100(2) : The Basic Field Equations

In the Maxwell Heaviside theory the homogeneous equation is given by:

$$d \wedge F = 0 \qquad (1)$$

and the inhomogeneous equation by:

$$d \wedge \tilde{F} = \mu_0 J \qquad (2)$$

in differential form notation. These are first translated into Vector notation as follows. Eq (1) is:

$$\partial_\rho F_{\mu\nu} + \partial_\nu F_{\rho\mu} + \partial_\mu F_{\nu\rho} = 0 \qquad (3)$$

and this is the same as:

$$\boxed{\partial_\mu \tilde{F}^{\mu\nu} = 0} \qquad (4)$$

where the tilde denotes Hodge dual:

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} . \qquad (5)$$

Example

$$\partial_0 F_{12} + \partial_2 F_{01} + \partial_1 F_{20} = 0, \qquad (6)$$

and:

$$F_{12} = \frac{1}{2} \epsilon_{1203} \tilde{F}^{03} = \tilde{F}^{03} \qquad (7)$$

$$F_{01} = \frac{1}{2} \epsilon_{0123} \tilde{F}^{23} = \tilde{F}^{23} \qquad (8)$$

$$F_{20} = \frac{1}{2} \epsilon_{2013} \tilde{F}^{13} = \tilde{F}^{13} \qquad (9)$$

so:

$$\partial_0 \tilde{F}^{03} + \partial_2 \tilde{F}^{23} + \partial_1 \tilde{F}^{13} = 0 \qquad (10)$$

Q.E.D.

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It follows that eq. (2) is:

$$\partial_\rho \tilde{F}_{\mu\nu} + \partial_\nu \tilde{F}_{\rho\mu} + \partial_\mu \tilde{F}_{\nu\rho} = \mu_0 \left(\dot{J}_{\rho\mu\nu} + \dot{J}_{\nu\rho\mu} + J_{\mu\nu\rho} \right) \quad -(11)$$

which is the same as:

$$\boxed{\partial_\mu F^{\mu\nu} = \mu_0 J^\nu} \quad -(12)$$

In vector notation eq. (4) becomes the eqns:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(13)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(14)$$

and eq. (12) becomes:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(15)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(16)$$

In the ECE theory eq. (1) becomes:

$$D \wedge F^a = A^{(0)} \left(R^a{}_b \wedge q^b \right) \quad -(17)$$

which is

$$\boxed{\partial_\mu \tilde{F}^{a\mu\nu} = -A^{(0)} \left(\tilde{R}^{a}{}_{\mu}{}^{\mu\nu} - \omega^a{}_{\mu b} \tilde{T}^{b\mu\nu} \right)} \quad -(18)$$

and eq. (2) becomes:

$$D \wedge \tilde{F}^a = A^{(0)} \left(\tilde{R}^a{}_b \wedge q^b \right) \quad -(19)$$

, which becomes:

$$\boxed{\partial_\mu F^{a\mu\nu} = -A^{(0)}\left(R^{a}{}_{\mu}{}^{\mu\nu} - \omega^{a}{}_{\mu b} T^{b\mu\nu}\right)} \quad -(20)$$

For all practical purposes in the laboratory eq. (18) is:

$$\partial_\mu \tilde{F}^{a\mu\nu} = 0 \quad -(21)$$

and eq. (20) is:

$$\partial_\mu F^{a\mu\nu} = -A^{(0)}\left(R^{a}{}_{\mu}{}^{\mu\nu}\right)_{grav} \quad -(22)$$

<u>Equations in the Base Manifold</u>

Eq. (18) is:

$$D_\mu \tilde{F}^{a\mu\nu} = -A^{(0)} \tilde{R}^{a}{}_{\mu}{}^{\mu\nu} \quad -(23)$$

and eq. (20) is:

$$D_\mu F^{a\mu\nu} = -A^{(0)} R^{a}{}_{\mu}{}^{\mu\nu} \quad -(24)$$

Here:

$$F^{a\mu\nu} = q^{a}{}_{\kappa} F^{\kappa\mu\nu} \quad -(25)$$

$$\tilde{F}^{a\mu\nu} = q^{a}{}_{\kappa} \tilde{F}^{\kappa\mu\nu} \quad -(26)$$

where $q^{a}{}_{\kappa}$ is <u>the Cartan tetrad</u>.

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The ~~metric~~ tetrad postulate is:

$$D_\mu q^a{}_\kappa = 0 \quad -(27)$$

and it follows that:

$$q^a{}_\kappa D_\mu \tilde{F}^{\kappa\mu\nu} = -A^{(0)} q^a{}_\kappa \tilde{R}^\kappa{}_\mu{}^{\mu\nu} \quad -(28)$$

and

$$q^a{}_\kappa D_\mu F^{\kappa\mu\nu} = -A^{(0)} q^a{}_\kappa R^\kappa{}_\mu{}^{\mu\nu} \quad -(29)$$

Particular solution of eqns. (28) and (29) are:

$$D_\mu \tilde{F}^{\kappa\mu\nu} = -A^{(0)} \tilde{R}^\kappa{}_\mu{}^{\mu\nu} \quad -(30)$$

$$D_\mu F^{\kappa\mu\nu} = -A^{(0)} R^\kappa{}_\mu{}^{\mu\nu} \quad -(31)$$

~~and~~ these are written in ~~the~~ base manifold, which is a 4-D spacetime with curvature and torsion. $-(32)$

For all practical purposes:

$$\boxed{\begin{aligned} \partial_\mu \tilde{F}^{\kappa\mu\nu} &= 0 \\ \partial_\mu F^{\kappa\mu\nu} &= -A^{(0)} \left(R^\kappa{}_\mu{}^{\mu\nu}\right)_{\text{grav.}} \end{aligned}} \quad (33)$$

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It is seen that the electromagnetic field in ECE theory becomes a rank-three tensor related to the rank three canonical energy-momentum density. In the vacuum:

$$R^{\kappa\ \mu\nu}_{\ \mu} = 0 \quad -(34)$$

because the vacuum metrics in gravitational GR are Ricci flat by construction. So, self-consistently, the vacuum e/m equations are:

$$\partial_\mu \tilde{F}^{\kappa\mu\nu} = 0 \quad -(35)$$

$$\partial_\mu F^{\kappa\mu\nu} = 0 \quad -(36)$$

Coulomb Law ($\nu = 0$ in eq. (33))

This is:

$$\partial_\mu F^{0\mu 0} = -A^{(0)}\left(R^{0\ \mu 0}_{\ \mu}\right) \quad -(37)$$

If the electric field vector is:

$$\underline{E} = E^{010}\,\underline{i} + E^{020}\,\underline{j} + E^{030}\,\underline{k}$$

$$= E_X\,\underline{i} + E_Y\,\underline{j} + E_Z\,\underline{k} \quad -(38)$$

eq. (37) is the Coulomb Law:

$$\underline{\nabla} \cdot \underline{E} = \frac{\rho}{\epsilon_0} \quad -(39)$$

 where the charge density ρ is proportional to:

$$R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30}. \quad -(40)$$

This result was tested by computer in paper 93 onwards for various metrics and it was found by computer that all vacuum metrics give:

$$R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30} = 0 \quad -(41)$$

correctly and self-consistently. So the electric field is defined by:

$$\boxed{\begin{aligned} E_X &= E^{010} \\ E_Y &= E^{020} \\ E_Z &= E^{030} \end{aligned}} \quad -(42)$$

in a generally covariant unified field theory. These are proportional to components of ~~the~~ orbital canonical angular momentum / energy density.

Ampère Maxwell Law $(\mu = 1, 2, 3)$

This is for $\mu = 1$:

$$\partial_0 F^{\kappa 01} + \partial_2 F^{\kappa 21} + \partial_3 F^{\kappa 31} \quad -(43)$$

$$= -A^{(0)} \left(R^{\kappa}{}_{0}{}^{01} + R^{\kappa}{}_{2}{}^{21} + R^{\kappa}{}_{3}{}^{31} \right)$$

For $\mu = 2$:

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$$\partial_0 F^{\kappa 0 2} + \partial_1 F^{\kappa 1 2} + \partial_3 F^{\kappa 3 2}$$
$$= -A^{(0)} \left(R^{\kappa}{}_{0}{}^{02} + R^{\kappa}{}_{1}{}^{12} + R^{\kappa}{}_{3}{}^{32} \right). \quad -(44)$$

Finally for $\nu = 3$:

$$\partial_0 F^{\kappa 0 3} + \partial_1 F^{\kappa 1 3} + \partial_2 F^{\kappa 2 3} \quad -(45)$$
$$= -A^{(0)} \left(R^{\kappa}{}_{0}{}^{03} + R^{\kappa}{}_{1}{}^{13} + R^{\kappa}{}_{2}{}^{23} \right)$$

These tensor equations can be condensed into the vector equation:

$$\boxed{\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J}} \quad -(46)$$

where:

$$J_X = -\frac{A^{(0)}}{\mu_0} \left(R^{1}{}_{0}{}^{01} + R^{1}{}_{2}{}^{21} + R^{1}{}_{3}{}^{31} \right)$$
$$J_Y = -\frac{A^{(0)}}{\mu_0} \left(R^{2}{}_{0}{}^{02} + R^{2}{}_{1}{}^{12} + R^{2}{}_{3}{}^{32} \right)$$
$$J_Z = -\frac{A^{(0)}}{\mu_0} \left(R^{3}{}_{0}{}^{03} + R^{3}{}_{1}{}^{13} + R^{3}{}_{2}{}^{23} \right)$$
$$-(47)$$

and

$$B_X = B^{123}$$
$$B_Y = B^{231} \quad -(48)$$
$$B_Z = B^{312}$$

This was again checked by computer.