Notes 105(4): Method for Calculating the Metric

Start from the geodesic equation:

$$\epsilon = -g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = \text{constant} \qquad -(1)$$

(see Carroll Chapter 7) In general:

$$-g_{00}\left(\frac{dt}{d\lambda}\right)^2 + g_{11}\left(\frac{dr}{d\lambda}\right)^2 + g_{22}\, r^2\left(\frac{d\phi}{d\lambda}\right)^2 = -\epsilon \qquad -(2)$$

Make no assumptions concerning the metric or connection.

As in Carroll the conserved quantities may be defined as:

$$E = g_{00}\frac{dt}{d\lambda}, \quad L = g_{22}\, r^2\frac{d\phi}{d\lambda}. \qquad -(3)$$

Therefore eq (2) is

$$-\frac{E^2}{2} + \frac{1}{2} g_{00}\, g_{11}\left(\frac{dr}{d\lambda}\right)^2 + \frac{g_{00}}{2}\left(g_{22}\, r^2\left(\frac{d\phi}{d\lambda}\right)^2 + \epsilon\right) = 0 \qquad -(4)$$

Define the quantity

$$V(r) = \frac{1}{2} g_{00}\left(g_{22}\, r\left(\frac{d\phi}{d\lambda}\right)^2 + \epsilon\right) \qquad -(5)$$

where:

$$\lambda = c\tau, \qquad -(6)$$

τ being the proper time.

2)

The quantity V in eq (5) is unitless. To write it in units of joules, and define it as potential energy:

$$V := \frac{1}{2} m c^2 g_{00} \left(g_{33} r^2 \left(\frac{d\phi}{d\lambda} \right)^2 + \epsilon \right) \quad -(7)$$

To consider light deflected by mass, the null geodesic is used, so:

$$\epsilon = 0 \quad -(8)$$

and

$$\boxed{V = \frac{1}{2} m c^2 g_{00} g_{33} r^2 \left(\frac{d\phi}{d\lambda} \right)^2} \quad -(9)$$

Without loss of generality it may be assumed that:

$$g_{33} = 1 \quad -(10)$$

so that the potential energy in joules is:

$$V = \frac{1}{2} m c^2 g_{00} r^2 \left(\frac{d\phi}{d\lambda} \right)^2$$

joules

$$\boxed{V = \frac{1}{2} m g_{00} r^2 \left(\frac{d\phi}{d\tau} \right)^2} \quad -(11)$$

This expression must now be related to the potential Φ of the ECE theory, and to the curvature, spin connection and torsion. From this, an explanation of g_{00} may be found from first principles.

3) In shorthand notation the ECE theory gives

$$\nabla \cdot \mathbf{g} = c^2 (R - \omega T) \quad (12)$$

$$c^2 k \rho_m = 8 \pi G \rho_m \quad (13)$$

where:

$$k = 1.86595 \times 10^{-26}\ N s^2 kg^{-2} \quad (14)$$

$$G = 6.6726 \times 10^{-11}\ N m^2 kg^{-2} \quad (15)$$

and ρ_m is mass density in $kg\ m^{-3}$.

Newtonian Limit

This is defined by

$$R \gg \omega T \quad (16)$$

i.e.

$$\omega \to 0 \quad (17)$$

In this limit

$$\mathbf{g} = -\nabla \Phi = -\frac{G m_2}{r^2}\,\mathbf{r} \quad (18)$$

and:

$$\mathbf{F} = m_1 \mathbf{g} = -\left(\frac{G m_1 m_2}{r^2}\right)\mathbf{r} \quad (19)$$

therefore

$$\nabla \cdot \mathbf{g} = \frac{2 G m_2}{r^3} \quad (20)$$

and:

4)

$$\rho_m = \frac{m_2}{4\pi r^3} : \frac{m_2}{3V} \quad \text{— (21)}$$

Therefore:

$$\boxed{R = \frac{k m_2}{3V} = R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30}} \quad \text{— (22)}$$

Inclusion of the Spin Connection

In this case

$$\boxed{\underline{g} = -(\nabla + \omega)\Phi} \quad \text{— (23)}$$

where:

$$V = m_2 \Phi. \quad \text{— (24)}$$

Basic Equations

These are therefore:

$$\boxed{\begin{aligned} V &= m_2\, g_{\omega}\, r^2 \left(\frac{d\phi}{d\tau}\right)^2 && \text{— (25)} \\ \underline{g} &= -(\nabla + \omega)\Phi && \text{— (26)} \\ \nabla \cdot \underline{g} &= c^2 (R - \omega T) && \text{— (27)} \\ V &= m_2 \Phi && \text{— (28)} \end{aligned}}$$

5) The mathematical problem is to solve this system of equations to give an expression for g_{00} in terms of R, ω and T. This expression for g_{00} may then be used to find the light deflection due to gravitation, perihelion advance, and so on.

From eq. (27):

$$g = c^2 \int (R - \omega T)\, dr \quad -(29)$$

From eq. (26):

$$g = \frac{\partial \Phi}{\partial r} - \omega \Phi \quad -(30)$$

where:

$$\Phi = g_{00}\, r^2 \left(\frac{d\phi}{d\tau}\right)^2 \quad -(31)$$

From eqs. (30) and (31):

$$g = -2 g_{00}\, r \left(\frac{d\phi}{d\tau}\right)^2 - \omega g_{00}\, r^2 \left(\frac{d\phi}{d\tau}\right)^2$$

$$= -g_{00}\, r \left(\frac{d\phi}{d\tau}\right)^2 (2 + \omega r) \quad -(32)$$

If we assume as in paper (3) that:

$$\omega = \frac{1}{r} \quad -(33)$$

then:

6)

$$\boxed{g = -g_{00}\left(\frac{d\phi}{d\tau}\right)^2 \quad -(34)}$$

$$\boxed{= c^2\int\left(R + \frac{T}{r}\right)dr \quad -(35)}$$

i.e. $-g_{00}\left(\dfrac{d\phi}{d\tau}\right)^2 = c^2\left(R + \dfrac{T}{r}\right)$

and:

$$\boxed{g_{00} = -c^2\left(R + \frac{T}{r}\right) \Big/ \left(\frac{d\phi}{d\tau}\right)^2} \quad -(36)$$

Experimental Result

This is given by

$$g_{00} = -\left(1 - \frac{2GM}{c^2 r}\right) \quad -(37)$$

It is seen that the structures of eqns (36) and (37) are identical. One possible solution is:

7)

$$R = \frac{1}{c^2}\left(\frac{d\phi}{d\tau}\right)^2 \quad (38)$$

and

$$T = -\frac{2GM}{c^4}\left(\frac{d\phi}{d\tau}\right)^2 \quad (39).$$

$$\boxed{T = -\frac{2GM}{c^2} R} \quad (40)$$

This is the condition that defines M. To be precisely correct

$$T = -\frac{2G\alpha}{c^2} R \quad (41)$$

so Schwarzschild's parameter is

$$\boxed{\alpha = -\frac{T c^2}{2GR}} \quad (42)$$