor. This leads to the Einstein equation for vacuum electromagnetism and develops a link between electromagnetic angular frequency, ω, and the gravitational constant k per unit volume V used to define mass density in gravitation. Thus electromagnetism and gravitation are shown to be parts of the same phenomenon in nature, and the elementary charge e is linked to elementary mass density, implying that the photon as particle is massive.

2. THE ANTISYMMETRIC RICCI TENSOR

It is well known that curvature of spacetime in general relativity is described by the Riemann tensor [11,12] $R^{\kappa}_{\lambda\mu\nu}$. In the theory of gravitation, contraction of this tensor to the symmetric Ricci tensor $R^{(S)}_{\lambda\nu}$ occurs by setting equal the indices κ and μ, and summing over repeated indices. The result is a symmetric Ricci tensor which is used to define Einstein's tensor,

$$G^{(E)}_{\lambda\nu} = R^{(S)}_{\lambda\nu} - \frac{1}{2} g_{\lambda\nu} R, \tag{1}$$

and the Einstein equation of gravitation,

$$G^{(E)}_{\lambda\nu} = \frac{8\pi k}{c^4} T_{\lambda\nu}. \tag{2}$$

Here $T_{\lambda\nu}$ is the canonical energy-momentum tensor, k is Einstein's gravitational constant, and c the speed of light. The constant k is universal and is positive, having an S.I. value [11],

$$k = 6.67 \times 10^{-11} m^3 kgm^{-1} s^{-2}. \tag{3}$$

he Bianchi identity of general relativity is essentially an equation of onservation, of general validity in physics, but is also an expression of eometry. All equations of physics are geometrical in origin [11,12], and this ias become a familiar feature on the landscape of twentieth century philosophy. t is well known furthermore that aspects of non-Abelian gauge theory [13] have ;everal things in common with those of general relativity. The discovery of :he experimentally verifiable $\boldsymbol{B}^{(3)}$ field unifies, as shown in this Letter and ılsewhere [14], the electromagnetic and gravitational fields within the same)verall Ricci tensor. Before the existence of $\boldsymbol{B}^{(3)}$ was realized, analogies)etween the two field theories were well known [13]. Gauge transformation in ılectromagnetism is analogous with coordinate transformation in gravitation; the gauge group in electromagnetism is the group of all coordinate transformations in gravitation; the gauge potential of electrodynamics is analogous to the connection coefficient (Christoffel symbol) in gravitation; the field strength tensor in electrodynamics is analogous to the Riemann curvature tensor in gravitation; finally the inhomogeneous part of the free space Maxwell equations is analogous with the Bianchi identity in gravitation.

However, the objections to unification prior to $\boldsymbol{B}^{(3)}$ are also well known [13]. Gravitational equations are non-linear, electromagnetic equations were thought to be linear. The realization [1–10] that electromagnetism in vacuo is based on cyclic relations, involving $\boldsymbol{B}^{(3)}$, between field components removes, however, the illusion of linearity given by an uncritical acceptance of the vacuum Maxwell equations without invalidating the superposition principle upon which is based quantum mechanics [1–10]. The other objection to unification was equally well known [13], and was based on the fact that gravitation is thought to be curvature of spacetime through Eq. (2), while electromagnetism appeared to have no such property. Evans and Vigier have shown, however [2], that the equations of vacuum electrodynamics are changed fundamentally by the existence of $\boldsymbol{B}^{(3)}$, *an experimental observable* [1–10]. In this section, the Evans-Vigier field equations [2] are recovered directly and simply from the familiar Bianchi identity,

$$D_\rho R^{\kappa}_{\lambda\mu\nu} + D_\nu R^{\kappa}_{\lambda\rho\mu} + D_\mu R^{\kappa}_{\lambda\nu\rho} = 0. \tag{4}$$

It has become habitual to link the Bianchi identity (4) immediately with the gravitational field. However, it is a geometrical relation which in principle has a more general significance, i.e., it should also be valid for other fields in physics.

Its reduction to the Evans-Vigier field equations [2] occurs by contracting the Riemann tensor $R^{\kappa}_{\lambda\mu\nu}$ to a novel *antisymmetric* Ricci tensor $R^{(A)}_{\mu\nu}$, a contraction which takes place by equating indices κ and λ, and summing over repeated indices. This procedure reduces the Bianchi identity (4) to the geometrical identity,

$$D_\rho R^{(A)}_{\mu\nu} + D_\nu R^{(A)}_{\rho\mu} + D_\mu R^{(A)}_{\nu\rho} = 0, \tag{5}$$

which becomes the Evans-Vigier field equations through the relation,

$$R^{(A)}_{\mu\nu} = \frac{e}{\hbar} G_{\mu\nu}, \tag{6}$$

where $G_{\mu\nu}$ is the antisymmetric, *electromagnetic* field strength tensor [2]. The proportionality factor $e/\hbar$ is the ratio of the quanta of charge and of action, and maintains dimensionality. The geometrical identity (5) becomes a physical equation of electrodynamics in vacuo,

$$D_\rho G_{\mu\nu} + D_\nu G_{\rho\mu} + D_\mu G_{\nu\rho} = 0, \tag{7}$$

in which appears covariant derivatives. The gauge group of this equation is the spacetime Poincaré group whose space component is O(3) [2]. Equation (7) is precisely the equation implied [2] by the existence in vacuo of the $\boldsymbol{B}^{(3)}$

ield.

. UNIFICATION OF ELECTROMAGNETISM WITH GRAVITATION

The field strength tensor $\boldsymbol{G}_{\mu\nu}$ can be expressed [2,14] in terms of the ommutator of rotation generators and the vector potential magnitude $\boldsymbol{A}^{(0)}$ as,

$$G_{\mu\nu} = \frac{e}{\hbar} A^{(0)2} [M_{\mu}, M_{\nu}], \tag{8}$$

nd, self consistently, contains within it the $\boldsymbol{B}^{(3)}$ *field.* In the complex asis [1–10] ((1), (2), (3)), and in vector notation,

$$\boldsymbol{B}^{(3)*} = -i \frac{e}{\hbar} \boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}. \tag{9}$$

This result is consistent with the minimal prescription applied to the free hoton [1–3] linear momentum $\hbar\boldsymbol{\kappa}$,

$$eA^{(0)} = \hbar\kappa. \tag{10}$$

From these considerations we find that

$$R_{\mu\nu}^{(A)} = \frac{\kappa^2}{\hbar} J_{\mu\nu}, \tag{11}$$

where $J_{\mu\nu}$ is an angular momentum expectation value. It is now possible to define the angular energy-angular momentum tensor,

$$T_{\mu\nu}^{(A)} = \omega J_{\mu\nu} = \frac{\hbar\omega}{\kappa^2} R_{\mu\nu}^{(A)}. \tag{12}$$

This is a type of Einstein equation for electromagnetism. We have shown that the $T_{\mu\nu}^{(A)}$ tensor of electromagnetism is the antisymmetric part, $R_{\mu\nu}^{(A)}$, of the Ricci tensor of curved spacetime within a factor $\hbar\omega / \kappa^2$.

The *complete*, second rank, Ricci tensor is the sum of its symmetric and antisymmetric components [15],

$$R_{\mu\nu} = R_{\mu\nu}^{(S)} - \frac{1}{2} g_{\mu\nu} R \quad + \quad R_{\mu\nu}^{(A)} .$$

gravitation electromagnetism (13)

e dimensions on the right hand side must be the same, so comparing Eqs. (2) ıd (12), and being careful to recall that the $T_{\mu\nu}$ tensor in gravitation is ːfined [16] as a density, occupying a volume V, we obtain,

$$\omega = \frac{8\pi k}{c^2 V} \hbar . \tag{14}$$

ıe physical meaning of this equation is made clear by assuming that

$$V = \lambda L^{*2} , \tag{15}$$

ıere λ is the wavelength and L^* a length to be determined. With $\omega = 2\pi f$, here f is electromagnetic frequency, Eq. (14) becomes,

$$\lambda f = \left(\frac{4k}{c^2 L^{*2}} \right) \hbar . \tag{16}$$

e note that if L^* is identified as *the Planck length* [16], Eq. (16) becomes, elf-consistently

$$\lambda f = c, \qquad L^* = \left(4 \frac{\hbar k}{c^3} \right)^{1/2} . \tag{17}$$

We have therefore *derived* a fundamental equation of vacuum electromag- ıetism, Eq. (17), *provided that*

$$\frac{c^3 V}{4k} = \hbar \lambda , \tag{18}$$

a relation which expresses the *electromagnetic* wavelength λ in terms of *gravitational* properties, and at the same time, quantizes the unified field.

4. DISCUSSION

The Planck length, L^*, is the dimension ($\sim 8 \times 10^{-35}$m) at which the smoothness of curved spacetime is broken down by quantization [16]. At distances of the order L^* there occur violent fluctuations in the geometry of curved spacetime, so that space develops a multiply connected, foam-like, character [16]. The Eq. (18) of the unified field also expresses the quantization of volume V *itself*. Quantum fluctuations develop in space itself, and these become measurable experimentally [16] as vacuum fluctuations in quantum electrodynamics. These conclusions are a direct, logical, consequence of the existence in the vacuum of the physical and measurable component $\boldsymbol{B}^{(3)}$ of the electromagnetic field. The most compact form of Eq. (18), and our final result, is

$$V = L^{*2} \lambda, \tag{19}$$

i.e the assumption (15) with L^* now identified as the Planck length.

If we take V in Eq. (19) to be the volume of the universe today, i.e., $V \sim 10^{78}\ m^3$ [16], and assume that this is a physical upper bound on V, then we obtain a corresponding maximum electromagnetic wavelength from Eq. (19),

$$\lambda_{\max} \sim 10^{10} m, \tag{20}$$

There are many more inferences of this nature which follow directly from that [1–10] of a non-zero $\boldsymbol{B}^{(3)}$ in the vacuum.

SUMMARY OF EXPERIMENTAL DATA

It is considered in this paper that experimental evidence [1–3] for the conjugate product of magneto-optics is also evidence for the $\boldsymbol{B}^{(3)}$ field in electromagnetism. Since $\boldsymbol{B}^{(3)}$ is derived from a fundamental spin angular momentum [3] it is observed in atomic absorption as the transfer of $\pm\hbar$ from photon to atom. The Einstein coefficients and Stokes parameters, for example, can be expressed in terms of $B^{(0)}\boldsymbol{B}^{(3)}$ [6]. Its radiation theory has been well developed [3] and its specific first order magnetizing influence can be observed under the right conditions [1,3]. In general, electrodynamics and spectroscopy can be reformulated and worked out entirely in terms of the $\boldsymbol{B}^{(3)}$ field and its magnitude $B^{(0)}$. The $\boldsymbol{B}^{(3)}$ field is therefore established empirically in as many ways as there are available for the transverse components of the received view. In other words $\boldsymbol{B}^{(3)}$ reduces electrodynamics to its simplest form: the transverse components become a consequence of $\boldsymbol{B}^{(3)}$ and vice versa.

ZITTERBEWEGUNG

A referee has made the suggestion that there may be zitterbewegung in the relativistic quantum field theory of the photon if massive. Although not directly related to the subject of this paper this is worth pursuing because the theory may lead to further experimental tests for photon mass, or lack thereof.

PHYSICAL SIGNIFICANCE OF THE NOVEL TENSOR CONTRACTION

The Riemann tensor $R^{\kappa}_{\lambda\mu\nu}$ is antisymmetric in $\mu\nu$ if $\kappa = \lambda$ [11,16]. Therefore a theory that uses the index contraction $\kappa = \lambda$ is a Riemannian theory that maintains the overall structure of the original four index Riemann tensor, giving the Ricci tensor $R^{(A)}_{\mu\nu}$. The commutator of affine connections appearing in $R^{(A)}_{\mu\nu}$ produces the longitudinal field $\boldsymbol{B}^{(3)}$ which is missing from the usual theory of electrodynamics. The usual theory therefore does not match the structure of $R^{(A)}_{\mu\nu}$, in which appears antisymmetric affine connections. Therefore the contraction represented by $\kappa = \lambda$ produces the magnetic field

$\boldsymbol{B}^{(3)}$ as part of $G_{\mu\nu}$. Such a contraction leads to a structure for the electromagnetic field tensor different from the usual $F_{\mu\nu}$ tensor of the received view, in which $\boldsymbol{B}^{(3)}$ is missing. The tensor $R_{\mu\nu}^{(A)}$ is connected with the Weyl conformal tensor [11,16] which perhaps can be used to produce a link between the antisymmetric and symmetric affine connections of $R_{\mu\nu}^{(A)}$ and $R_{\mu\nu}^{(S)}$ in our simple theory. However that would probably lead to a more intricate structure for electromagnetism, one which may however have physical significance.

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REFERENCES

[1] M. W. Evans and J.-P. Vigier, *The Enigmatic Photon, Vol. 1: The Field* $\boldsymbol{B}^{(3)}$ (Kluwer Academic, Dordrecht, 1994).

[2] M. W. Evans and J.-P. Vigier, *The Enigmatic Photon, Vol. 2: Non-Abelian Electrodynamics* (Kluwer Academic, Dordrecht, 1995).

[3] M. W. Evans, J.-P. Vigier, S. Roy, and S. Jeffers, *The Enigmatic Photon, Vol. 3: Theory and Practice of* $\boldsymbol{B}^{(3)}$ (Kluwer Academic, Dordrecht, in prep.).

[4] M. W. Evans, *Physica B* **182**, 227, 237 (1992); **183**, 103 (1993); **190**, 310 (1993); *Physica A* **214**, 605 (1995).

[5] M. W. Evans, *Found. Phys. Lett.* **7**, 67, 209, 379, 467, 577 (1994); **8** (1995); **24**, 1519, 1671 (1994); **25**, 175, 383 (1995); S. Roy and M. W. Evans, *Found. Phys.* in press; ibid., submitted.

[6] M. W. Evans and A. A. Hasanein, The Photomagneton in Quantum Field Theory (World Scientific, Singapore, 1994); M. W. Evans, The Photon's Magnetic Field (World Scientific, Singapore, 1992); M. W. Evans and S. Kielich, eds., *Modern Nonlinear Optics*, Vol. 85(2) of *Advances in Chemical Physics*, I. Prigogine and S. A. Rice, eds., (Wiley Interscience, New York, 1993).

[7] M. W. Evans, S. Roy and S. Jeffers, *Lett. Nuovo Cim.* in press.

[8] M. W. Evans, *Mod. Phys. Lett.* **7B**, 1247 (1993); *J. Mol. Liq.* **55**, 127 (1993).

[9] M. W. Evans, J.-P. Vigier, S. Roy, and S. Jeffers, *The Enigmatic Photon, Vol. 4: Unification of Electrodynamics and Gravitation with* $\boldsymbol{B}^{(3)}$ (Kluwer, Dordrecht, in prep.).

[10] M. W. Evans, *Found. Phys. Lett.* and *Opt. Lett.*, submitted for publication.

[11] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* 4th edn. (Pergamon, Oxford, 1975).

[12] S. K. Bose, *An Introduction to General Relativity.* (Wiley Eastern, New Delhi, 1980).

[13] L. H. Ryder, *Quantum Field Theory* 2nd edn. (Cambridge University Press, Cambridge, 1987).

[14] M. W. Evans, *Found. Phys. Lett.*, submitted for publication.

[15] I. S. Sokolnikoff, *Tensor Analysis: Theory and Applications* (Wiley, London, 1951).

[16] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).