# Notes 106(1): Orbital Precession in ECE Theory.

The standard theory uses the line element:

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\phi^2 \quad -(1)$$

where:

$$r_s = \frac{2GM}{c^2} \quad -(2)$$

In ECE theory:

$$r_s = -\frac{1}{R} \quad -(3)$$

In the standard (EH) theory this is always called "the Schwarzschild radius", but it was not in fact given by Schwarzschild in 1916. In ECE theory it is the ratio of torsion to curvature.

Two constants of motion are defined from the line element (1) as follows:

$$E = mc^2 \left(1 - \frac{r_s}{r}\right) \frac{dt}{d\tau}, \quad -(4)$$

$$L = mr^2 \frac{d\phi}{d\tau}. \quad -(5)$$

Therefore from eqs. (1), (4) and (5):

$$\boxed{\left(\frac{dr}{d\tau}\right)^2 = \frac{E}{m^2c^2} - \left(1 - \frac{r_s}{r}\right)\left(c^2 + \frac{L^2}{m^2r^2}\right)} \quad -(6)$$

This is the equation of motion of a planet of

2)

mass $m$ orbiting a mass $M$. The equation may be written as:

$$\frac{1}{2} m \left(\frac{dr}{d\tau}\right)^2 = \left(\frac{E^2}{2mc^2} - \frac{1}{2} mc^2\right) + \frac{GMm}{r} - \frac{L^2}{2mr^2} + \frac{GML^2}{c^2 m r^3} \quad -(7)$$

This is the equation of a particle of mass $m$ in a one-dimensional effective potential:

$$V(r) = -\frac{GMm}{r} + \frac{L^2}{2mr^2} - \frac{GML^2}{c^2 m r^3} \quad -(8)$$

The first two terms are classical, the Newtonian attraction and the centripetal repulsion. The third term is the correction from general relativity. This leads to a precession of elliptical orbits by an angle per revolution of:

$$\delta\phi = \frac{6\pi GM}{c^2 A (1-e^2)} \quad -(9)$$

where $A$ is the semi-major axis and $e$ is the eccentricity. In ECE theory:

$$\delta\phi = -\left(\frac{3\pi r}{A(1-e^2)}\right)\frac{T}{R} \quad -(10)$$

i.e.

$$\delta\phi \propto r\frac{T}{R} \quad -(11)$$

3)

## Notes

1) If $2GM \neq rc^2$ exactly, $\delta\phi$ will deviate from the EH prediction. This is probably what is happening in the pioneer anomaly. In the solar system, $\delta\phi$ is very small, but in a binary pulsar, $\delta\phi$ is many orders of magnitude greater, so a pioneer type anomaly will be much larger. So precision tests of ECE vs. EH such large precession systems are needed.

2) The above theory is true for $m \ll M$, i.e. $M$ is regarded as stationary. It must be checked that it is still valid in a binary pulsar, where $m \sim M$.

3) Light deflection is predicted by eliminating the proper time as follows:

$$\left(\frac{dr}{d\phi}\right)^2 = \left(\frac{dr}{d\tau}\right)^2 \left(\frac{d\tau}{d\phi}\right)^2 = \left(\frac{dr}{d\tau}\right)^2 \left(\frac{mr^2}{L}\right)^2 \quad -(12)$$

$$\Rightarrow \left(\frac{dr}{d\phi}\right)^2 = \frac{r^4}{b^2} - \left(1 - \frac{r_s}{r}\right)\left(\frac{r^4}{a^2} + r^2\right) \quad -(13)$$

which is the orbital equation

4) Here:

$$a := \frac{L}{mc}, \quad b := \frac{cL}{E} \qquad (14)$$

In ECE theory orbital equation is still true, but $r_s$ is defined by eq (3) for all orbits, are defined by the ratio of $T$ to $R$.

From eq (13):

$$\phi = \int \left( r^2 \left( \frac{1}{b^2} - \left(1 - \frac{r_s}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)\right)^{-1} dr \qquad (15)$$

The well known formula for light deflection is obtained as $m \to 0$ — (16)

i.e.

$$\phi \to \int \left( r^2 \left( \frac{1}{b^2} - \left(1 - \frac{r_s}{r}\right)\frac{1}{r^2}\right)\right)^{-1} dr \qquad (17)$$

Expanding in powers of $r_s / r$ gives:

$$\delta\phi \sim \frac{2 r_s}{b} = \frac{4GM}{c^2 b} = -\frac{2T}{R} \cdot \frac{1}{b} \qquad (18)$$

and $b$ is identified with the distance of closest approach.

Any deviation means ECE is preferred to EH.