116(3): Structure of the Inhomogeneous Laws

For electrodynamics the basic structure is:

$$\partial_\mu F^{\kappa\mu\nu} = J^{\kappa\nu} / \epsilon_0 \quad -(1)$$

where:

$$F^{\kappa\mu\nu} = \begin{bmatrix} 0 & -E_X^\kappa & -E_Y^\kappa & -E_Z^\kappa \\ E_X^\kappa & 0 & -cB_Z^\kappa & cB_Y^\kappa \\ E_Y^\kappa & cB_Z^\kappa & 0 & -cB_X^\kappa \\ E_Z^\kappa & -cB_Y^\kappa & cB_X^\kappa & 0 \end{bmatrix} \quad -(2)$$

Coulomb's Law

This is given by:

$$\kappa = \nu = 0 \quad -(3)$$

Ampère Maxwell Law

This is given by:

$$\nu = 1, 2, 3 \quad -(4)$$

as follows:

$$\partial_0 F^{\kappa01} + \partial_2 F^{\kappa21} + \partial_3 F^{\kappa31} = J^{\kappa1}/\epsilon_0 \quad -(5)$$

$$\partial_0 F^{\kappa02} + \partial_1 F^{\kappa12} + \partial_3 F^{\kappa32} = J^{\kappa2}/\epsilon_0 \quad -(6)$$

$$\partial_0 F^{\kappa03} + \partial_1 F^{\kappa13} + \partial_2 F^{\kappa23} = J^{\kappa3}/\epsilon_0 \quad -(7)$$

i.e.

$$-\frac{1}{c}\frac{\partial E_X^\kappa}{\partial t} + c\left(\frac{\partial B_Z^\kappa}{\partial Y} - \frac{\partial B_Y^\kappa}{\partial Z}\right) = \frac{1}{\epsilon_0} J_X^\kappa \quad -(8)$$

$$-\frac{1}{c}\frac{\partial E_Y^\kappa}{\partial t} + c\left(\frac{\partial B_X^\kappa}{\partial Z} - \frac{\partial B_Z^\kappa}{\partial X}\right) = \frac{1}{\epsilon_0} J_Y^\kappa \quad -(9)$$

$$-\frac{1}{c}\frac{\partial E_Z^\kappa}{\partial t} + c\left(\frac{\partial B_Y^\kappa}{\partial X} - \frac{\partial B_X^\kappa}{\partial Y}\right) = \frac{1}{\epsilon_0} J_Z^\kappa \quad -(10)$$

2)

Choice of κ Index

The choice of κ index is determined by the fact that the charge current density is a four vector. The charge density is time-like and the index:

$$\kappa = 0 \quad - (11)$$

is chosen as the time like index. The three components of the current density vector are defined by:

$$\kappa = 1, 2, 3 \quad - (12)$$

so:

$$\underline{J} = J_x^1 \underline{i} + J_Y^2 \underline{j} + J_Z^3 \underline{k} \quad - (13)$$

Therefore in eq. (8), (9) and (10), $\kappa = 1, 2, 3$ respectively. This choice of κ index means that:

$$\underline{E} = E^{110} \underline{i} + E^{220} \underline{j} + E^{330} \underline{k} \quad - (14)$$

and

$$\underline{B} = B^{332} \underline{i} + B^{113} \underline{j} + B^{221} \underline{k} \quad - (15)$$

as summarized in Table 1 of paper 100.

Therefore in general relativity, the electric and magnetic field components of the Ampère Maxwell law are made up of components of rank ~~three~~ tensors, these are related to components of angular momentum / energy density.

3) In dynamics it is possible to adopt a similar procedure. The basic tensorial structure is:

$$\partial_\mu T^{\kappa\mu\nu} = j^{\kappa\nu} \quad -(16)$$

The field tensor analogous to eq. (2) is:

$$G^{\kappa\mu\nu} = c^2 T^{\kappa\mu\nu} = \begin{bmatrix} 0 & -g_X^\kappa & -g_Y^\kappa & -g_Z^\kappa \\ g_X^\kappa & 0 & -h_Z^\kappa & h_Y^\kappa \\ g_Y^\kappa & h_Z^\kappa & 0 & -h_X^\kappa \\ g_Z^\kappa & -h_Y^\kappa & h_X^\kappa & 0 \end{bmatrix} \quad -(17)$$

leading to the inhomogeneous equation of relativistic classical dynamics:

$$\underline{\nabla} \cdot \underline{g} = 4\pi G \rho_m \quad -(18)$$

and

$$\underline{\nabla} \times \underline{h} - \frac{1}{c}\frac{\partial \underline{g}}{\partial t} = 4\pi G \underline{J}_m \quad -(19)$$

<u>Newtonian Dynamics</u>

These are a limit of the relativistic equation (18). The acceleration due to gravity in eq. (18) is:

$$\underline{g} = c^2 \underset{\sim}{T} \quad -(20)$$

where:

$$\underset{\sim}{T} = T^{010}\,\underline{i} + T^{020}\,\underline{j} + T^{030}\,\underline{k} \quad -(21)$$

is the orbital torsion vector. The mass density in eq. (18) is:

$$\rho_m = \frac{c^2}{4\pi G}\left(R - \omega T\right) \quad -(22)$$

where:

$$j^{00} = R - \omega T \quad -(23)$$

$$= R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30} - 4\left(\omega^{0}{}_{1\lambda}T^{\lambda 10} + \omega^{0}{}_{2\lambda}T^{\lambda 20} + \omega^{0}{}_{3\lambda}T^{\lambda 30}\right) \quad -(24)$$

If the Newtonian limit is considered to be the limit when flat spacetime is approached, then:

$$R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30} \rightarrow \frac{4\pi G}{c^2}\rho_m \quad -(25)$$

It is well known that eq. (18) leads to the Newtonian inverse square law when ρ_m is a constant. In situations where torsion is significant, as in spiral galaxies, there are well known deviations.

5) The new law (19) has no Newtonian equivalent, but is related to Eulerian dynamics. The quantity $\underline{h}$ has the S.I. units of acceleration ($m s^{-2}$), and is analogous to the magnetic field in electrodynamics. Eqs. (18) and (19) can be considered as the correctly relativistic gravitomagnetic equations.

In eq. (19):

$$\underline{g} = c^2 \left(T^{110} \, \underline{i} + T^{220} \, \underline{j} + T^{330} \, \underline{k} \right) \quad -(26)$$

and

$$\underline{h} = c^2 \left(T^{332} \, \underline{i} + T^{113} \, \underline{j} + T^{221} \, \underline{k} \right) \quad -(27)$$

From a comparison of eqs. (21) and (26) it is seen that $\underline{g}$ in the former is orbital and in the latter is spin. The acceleration $\underline{h}$ is another type of "spin acceleration."

"Dark Matter"

This ad hoc, or empirical, postulate may be replaced by solutions of eqs (18) and (19).