1) Paper 105 Notes 2 :

Light Deflection due to the Spin Connection

The light deflection due to gravitation is considered to be due to the spin connection. The starting equation is the Hodge dual of the Bianchi identity:

$$d \wedge \tilde{T} + \omega \wedge \tilde{T} = - q \wedge \tilde{R} \quad -(1)$$

which is the equation:

$$\partial_\mu T^{a\mu\nu} = -\left(R^{a}{}_{\mu}{}^{\mu\nu} + \omega^{a}{}_{\mu b} T^{b\mu\nu} \right) \quad -(2)$$

A particular solution in the base manifold is :

$$\partial_\mu T^{\kappa\mu\nu} = -\left(R^{\kappa}{}_{\mu}{}^{\mu\nu} + 4\,\omega^{\kappa}{}_{\mu\lambda} T^{\lambda\mu\nu} \right). \quad -(3)$$

The Newtonian limit is considered to be the limit in which the spin connection is zero asymptotically i.e.

$$\partial_\mu T^{\kappa\mu\nu} = - R^{\kappa}{}_{\mu}{}^{\mu\nu} \quad -(4)$$

As in note 105(1):

$$\underline{g} = - c^2 \left(T^{010}\, \underline{i} + T^{020}\, \underline{j} + T^{030}\, \underline{k} \right) \quad -(5)$$

$$k \rho_m = - \left(R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30} \right) \quad -(6)$$

The minus sign has been introduced to adhere to the convention of the Einstein postulate:

2)

$$R = -kT \quad -(7)$$

where R is scalar curvature and T is index contracted energy - momentum density. This convention is used to derive the ECE wave equation:

$$(\square + kT)\, q^a_\mu = 0 \quad -(8)$$

Therefore in Newtonian physics:

$$\underline{\nabla} \cdot \underline{g} = c^2 k \rho_m \quad -(9)$$

whose structure is the same as the Coulomb law of electrodynamics

$$\underline{\nabla} \cdot \underline{E} = \rho_e / \epsilon_0 \quad -(10)$$

The two laws (9) and (10) are derived in ECE theory from the same Cartan geometry (1).

In the Newtonian limit $k\rho_m$ of eq. (9) is defined by the curvature tensor elements of eq. (8). More generally however:

$$\boxed{k\rho_m = -\left(R^{0\ \ 10}_{\ 1} + R^{0\ \ 20}_{\ 2} + R^{0\ \ 30}_{\ 3} + 4\left(\omega^0_{\ 1\lambda} T^{\lambda 10} + \omega^0_{\ 2\lambda} T^{\lambda 20} + \omega^0_{\ 3\lambda} T^{\lambda 30} \right)\right)} \quad -(11)$$

and for a non-zero spin connection the effective mass density ρ_m may be increased as in eq. (11). Therefore from eq. (9) the effective g is also increased. Alternatively, if the sign of the second (spin connection) term in eq. (11) is negative, the effective g may be decreased. These phenomena occur in regions of spacetime where the spin-connection is non-zero. In the laboratory the spacetime is Minkowski spacetime, and Newtonian dynamics are operative.

In light deflection around the sun, however, it is known that the effective g is twice the Newtonian g. Thus:

$$\boxed{R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30} = 4\left(\omega^{0}{}_{1\lambda}T^{\lambda 10} + \omega^{0}{}_{2\lambda}T^{\lambda 20} + \omega^{0}{}_{3\lambda}T^{\lambda 30}\right)} \quad -(12)$$

There seems to be no a priori reason why light deflection must always be twice the Newtonian result. This is the case for the solar system, but for other systems it may not be the case. Again, the spin connection governs the deflection in general.