Notes 113(7): The Force Law of Binary Pulsar Orbits

The potential energy in this case is:

$$V = \frac{1}{2}mc^2 + \frac{1}{2}mc^2\frac{\mu}{r} + \frac{1}{2}m\frac{L^2}{r^2} + \frac{1}{2}m\frac{L^2\mu}{r^3} \quad -(1)$$

where:

$$\mu = -\left(\frac{2MG}{c^2} + \frac{a}{r}\right). \quad -(2)$$

The perturbation term a/r generates an additional attraction potential:

$$\Delta V = -\frac{1}{2}ma\left(\frac{c^2}{r^2} + \frac{L^2}{r^4}\right) \quad -(3)$$

and an additional force of attraction:

$$\Delta F = -\frac{\partial V}{\partial r} = -ma\left(\frac{c^2}{r^3} + \frac{2L^2}{r^5}\right). \quad -(4)$$

The force law for binary pulsars is therefore:

$$\boxed{F = -\frac{mMG}{r^2} + \frac{m}{r^3}\left(L^2 - ac^2\right) - \frac{3LmMG}{r^4} - \frac{2amL^2}{r^5}} \quad -(5)$$

As shown in paper 108, this force law (5) produces a relativistic orbit that slowly spirals inwards by a few millimetres per revolution.
