FLOW CHART ONE

FIRST CARTAN STRUCTURE EQUATION

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b$$

$$T^a{}_{\mu\nu} = \partial_\mu q^a{}_\nu - \partial_\nu q^a{}_\mu + \omega^a{}_{\mu b} q^b{}_\nu - \omega^a{}_{\nu b} q^b{}_\mu$$

$D_\mu q^a{}_\nu = 0$, Tetrad Postulate $\rightarrow$ Links Cartan and Riemann Geometry

$$\partial_\mu q^a{}_\nu = q^a{}_\lambda \Gamma^\lambda{}_{\mu\nu} - q^b{}_\nu \omega^a{}_{\mu b},$$

$$\partial_\nu q^a{}_\mu = q^a{}_\lambda \Gamma^\lambda{}_{\nu\mu} - q^b{}_\mu \omega^a{}_{\nu b}$$

$$T^a{}_{\mu\nu} = q^a{}_\lambda \left(\Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu} \right)$$

$$= q^a{}_\lambda T^\lambda{}_{\mu\nu}$$

Torsion Tensor of Riemann Geometry

FLOW CHART TWO

THE BIANCHI IDENTITY

$$\boxed{D \wedge T := R \wedge q \neq 0}$$

$$R^{\lambda}{}_{\rho\mu\nu} + R^{\lambda}{}_{\mu\nu\rho} + R^{\lambda}{}_{\nu\rho\mu}$$

$$:= \partial_{\mu}\Gamma^{\lambda}_{\nu\rho} - \partial_{\nu}\Gamma^{\lambda}_{\mu\rho} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\nu\rho} - \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\mu\rho}$$

$$+ \partial_{\nu}\Gamma^{\lambda}_{\rho\mu} - \partial_{\rho}\Gamma^{\lambda}_{\nu\mu} + \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\rho\mu} - \Gamma^{\lambda}_{\rho\sigma}\Gamma^{\sigma}_{\nu\mu}$$

$$+ \partial_{\rho}\Gamma^{\lambda}_{\mu\nu} - \partial_{\mu}\Gamma^{\lambda}_{\rho\nu} + \Gamma^{\lambda}_{\rho\sigma}\Gamma^{\sigma}_{\mu\nu} - \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\rho\nu} .$$

$$\neq 0$$

$$\boxed{T^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}}$$

$$\boxed{T^{a} = d \wedge q^{a} + \omega^{a}{}_{b} \wedge q^{b}}$$

$$\boxed{T = D \wedge q}$$

FLOW CHART THREE

HOMOGENEOUS ECE FIELD EQUATION

$$D \wedge T := R \wedge q$$

$$D \wedge (D \wedge T) := D \wedge (R \wedge q)$$

$$D \wedge T^a := R^a{}_b \wedge q^b$$

$$d \wedge T^a = R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b$$

$$d \wedge T^a = 0, \text{ F.A.P.P.}$$

$$R \wedge q = \omega \wedge T$$

$$d \wedge T^a = -\left(q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b\right)$$

$$\partial_\rho T^a{}_{\mu\nu} + \partial_\nu T^a{}_{\rho\mu} + \partial_\mu T^a{}_{\nu\rho} = -\left(R^a{}_{\rho\mu\nu} + R^a{}_{\nu\rho\mu} + R^a{}_{\mu\nu\rho} + \omega^a{}_{b\rho} T^b{}_{\mu\nu} + \omega^a{}_{b\nu} T^b{}_{\rho\mu} + \omega^a{}_{b\mu} T^b{}_{\nu\rho}\right)$$

Sums of two-forms on both sides

Hodge dual on both sides

$$\partial_\mu \tilde{T}^{a\mu\nu} = -\left(\tilde{R}^a{}_\mu{}^{\mu\nu} + \omega^a{}_{\mu b} \tilde{T}^{b\mu\nu}\right)$$

$$\partial_\mu \tilde{F}^{a\mu\nu} = -A^{(0)}\left(\tilde{R}^a{}_\mu{}^{\mu\nu} + \omega^a{}_{\mu b} \tilde{T}^{b\mu\nu}\right)$$

$$\partial_\mu \tilde{F}^{a\mu\nu} = 0,$$

F.A.P.P.

Free electromagnetic field

$$\partial_\mu \tilde{F}^{\kappa\mu\nu} = 0$$

$$\underline{\nabla} \cdot \underline{B} = 0,$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0}$$

FLOW CHART FOUR

INHOMOGENEOUS ECE FIELD EQUATION

$$D \wedge \tilde{T} := \tilde{R} \wedge q$$

$$D \wedge \tilde{T}^a := \tilde{R}^a{}_b \wedge q^b$$

$$\partial_\rho \tilde{T}^a{}_{\mu\nu} + \partial_\nu \tilde{T}^a{}_{\rho\mu} + \partial_\mu \tilde{T}^a{}_{\nu\rho} = -\left(\tilde{R}^a{}_{\rho\mu\nu} + \tilde{R}^a{}_{\nu\rho\mu} + \tilde{R}^a{}_{\mu\nu\rho} + \omega^a{}_{\rho b} \tilde{T}^b{}_{\mu\nu} + \omega^a{}_{\nu b} \tilde{T}^b{}_{\rho\mu} + \omega^a{}_{\mu b} \tilde{T}^b{}_{\nu\rho} \right)$$

$$\partial_\mu T^{a\mu\nu} = -\left(R^a{}_\mu{}^{\mu\nu} + \omega^a{}_{\mu b} T^{b\mu\nu} \right)$$

$$\partial_\mu F^{a\mu\nu} = -A^{(0)} \left(R^a{}_\mu{}^{\mu\nu} + \omega^a{}_{\mu b} T^{b\mu\nu} \right)$$

$$\partial_\mu F^{a\mu\nu} = -A^{(0)} \left(R^a{}_\mu{}^{\mu\nu} \right)_{EH}$$

$$\partial_\mu F^{\kappa\mu\nu} = -A^{(0)} \left(R^\kappa{}_\mu{}^{\mu\nu} \right)_{EH}$$

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 ,$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J}$$

$$D \wedge (D \wedge \tilde{T}) := D \wedge (\tilde{R} \wedge q)$$

Ricci flat vacuum

$$\partial_\mu F^{\kappa\mu\nu} = 0,$$

$$R^\kappa{}_\mu{}^{\mu\nu} = 0 .$$

$$\underline{\nabla} \cdot \underline{E} = 0,$$

$$\underline{\nabla} \times \underline{B} = \frac{1}{c^2} \frac{\partial E}{\partial t},$$

$$G_{\mu\nu} = 0 \quad (EH)$$

No sources for fields, unphysical!

FLOW CHART FIVE

THE BASIC FIELD EQUATIONS

HOMOGENEOUS

$$D_\mu \tilde{T}^{\kappa\mu\nu} = -\tilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu}$$

$$D_\mu \tilde{T}^{a\mu\nu} = -\tilde{R}^{a}{}_{\mu}{}^{\mu\nu}$$

$$D_\rho T^a_{\mu\nu} + D_\nu T^a_{\rho\mu} + D_\mu T^a_{\nu\rho} = -\left(R^a_{\rho\mu\nu} + R^a_{\nu\rho\mu} + R^a_{\mu\nu\rho}\right)$$

$$D \wedge T := -q \wedge R$$

$$D \wedge \tilde{T} := -q \wedge \tilde{R}$$

$$D_\rho \tilde{T}^a_{\mu\nu} + D_\nu \tilde{T}^a_{\rho\mu} + D_\mu \tilde{T}^a_{\nu\rho} = -\left(\tilde{R}^a_{\rho\mu\nu} + \tilde{R}^a_{\nu\rho\mu} + \tilde{R}^a_{\mu\nu\rho}\right)$$

$$D_\mu T^{a\mu\nu} = -R^{a}{}_{\mu}{}^{\mu\nu}$$

$$D_\mu T^{\kappa\mu\nu} = -R^{\kappa}{}_{\mu}{}^{\mu\nu}$$

INHOMOGENEOUS

FLOW CHART SIX

APPROXIMATIONS TO THE BASIC FIELD EQUATIONS

No interaction between e/m and gravitation.

$$\underline{\nabla} \cdot \underline{B} = \underline{0}$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0}$$

$$\partial_\mu \tilde{F}^{\kappa\mu\nu} \doteq 0$$

$$\partial_\mu \tilde{F}^{\kappa\mu\nu} = -A^{(0)}\left(\tilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu} + \omega^{\kappa}{}_{\mu b}\tilde{T}^{b\mu\nu}\right)$$

$$G_{\mu\nu} = 0$$

$$T_{\mu\nu} = 0$$

$$D_\mu \tilde{T}^{\kappa\mu\nu} = -\tilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu}$$

$$D_\mu T^{\kappa\mu\nu} = -R^{\kappa}{}_{\mu}{}^{\mu\nu}$$

$$F_{\kappa\mu\nu} = 0$$

$$J_{\kappa\mu\nu} = 0$$

$$\partial_\mu F^{\kappa\mu\nu} = -A^{(0)}\left(R^{\kappa}{}_{\mu}{}^{\mu\nu} + \omega^{\kappa}{}_{\mu b}T^{b\mu\nu}\right)$$

$$\partial_\mu F^{\kappa\mu\nu} \doteq -A^{(0)}\left(R^{\kappa}{}_{\mu}{}^{\mu\nu}\right)_{grav.}$$

$$\underline{E} = \underline{0}, \quad \underline{B} = \underline{0}, \quad T_{\mu\nu} = 0$$

$$\underline{\nabla} \cdot \underline{E} = 0,$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \underline{0},$$

for a Ricci flat vacuum where $\left(R^{\kappa}{}_{\mu}{}^{\mu\nu}\right)_{grav} = 0$

FLOW CHART SEVEN

HODGE DUAL OF THE BIANCHI IDENTITY

$$[D_\mu, D_\nu] V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\lambda{}_{\mu\nu} D_\lambda V^\rho$$

$$\downarrow$$

$$D \wedge T := R \wedge q$$

$$\downarrow$$

$$[D^\mu_\mu, \tilde{D}_\nu]_{HD} V^\rho = \tilde{R}^\rho{}_\sigma{}^{\mu\nu} V^\sigma - \tilde{T}^{\lambda\mu\nu} D_\lambda V^\rho$$

$$\downarrow$$

$$g_{\mu\alpha} g_{\nu\beta} [D^\alpha, D^\beta]_{HD} V^\rho = g_{\mu\alpha} g_{\nu\beta} \left(\tilde{R}^\rho{}_\sigma{}^{\mu\nu} V^\sigma - \tilde{T}^{\lambda\mu\nu} D_\lambda V^\rho \right)$$

$$\downarrow$$

$$g^{\mu\alpha} g^{\nu\beta} [D_\alpha, D_\beta]_{HD} V^\rho = g^{\mu\alpha} g^{\nu\beta} \left(\tilde{R}^\rho{}_{\sigma\alpha\beta} V^\sigma - \tilde{T}^\lambda{}_{\alpha\beta} D_\lambda V^\rho \right)$$

$$\downarrow$$

$$[D_\mu, D_\nu]_{HD} V^\rho = \tilde{R}^\rho{}_{\sigma\mu\nu} V^\sigma - \tilde{T}^\lambda{}_{\mu\nu} D_\lambda V^\rho$$

$$\downarrow$$

$$D \wedge \tilde{T} := \tilde{R} \wedge q$$

$$\downarrow$$

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu}$$

$$\downarrow$$

Einstein Hilbert equation is self-inconsistent.