

# 115(a): Lorentz Transform of Free Space MH Equations

These are:

$$\left( \partial_\alpha \tilde{F}^{\alpha\beta} \right)' = \left( \frac{\partial x^{\beta'}}{\partial x^\alpha} \right) \partial_\mu \tilde{F}^{\mu\nu} = 0 \quad - (1)$$

$$\left( \partial_\alpha F^{\alpha\beta} \right)' = \left( \frac{\partial x^{\beta'}}{\partial x^\alpha} \right) \partial_\mu F^{\mu\nu} = 0 \quad - (2)$$

$$\Lambda^{\beta'}_\alpha = \left( \frac{\partial x^{\beta'}}{\partial x^\alpha} \right) \quad - (3)$$

These free space laws are invariant under any Lorentz boost a relation. They are the same in frames  $K$  and  $K'$ !

$$\begin{aligned} \underline{\nabla} \cdot \underline{B} &= 0 \\ \underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} &= 0 \\ \underline{\nabla} \cdot \underline{E} &= 0 \\ \underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} &= 0 \end{aligned}$$



$$\begin{aligned} (\underline{\nabla} \cdot \underline{B})' &= 0 \\ (\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t})' &= 0 \\ (\underline{\nabla} \cdot \underline{E})' &= 0 \\ (\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t})' &= 0 \end{aligned} \quad - (4)$$

However, the individual fields transform as follows to first order in  $\underline{v}$ :

$$\underline{E}' = \gamma (\underline{E} + \underline{v} \times \underline{B}) \quad - (5)$$

$$\underline{B}' = \gamma \left( \underline{B} - \frac{1}{c^2} \underline{v} \times \underline{E} \right) \quad - (6)$$

$$\underline{E} = \gamma (\underline{E}' - \underline{v} \times \underline{B}') \quad - (7)$$

$$\underline{B} = \gamma \left( \underline{B}' + \frac{1}{c^2} \underline{v} \times \underline{E}' \right) \quad - (8)$$

and charge.

2) Remarks

Although the individual fields change from  $K$  to  $K'$  and vice-versa, the four laws of free-space electrodynamics in the standard model remain the same. The four laws of free space electrodynamics are Lorentz invariant. Therefore no explanation can be found for either the Faraday or Sagnac paradoxes in terms of these laws. In order to explain the Faraday disk of 1831 and the Sagnac effect of 1913, it is necessary to consider electrodynamics as part of a generally covariant unified field theory.

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