III(1): The Coriolis and Centripetal Accelerations

The classical expression can be found in Marion and Thornton chapter 9:

$$\underset{\sim}{F}_{eff} = m\underline{a}_r = \underset{\sim}{F} - m\ddot{\underset{\sim}{R}}_f - m\dot{\underline{\omega}} \times \underline{r} - m\underline{\omega} \times (\underline{\omega} \times \underline{r}) - 2m\underline{\omega} \times \underline{v}_r \quad — (1)$$

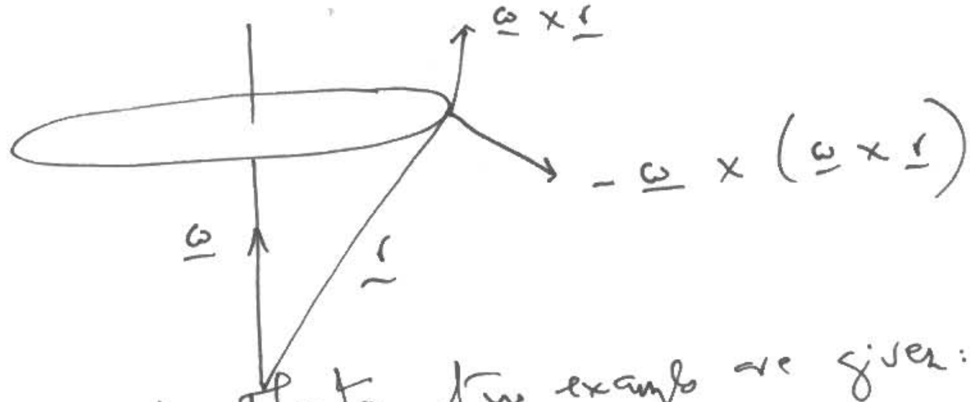

Fig (1)

On p. 343 of Marion and Thornton two examples are given:

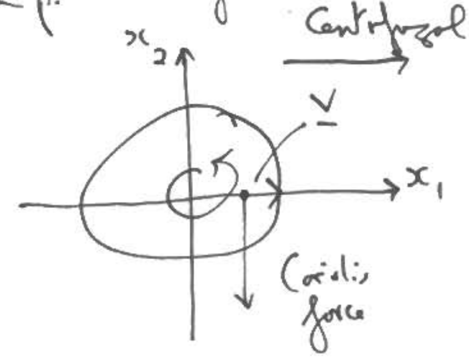

Fig. (2)

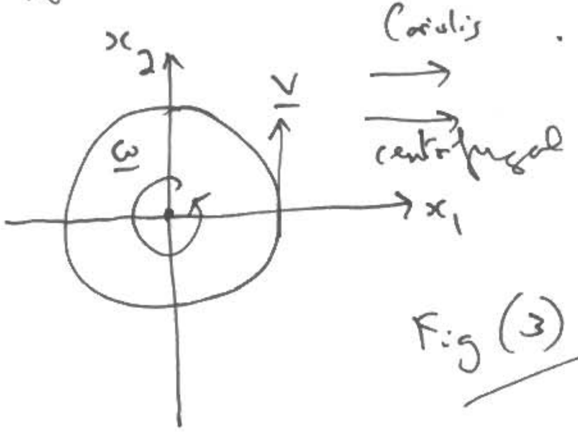

Fig (3)

$\underline{\omega} = \omega \underline{e}_3,$

$\underline{v} = v\underline{e}_1, \ \underline{r} = r\underline{e}_1$

$\underline{\omega} = \omega \underline{e}_3,$

$\underline{v} = v\underline{e}_2, \ \underline{r} = r\underline{e}_1$

$$\underline{F}_{eff} = m\omega^2 r\underline{e}_1 - 2m\omega v\underline{e}_2$$

$$\underline{F}_{eff} = m\omega^2 r\underline{e}_1 + 2m\omega v\underline{e}_1$$

In Fig (3) for example the tangential velocity $\underline{v}$ is:

$$\underline{v} = \underline{\omega} \times \underline{r} \quad — (2)$$

and

$$|\underline{v}| = \omega r = v \quad — (3)$$

as used in paper 110.

2)

The acceleration in the Foucault pendulum is:

$$\underline{a}_r = \underline{g} + \frac{1}{m}\underline{T} - 2\underline{\omega} \times \underline{v} \quad - (4)$$

and the plane of rotation of the pendulum rotates with frequency:

$$\omega_z = \omega \sin\lambda \quad - (5)$$

where the angle of rotation is:

$$\theta = \omega_z t. \quad - (6)$$

As in ~~note~~ paper 110 the relativistic correction is

$$\Omega_z = \omega_z \left(1 - \frac{v^2}{c^2}\right)^{-1} \quad - (7)$$

$$\sim \omega_z \left(1 - \frac{v^2}{c^2} + \cdots\right) \quad - (8)$$

for

$$v << c. \quad - (9)$$

<u>Rotational Frame Dragging</u>

One example is given in Fig (3), where the acceleration is:

$$\underline{a}_{eff} = \left(\omega^2 r + 2\omega v\right) \underline{e}_1 \quad - (10)$$

$$\boxed{\underline{a}_{eff} = 3\omega^2 r\, \underline{e}_1} \quad - (11)$$

In the case of the Thomas precession this is given by:

3)

$\underline{a}_{eff}$ (relativistic)

$$= 3\Omega^2 r \, \underline{e}_1 \quad - (12)$$

$$= 3\omega^2 r \left(1 - \frac{v^2}{c^2}\right)^{-2} \underline{e}_1 \quad - (13)$$

$$- (14)$$

i.e. $$\boxed{\underline{a}_{eff}(\text{relativistic}) \sim 3\omega^2 r \left(1 - 2\left(\frac{v}{c}\right)^2 + \dots\right) \underline{e}_1}$$

This is rotational frame dragging from the Minkowski metric. The correction is order $(v/c)^2$ so is very small..

The next stage is to evaluate the rotational frame dragging from a rotated so-called Schwarzschild metric:

$$ds^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2 \quad - (15)$$

$X = r \sin\theta \cos\phi,$

$Y = r \sin\theta \sin\phi,$

$Z = r \cos\theta$

We now consider a rotation about Z:

$$\phi' = \phi + \omega t \quad - (16)$$