1) 136(1): Development of the Tetrad Postulate and ECE Lemma in SU(2) Representation Space.

The tetrad postulate is:

$$D_\mu q^a_\nu = 0 \quad -(1)$$

and is fundamental to geometry. It can be re-expressed as the ECE Lemma:

$$\Box q^a_\nu := R q^a_\nu \quad -(2)$$

where:

$$R = q^\lambda_a \partial^\mu (\Gamma^a_{\mu\lambda} - \omega^a_{\mu\lambda}) \quad -(3)$$

In paper 135 it was shown that eq. (2) is

$$\sigma^\mu p_\mu q^R_1 = \sigma^0 p_0 q^L_1 \quad -(4)$$

$$\sigma^\mu p_\mu q^R_2 = \sigma^0 p_0 q^L_2 \quad -(5)$$

where

$$q^a_\mu = \begin{bmatrix} q^R_1 & q^R_2 \\ q^L_1 & q^L_2 \end{bmatrix}, \quad -(6)$$

$$\sigma^\mu = (\sigma^0, \sigma^i) \quad -(7)$$

$$p_\mu = (p_0, -p_i). \quad -(8)$$

The Pauli matrices form an SU(2) Lie algebra:

$$\left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2} \right] = i \frac{\sigma_k}{2} \quad -(9)$$

et cyclicum

2) Eqs. (4) and (5) are equations of the unified field in SU(2) representation space. This means that any component of the unified field can be developed in SU(2) representation space. This argument can be extended to an SU(n) representation space. The unified field can therefore be developed in any representation space.

To date, the development of field theory in SU(2) has been restricted to the free fermion, where:

$$R = -\left(mc/\hbar\right)^2 \quad -(10)$$

$$\lambda_c = \hbar/mc \quad -(11)$$

where λ_c is the Compton wavelength. The free fermion is the fermion free of the influence of any other component of the unified field, such as gravitation or electromagnetism. When the fermion interacts with any other component of the unified field, R is defined by eq. (3).

Now use in eqs. (4) and (5):

$$p_\mu = i\hbar\,\partial_\mu \quad -(12)$$

to obtain:

$$i\sigma^\mu \partial_\mu \phi_1^R = |R_0|^{1/2} \sigma^0 \phi_1^L \quad -(13)$$

$$i\sigma^\mu \partial_\mu \phi_2^R = |R_0|^{1/2} \sigma^0 \phi_2^L \quad -(14)$$

where:

$$|R_0|^{1/2} = mc/\hbar \quad -(15)$$

and:

$$p_0 = mc = \hbar|R_0|^{1/2} \quad -(16)$$

Write eq. (16) as:

$$p_0 = \hbar|R_0|^{1/2} e_0 \quad -(17)$$

where the e_0 timelike unit vector is part of:

$$e_\mu = (e_0, -e_i) \quad -(18)$$

Finally generalize eq. (17) to:

$$\boxed{p_\mu = \hbar|R|^{1/2} e_\mu} \quad -(19)$$

From eqs. (12) and (19):

$$\boxed{\partial_\mu = -i|R|^{1/2} e_\mu} \quad -(20)$$

However, ∂_μ is the basis vector for the coordinate adapted representation of ~~Car~~ differential geometry. Eq. (20) therefore traces the origin of quantum mechanics to the curvature R of the ECE lemma. The fermion equation is:

$$\boxed{\begin{array}{l} \sigma^\mu e_\mu |R|^{1/2} \phi_1^R = \sigma^0 e_0 |R_0|^{1/2} \phi_1^L \\ \sigma^\mu e_\mu |R|^{1/2} \phi_2^R = \sigma^0 e_0 |R_0|^{1/2} \phi_2^L \end{array}} \quad -(21)$$

Eq. (21) is straightforwardly extended to SU(2)

#1)

136(2): Development of the Minimal Prescription in ECE Theory.

As in previous notes it has been shown that the tetrad postulate of Cartan geometry may be written as:

$$\boxed{i\sigma^\mu \partial_\mu \phi^R = \sigma^0 \kappa \phi^L} \quad -(1)$$

in the SU(2) representation space. This is an equation of fundamental geometry. The wavenumber κ is defined by

$$\kappa = mc / \hbar \quad -(2)$$

using the postulate:

$$|R|^{1/2} = \kappa. \quad -(3)$$

This means that quantum electrodynamics may be developed as:

$$\boxed{i\sigma^\mu \partial_\mu A^R = \sigma^0 \kappa A^L} \quad -(4)$$

giving a novel fermionic description of the photon of mass m. In the standard model the photon is considered to be a massless boson. This standard description is inconsistent with the fact that light is deflected by gravity, and inconsistent in many other ways. Eq. (4) means that the photon may have left and right handed states, in the sense that an electron has left and right handed states.

2)

This development is based on the fact that any of the unified field may be developed in an SU(2) representation space because geometry itself may be developed in an SU(2) representation space. The development suggests that linear momentum may be developed as a tetrad:

$$p^a_\mu = \begin{bmatrix} p^R_1 & p^L_1 \\ p^R_2 & p^L_2 \end{bmatrix} \quad -(5)$$

so that:

$$\boxed{i\sigma^\mu \partial_\mu p^R = \sigma^0 \kappa p^L} \quad -(6)$$

The minimal prescription is thus:

$$\boxed{p^a_\mu \rightarrow p^a_\mu + eA^a_\mu} \quad -(7)$$

Now use:

$$p^a_\mu p^\mu_a = p_0^2 \quad -(8)$$

then the Einstein energy equation is developed as:

$$\boxed{p^a_\mu p^\mu_a = p_0^2 = m^2c^2} \quad -(9)$$

If $\underline{p}$ is real valued then:

$$\boxed{(\underline{\sigma} \cdot \underline{p})(\underline{\sigma} \cdot \underline{p}) = p^a_\mu p^\mu_a = p_0^2 = m^2c^2} \quad -(10)$$

where m is the mass of the photon.

Eq. (10) is based on the geometry:

$$(\underline{\sigma}\cdot\underline{e})(\underline{\sigma}\cdot\underline{e}) = q^a_\mu\, q^\mu_a = 1. \quad -(11)$$

Now use eqs. (13) to (15) of paper 129:

$$\underline{\sigma}^1 = \sigma^1\underline{i},\ \underline{\sigma}^2 = \sigma^2\underline{j},\ \underline{\sigma}^3 = \sigma^3\underline{k}. \quad -(12)$$

Thus:

$$\underline{\sigma}^1\cdot\underline{i} = \sigma^1,\ \underline{\sigma}^2\cdot\underline{j} = \sigma^2,\ \underline{\sigma}^3\cdot\underline{k} = \sigma^3 \quad -(13)$$

In paper 129 it was shown that:

$$\sigma^1 = q^1_X,\ \sigma^2 = q^2_Y,\ \sigma^3 = q^3_Z, \quad -(14)$$

so:

$$q^1_X = \underline{\sigma}^1\cdot\underline{i},\ q^2_Y = \underline{\sigma}^2\cdot\underline{j},\ q^3_Z = \underline{\sigma}^3\cdot\underline{k} \quad -(15)$$

Define:

$$\underline{e} = \underline{i} + \underline{j} + \underline{k} \quad -(16)$$

$$\underline{\sigma} = \sigma^1\underline{i} + \sigma^2\underline{j} + \sigma^3\underline{k} \quad -(17)$$

so

$$\boxed{\underline{\sigma}\cdot\underline{e} = q^1_X + q^2_Y + q^3_Z} \quad -(18)$$

In eq. (11):

4)

$$q_\mu^{a} q_a^{\mu} = q_x^1 q_1^x + q_y^2 q_2^y + q_z^3 q_3^z \quad -(19)$$

By definition:

$$\sigma^1 = (\sigma^1)^{-1} \text{ etc.} \quad -(20)$$

so: $$q_x^1 = q_1^x, \quad q_y^2 = q_2^y, \quad q_z^3 = q_3^z \quad -(21)$$

and so: $$(\underline{\sigma}\cdot p)(\underline{\sigma}\cdot p) = (p_x^1)^2 + (p_y^2)^2 + (p_z^3)^2 \quad -(22)$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} p_0^2 \quad -(23)$$

In this notation:

$$p_x^1 = p_0 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad p_y^2 = p_0 \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad p_z^3 = p_0 \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

i.e. $$p_x^1 = p_0 q_x^1, \quad p_y^2 = p_0 q_y^2, \quad p_z^3 = p_0 q_z^3 \quad -(24)$$

This is the linear momentum of the photon in SU(2) representation space.

Similarly the electromagnetic potential in SU(2) representation space is:

$$A_x^1 = A^{(0)} q_x^1 \text{ etc.} \quad -(25)$$

Eq. (25) is an example of:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(26)$$

Eq. (24) is an example of:

$$\Phi^a_\mu = \Phi^{(0)} q^a_\mu . \quad -(27)$$

Using eq. (10), the interaction of a photon of mass m with a magnetic field may be evaluated. In the non-relativistic limit, the hamiltonian is:

$$H = \frac{1}{2m} \underline{\sigma} \cdot (\underline{p} + e\underline{A}) \underline{\sigma} \cdot (\underline{p} + e\underline{A}) + V \quad -(28)$$

where:

$$\boxed{\begin{aligned}&(\underline{\sigma} \cdot (\underline{p} + e\underline{A}))(\underline{\sigma} \cdot (\underline{p} + e\underline{A})) \\ &= (p^a_\mu + eA^a_\mu)(p^\mu_a + eA^\mu_a)\end{aligned}} \quad -(29)$$

Use this in the Schrödinger eqn.:

$$H\phi = E\phi \quad -(30)$$

to find:

$$H\phi = \frac{e\hbar}{2m} (\underline{\sigma} \cdot \underline{B}) \phi \quad -(30)$$

$$+ \cdots$$

The ESR frequency is:

$$\boxed{\omega_{res} = \frac{e\hbar B}{m}} \quad -(31)$$

1)

136 (3): Resonance Frequency Calculation

The resonance frequency is:

$$\omega_{res} = \frac{e\hbar}{m} B \quad -(1)$$

where

$$p = eA^{(0)} = \hbar \kappa \quad -(2)$$

These are both ECE and standard model equations, eq. (2) being an application of the minimal prescription to the photon. Now use:

$$I = \epsilon_0 c E^{(0)2} \quad -(3)$$

in ~~volts~~ watts per square metre. Here:

$$E^{(0)} = cB^{(0)} = c\kappa A^{(0)} = \omega A^{(0)} \quad -(4)$$

Therefore:

$$I = \epsilon_0 c \omega^2 A^{(0)2} \quad -(5)$$

and

$$\omega_{res} = \left(\frac{\hbar^2 \kappa}{A^{(0)} m} \right) B \quad -(6)$$

$$= \frac{\hbar^2 (\epsilon_0 c)^{1/2}}{c} \cdot \frac{1}{m} \cdot \frac{\omega^2}{I^{1/2}} \cdot B$$

$$\boxed{\omega_{res} = \hbar^2 \left(\frac{\epsilon_0}{c} \right)^{1/2} \left(\frac{1}{m} \frac{\omega^2}{I^{1/2}} \right) B} \quad -(7)$$

$$\boxed{\omega_{res} = \frac{1.91 \times 10^{-78}}{m} \left(\frac{\omega^2}{I^{1/2}} \right) B} \quad -(8)$$

2)

where:

$\underline{I}$ = e/m power density in watts per sq. m.

ω = e/m angular frequency in rad s^{-1}

m = photon mass in kilograms

B = flux density of magnet in tesla

If for example:

$$\omega = 10^{15} \text{ rad } s^{-1}$$

$$\underline{I} = 1.0 \text{ watt per sq m}$$

$$\omega_{res} = \frac{1.91 \times 10^{-48}}{m} B \quad -(9)$$

If the photon mass is, say, 10^{-60} kilograms for the sake of illustration only, and if B is one tesla, then:

$$\omega_{res} = 10^{12} \text{ rad } s^{-1} \quad -(10)$$

and within range of an ordinary ESR or NMR spectrometer.

Remarks

1) If ω_{res} is observed it gives the photon mass.

2) If ω_{res} is not observed there is no photon mass, a deep crisis for physics.

1)

136(4): Gyromagnetic Ratio of the Photon and the Proca Equation

In previous notes it was shown that if the photon has mass m and charge e defined by:

$$p = \hbar \kappa = e A^{(0)} \quad -(1)$$

then one photon may interact with a permanent magnet of flux density B to give the resonance frequency:

$$\omega = g \hbar B \quad -(2)$$

where

$$g = e / m \quad -(3)$$

is the gyromagnetic ratio of the photon.

The standard model of physics essentially ignored the photon mass m throughout the twentieth century, and therefore ignored g. In the standard physics, there is no theory that predicts that the photon will interact with a permanent magnet. However, in the nineteen twenties, Proca devised a wave equation for photon mass:

$$(\Box + \kappa^2) A_\mu = 0 \quad -(4)$$

where

$$\kappa = mc / \hbar \quad -(5)$$

The standard model assumes that m is zero, and uses the d'Alembert equation:

$$\Box A_\mu = 0 \quad -(6)$$

The old physics is full of contradictions, one of

2)

is that photon mass m is used in the theory of light deflection by gravitation, and if m were zero there would be no deflection. Another is that the Proca equation is not compatible at all with gauge theory.

The Proca wave equation (4) has the same structure as the wave equation of the fermion:

$$(\Box + \kappa^2)\phi = 0 \quad -(7)$$

The electromagnetic potential A_μ in eq. (4) is defined in Minkowski spacetime and has four components. The fermion wave function ϕ also has four components. So:

$$A_\mu = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{bmatrix}, \quad \phi = \begin{bmatrix} \phi_1^R \\ \phi_2^R \\ \phi_1^L \\ \phi_2^L \end{bmatrix} \quad -(8)$$

In the old physics ϕ^R and ϕ^L are the Pauli spinors:

$$\phi^R = \begin{bmatrix} \phi_1^R \\ \phi_2^R \end{bmatrix}, \quad \phi^L = \begin{bmatrix} \phi_1^L \\ \phi_2^L \end{bmatrix} \quad -(9)$$

in SU(2) representation space. The spacelike part of A_μ is in O(3) representation space.

Eqs. (4) and (7) are shorthand notation for:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} (\Box + \kappa^2) A_\mu = 0 \quad -(10)$$

and similarly for eqn. (7). Eq. (10) means that each component of A_μ and ϕ obeys the wave eqn., i.e.:

3)

$$\left.\begin{aligned}(\Box+\kappa^2)A_0&=0\\(\Box+\kappa^2)A_1&=0\\(\Box+\kappa^2)A_2&=0\\(\Box+\kappa^2)A_3&=0\end{aligned}\right],\quad\left.\begin{aligned}(\Box+\kappa^2)\phi_1^R&=0\\(\Box+\kappa^2)\phi_2^R&=0\\(\Box+\kappa^2)\phi_1^L&=0\\(\Box+\kappa^2)\phi_2^L&=0\end{aligned}\right] \quad -(11)$$

The operator in eq. (10) may be factorized to give more information than eq. (11). In the old physics the factorization used the Dirac 4 × 4 matrices:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}(\Box+\kappa^2) = -(i\gamma_\mu\partial^\mu+\kappa)(i\gamma^\mu\partial_\mu-\kappa) \quad -(12)$$

This is an operator factorization which is true for both A_μ and ϕ, or any wavefunction. The right hand side of eq. (12) is:

$$(\gamma_\mu\partial^\mu)(\gamma^\mu\partial_\mu)+\kappa^2 := RHS \quad -(13)$$

$$= (\partial^\mu\partial_\mu)(\gamma_\mu\gamma^\mu)+\kappa^2 \quad -(14)$$

$$= \gamma^\mu\gamma_\mu\,\Box+\kappa^2.$$

Now use:

$$\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu = 2g^{\mu\nu} \quad -(15)$$

where

$$g^{\mu\nu} = g_{\mu\nu} = \begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix} \quad -(16)$$

is the Minkowski metric. We have:

$$\gamma_\nu = g_{\nu\mu}\gamma^\mu \quad -(17)$$

$$\gamma^\nu = g^{\nu\mu}\gamma_\mu \quad -(18)$$

So in eq. (15):

$$g^{\nu\mu}\gamma^\mu\gamma_\mu + g^{\nu\mu}\gamma^\mu\gamma_\mu = 2g^{\mu\nu} \quad -(19)$$

When:

$$\mu = \nu \quad -(20)$$

$$2g^{\mu\mu}(\gamma^\mu\gamma_\mu) = 2g^{\mu\mu} \quad -(21)$$

so

$$\gamma^\mu\gamma_\mu = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad -(22)$$

The shorthand notation of eq. (12) means that:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}(\Box + \kappa^2) = \gamma^\mu\gamma_\mu\Box + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\kappa^2 \quad -(23)$$

which is true by eqn. (22), QED.

Therefore eqn. (7) may be written as the Dirac equation:

$$(i\gamma^\mu\partial_\mu - \kappa)\psi = 0 \quad -(24)$$

as is well known. However, eqn. (10) may also be written as:

5)

$$\boxed{\left(i\gamma^\mu \partial_\mu - \kappa\right) A_\nu = 0} \quad -(25)$$

which is the old physics, is unknown. When written out in full, eqns. (24) and (25) are:

$$\left(i\gamma^\mu \partial_\mu - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \kappa\right) \phi = 0 \quad -(26)$$

$$\left(i\gamma^\mu \partial_\mu - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \kappa\right) A_\nu = 0 \quad -(27)$$

Photon mass on therefore makes a profound difference to electromagnetic theory, because eqn. (27) provides inter-relation between components of A_μ in the same way as eqn. (26) gives inter-relation between the components of a fermion.

Advances Made by ECE Theory

These are summarized in papers 129, 130 and 135. Notably, both A_μ and ϕ are recognized as tetrads, and eqns. (26) and (27) are simplified to equation in 2 x 2 Pauli matrices instead of Dirac 4 x 4 matrices. Thus:

$$\phi = \begin{bmatrix} q_1^R & q_2^R \\ q_1^L & q_2^L \end{bmatrix} \quad -(28)$$

suggesting a similar structure for A_μ^a, (next note)

6)

As in previous notes, eq. (28) means:

$$i\sigma^\mu \partial_\mu \psi_1^R = \sigma^0 \left(\frac{mc}{\hbar}\right) \psi_1^L \quad -(28)$$

$$i\sigma^\mu \partial_\mu \psi_2^R = \sigma^0 \left(\frac{mc}{\hbar}\right) \psi_2^L \quad -(29)$$

New equations for the electromagnetic field may now be written down by ~~writ~~ writing:

$$\underset{\sim}{A} = \begin{bmatrix} A_0 & A_1 \\ A_2 & A_3 \end{bmatrix} \quad -(30)$$

so

$$\boxed{\begin{aligned} i\sigma^\mu \partial_\mu A_0 &= \sigma^0 \left(\frac{mc}{\hbar}\right) A_2 \\ i\sigma^\mu \partial_\mu A_1 &= \sigma^0 \left(\frac{mc}{\hbar}\right) A_3 \end{aligned}} \quad -(31)$$

In ECE theory, these are true for each polarization of $\underset{\sim}{A}^a$, where:

$$a = (0), (1), (2), (3) \quad -(32)$$

Eqns. (31) follow from the factorization:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Box = -\frac{i}{2}\left(\sigma^\mu \partial_\mu\right)\left(i\sigma_\mu \partial^\mu\right) \quad -(33)$$

$$= -\frac{1}{2}\left(\sigma^\mu \sigma_\mu\right)\left(\partial^\mu \partial_\mu\right)$$

where

$$\Box = \partial^\mu \partial_\mu \quad -(34)$$

and:

$$\sigma^\mu \sigma_\mu = \left(\sigma^0\right)^2 - \left(\sigma^i\right)^2$$

7)

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \underline{\sigma} \cdot \underline{\sigma}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1-i \\ 1+i & -1 \end{bmatrix} \begin{bmatrix} 1 & 1-i \\ 1+i & -1 \end{bmatrix}$$

$$= -2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{QED}$$

So:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Box = \frac{1}{2} \left(i \sigma_\mu \partial^\mu \right) \left(i \sigma^\mu \partial_\mu \right) \quad -(34)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Box = - \left(i \gamma_\mu \partial^\mu \right) \left(i \gamma^\mu \partial_\mu \right) \quad -(35)$$

In previous notes it was shown that the fermion equation is:

$$i \sigma^\mu \partial_\mu \phi^R = \frac{mc}{\hbar} \sigma^0 \phi^L \quad -(36)$$

which expands to:

$$i \left(\sigma^0 \partial_0 - \sigma^i \partial_i \right) \phi^R = \kappa \sigma^0 \phi^L \quad -(37)$$

where: $\phi^R = [q_1^R \; q_2^R]$, $\phi^L = [q_1^L \; q_2^L]$

Applying the parity operator $\hat{P}$ to eq. (37) gives:

$$i \left(\sigma^0 \partial_0 + \sigma^i \partial_i \right) \phi^L = \kappa \sigma^0 \phi^R \quad -(38)$$

8)

Therefore there exist novel equations of quantum electrodynamics:

$$\boxed{\begin{aligned} i\left(\sigma^0\partial_0-\sigma^i\partial_i\right)A^R &= \kappa\sigma^0 A^L \\ i\left(\sigma^0\partial_0+\sigma^i\partial_i\right)A^L &= \kappa\sigma^0 A^R \end{aligned}} \quad -(39)$$

where:

$$A^R = [A_0, A_1], \quad A^L = [A_2, A_3] \quad -(40)$$

for each state a of polarization. The equivalent fermion equations are:

$$i\left(\sigma^0\partial_0-\sigma^i\partial_i\right)\phi^R = \kappa\sigma^0\phi^L \quad -(41)$$

$$i\left(\sigma^0\partial_0+\sigma^i\partial_i\right)\phi^L = \kappa\sigma^0\phi^R \quad -(42)$$

In paper 130 it was shown that these can be written as:

$$\left(\sigma^0 E-\sigma^3 c\,\underline{\sigma}\cdot\underline{p}\right)\phi = mc^2\sigma^1\phi \quad -(43)$$

where

$$p^\mu = \left(\frac{E}{c}, \underline{p}\right) = i\hbar\partial^\mu$$

$$= i\hbar\left(\frac{1}{c}\frac{\partial}{\partial t}, -\underline{\nabla}\right)$$

so

$$E = i\hbar\frac{\partial}{\partial t}, \quad \underline{p} = -i\hbar\underline{\nabla}$$

and

$$\boxed{\begin{aligned} i\left(\frac{\sigma^0}{c}\frac{\partial}{\partial t}+\sigma^3\underline{\sigma}\cdot\underline{\nabla}\right)\phi &= \kappa\sigma^1\phi \\ i\left(\frac{\sigma^0}{c}\frac{\partial}{\partial t}+\sigma^3\underline{\sigma}\cdot\underline{\nabla}\right)\underline{A}^a &= \kappa\sigma^1\underline{A}^a \end{aligned}} \quad -(44)$$

136(5): Simple Proof of the Hodge Dual Identity.

The commutator of covariant derivatives acts on the vector V^ρ to define simultaneously the curvature tensor $R^\rho{}_{\sigma\mu\nu}$ and the torsion tensor $T^\lambda{}_{\mu\nu}$:

$$[D_\mu, D_\nu] V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\lambda{}_{\mu\nu} D_\lambda V^\rho. \quad -(1)$$

The commutator operator $[D_\mu, D_\nu]$ is the Hodge dual of the commutator operator $[D_\alpha, D_\beta]$:

$$[D_\mu, D_\nu] = \frac{1}{2} \|g\|^{1/2} \epsilon_{\mu\nu}{}^{\alpha\beta} [D_\alpha, D_\beta] \quad -(2)$$

in four dimensions. Here $|g|$ is the determinant of the metric, and the 4-D antisymmetric tensor $\epsilon_{\mu\nu}{}^{\alpha\beta}$ is defined in Minkowski spacetime. So:

$$\epsilon_{\mu\nu}{}^{\alpha\beta} \left([D_\alpha, D_\beta] V^\rho \right) \quad -(3)$$
$$= \epsilon_{\mu\nu}{}^{\alpha\beta} \left(R^\rho{}_{\sigma\alpha\beta} V^\sigma - T^\lambda{}_{\alpha\beta} D_\lambda V^\rho \right)$$

A solution of eq. (3) is:

$$[D_\alpha, D_\beta] V^\rho = R^\rho{}_{\sigma\alpha\beta} V^\sigma - T^\lambda{}_{\alpha\beta} D_\lambda V^\rho \quad -(4)$$

Eq. (1) defines the Cartan Bianchi identity, eq. (4) defines the Hodge dual of the identity. Both eqns. (1) and (4) are examples of the same identity. Eqn. (1) implies:

$$D_\alpha T^{\lambda\alpha\beta} := R^\lambda{}_\alpha{}^{\alpha\beta} \quad -(5)$$

and eq. (4) implies:

$$D_\mu T^{\lambda\mu\nu} := R^\lambda{}_\mu{}^{\mu\nu} \quad -(6)$$

Q.E.D.

2)

Computer algebra has been used in papers 93, 95, 96, 107 and 120 to show conclusively that metrics of the Einstein equation do not obey eqns. (5) and (6). It was found that:

$$R^{\lambda}{}_{\mu}{}^{\mu\nu} \neq 0 \quad -(7)$$

in general, while $D_\mu T^{\lambda\mu\nu} = 0$ by definition. Therefore the theory behind CERN and other projects of the standard model is incorrect in many ways.

In differential form notation eq. (5) is:

$$D \wedge T := R \wedge q \quad -(8)$$

and eq. (6) is:

$$D \wedge \tilde{T} := \tilde{R} \wedge q \quad -(9)$$

Here T and $\tilde{T}$ are examples of the same tensor, but with different indices. Eq. (8) is:

$$d \wedge T := R \wedge q - \omega \wedge T = j \quad -(10)$$

and eq. (9) is:

$$d \wedge \tilde{T} := \tilde{R} \wedge q - \omega \wedge \tilde{T} = \tilde{j} \quad -(11)$$

Thus:

$$\boxed{\begin{aligned} d \wedge T &:= j \\ d \wedge \tilde{T} &:= \tilde{j} \end{aligned}} \quad -(12)$$

These are fundamental equations of differential geometry. The geometry used by Einstein was:

$$j =? \, 0 \quad -(13)$$

3)

and

$$T = ? \; 0 \quad -(14)$$

The existence of $\tilde{j}$ was not considered by Einstein. The computer found that metrics of the Einstein equation gave:

$$\tilde{R} \wedge q \neq 0 \quad -(15)$$

and:

$$R \wedge q = 0 \quad -(16)$$

which is inconsistent with the fundamental geometry.

Summary.

1) In Einsteinian physics and cosmology:

$$T = 0, \; \tilde{T} = 0, \; j = 0, \; \tilde{j} \neq 0 \quad -(17)$$

which is incorrect.

2) The correct field equations must be based on eqn. (12).

a) Free field Equations

$$\boxed{\begin{aligned} d \wedge T &= 0 \\ d \wedge \tilde{T} &= 0 \end{aligned}} \quad -(18)$$

but

$$T \neq 0, \; \tilde{T} \neq 0. \quad -(19)$$

b) Field Matter Interaction

The equations are (12), $j \neq 0$, $\tilde{j} \neq 0$.

The fundamental ECE hypothesis is:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(20)$$

4)

which implies:
$$F^a_{\mu\nu} = A^{(0)} T^a_{\mu\nu} \quad -(21)$$

So:
$$\boxed{\left.\begin{array}{l} d \wedge F = 0 \\ d \wedge \tilde{F} = 0 \end{array}\right] \text{ free field}} \quad -(22)$$

$$\boxed{\begin{array}{ll} d \wedge F = A^{(0)} \tilde{j} & \text{field / matter} \\ d \wedge \tilde{F} = A^{(0)} j & \text{interaction} \end{array}} \quad -(23)$$

The traditional construction of equations in electrodynamics is to write down one free field equation and one field matter equation. The Maxwell Heaviside equations for example are:

$$d \wedge F = 0 \quad -(24)$$
$$d \wedge \tilde{F} = J / \epsilon_0 \quad -(25)$$

In ECE, (shorthand notation) these become:

$$d \wedge F = 0 \quad -(26)$$
$$d \wedge \tilde{F} = A^{(0)} j \quad -(27)$$

and in standard notation:

$$d \wedge F^a = 0 \quad -(28)$$
$$d \wedge \tilde{F}^a = A^{(0)} j^a \quad -(29)$$

Eq. (28) is for the free field, and eq. (29) for the field matter interaction in ECE theory:

5)

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \quad -(30)$$

and in Maxwell Heaviside theory:

$$F = d \wedge A \quad -(31)$$

and the spin connection is missing.

Experimental Evidence for Eq. (30)

a) The inverse Faraday effect shows the existence of $\omega^a{}_b \wedge A^b$ and the existence of the spin connection in electrodynamics. This links up with $B^{(3)}$ field theory and the harmonic theories of $B^{(3)}$ by Horwitz et al., Barrett, Lehnert et al., and Hermuth et al.

b) A lot more evidence is collected on www.aias.us

136(6): Majorana and SU(3) Electrodynamics

In 1811 Arago discovered the right and left circular states of polarization of light. Taking the (1) polarization of ECE theory, the right and left handed potentials are:

$$\underline{A}_R^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (i\underline{i} + \underline{j}) e^{i(\omega t - \kappa Z)} \quad -(1)$$

$$\underline{A}_L^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (i\underline{i} - \underline{j}) e^{i(\omega t - \kappa Z)} \quad -(2)$$

In free space (free field):

$$\kappa = \omega / c \quad -(3)$$

It follows that:

$$\underline{\nabla} \times \underline{A}_R^{(1)} + \frac{i}{c} \frac{\partial \underline{A}_R^{(1)}}{\partial t} = \underline{0} \quad -(4)$$

$$\underline{\nabla} \times \underline{A}_L^{(1)} - \frac{i}{c} \partial \underline{A}_L^{(1)} = \underline{0} \quad -(5)$$

(GCUFT 2, pp 149 ff).

The Beltrami equations are:

$$\underline{\nabla} \times \underline{A}_R^{(1)} = \kappa \underline{A}_R^{(1)} \quad -(6)$$

$$\underline{\nabla} \times \underline{A}_L^{(1)} = -\kappa \underline{A}_L^{(1)} \quad -(7)$$

As shown in GCUFT 2, pp 149 ff eqs. (4) and (5) are the ~~Beltrami~~ Majorana equations:

2.

$$\left(\frac{E_n}{c} + \underline{\alpha} \cdot \underline{p}\right) \phi_R^{(1)} = 0 \quad -(8)$$

$$\left(\frac{E_n}{c} - \underline{\alpha} \cdot \underline{p}\right) \phi_L^{(1)} = 0 \quad -(9)$$

which are interchangeable under parity. Here, the three spinor is:

$$\phi_R^{(1)} = \begin{bmatrix} A_{RX}^{(1)} \\ A_{RY}^{(1)} \\ A_{RZ}^{(1)} \end{bmatrix} \quad -(10)$$

and:

$$\underline{\alpha} \cdot \underline{p} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix} p_X + \begin{bmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{bmatrix} p_Y + \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} p_Z \quad -(11)$$

and:

$$\left.\begin{aligned} p^\mu &= \left(\frac{E_n}{c}, \underline{p}\right) = i\hbar \partial^\mu \\ &= i\hbar \left(\frac{1}{c}\frac{\partial}{\partial t}, -\underline{\nabla}\right) \end{aligned}\right\} \quad -(12)$$

It is seen that eqs. (8) and (9) have the same overall structure as the SU(2) fermion equations for a massless particle:

$$i \sigma^\mu \partial_\mu \phi^R = 0 \quad -(13)$$

$$i \sigma^\mu \partial_\mu \phi^L = 0 \quad -(14)$$

where σ^μ is the four vector of Pauli matrices.

The α matrices in eq. (11) are matrices of the SU(3) group.

3)

The $SU(3)$ group is made up of 8 Gell-Mann matrices and is defined by:

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = i f_{abc} \frac{\lambda_c}{2} \quad -(15)$$

where the group structure factor f_{abc} is totally antisymmetric and defined by:

$$\left.\begin{aligned} f_{123} &= 1 \\ f_{147} &= -f_{156} = f_{246} = f_{257} \\ &= f_{345} = -f_{367} = \frac{1}{2} \\ f_{458} &= f_{678} = \frac{\sqrt{3}}{2} \end{aligned}\right] \quad -(16)$$

Here: $\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,

$\lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$,

$\lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}$, $\lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$,

$\lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}$, $\lambda_8 = \begin{bmatrix} 1/\sqrt{3} & 0 & 0 \\ 0 & 1/\sqrt{3} & 0 \\ 0 & 0 & -2/\sqrt{3} \end{bmatrix}$

$-(17)$

Therefore eq. (11) is:

4)

$$\underline{\alpha} \cdot \underline{p} = \lambda_7 p_x + \lambda_5 p_y + \lambda_2 p_z \quad -(18)$$

where

$$\left[\frac{\lambda_2}{2}, \frac{\lambda_5}{2} \right] = \frac{i}{2} \frac{\lambda_7}{2} \quad -(19)$$

i.e.

$$[\lambda_2, \lambda_5] = i \lambda_7 \quad -(20)$$

et cyclicum

Here:

$$\lambda_2 = J_Z, \; \lambda_5 = J_Y, \; \lambda_7 = J_X \quad -(21)$$

where J_X, J_Y and J_Z are the infinitesimal rotation generators, angular momenta within $\underline{\Gamma}$.

Remarks

1) In notes for paper 136 it has been shown that electrodynamics may be expressed in both an SU(2) and SU(3) representation space.

2) The Majorana three-spinor (10) is a tetrad formulation in ECE. Instead of a column vector it is expressed as a row vector of the complete 3×3 tetrad, the ~~right~~ (1) (2) and (3) polarization for both the right and left handed states of handedness. If we include the (0) state it becomes a 4×4 tetrad.

3) The Majorana equations (8) and (9) are for a massless photon and are incomplete. They must be extended for use with a massive photon.

4) The Gell-Mann matrices are basis elements of SU(3) electrodynamics. Eq. (18) is one example only.

136(7): Refutation of the Einstein Field Equation.

Consider the basic theorems of geometry.

$$[D_\mu, D_\nu] V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\lambda{}_{\mu\nu} D_\lambda V^\rho \quad -(1)$$

and

$$[D_\mu, D_\nu]_{HD} V^\rho = \tilde{R}^\rho{}_{\sigma\mu\nu} V^\sigma - \tilde{T}^\lambda{}_{\mu\nu} D_\lambda V^\rho \quad -(2)$$

where HD and the tilde denote Hodge dual. Eq. (1) implies the Cartan Bianchi identity:

$$D_\mu T^a{}_{\nu\rho} + D_\rho T^a{}_{\mu\nu} + D_\nu T^a{}_{\rho\mu} := R^a{}_{\mu\nu\rho} + R^a{}_{\rho\mu\nu} + R^a{}_{\nu\rho\mu} \quad -(3)$$

and eq. (2) implies the Cartan Evans identity:

$$D_\mu \tilde{T}^a{}_{\nu\rho} + D_\rho \tilde{T}^a{}_{\mu\nu} + D_\nu \tilde{T}^a{}_{\rho\mu} := \tilde{R}^a{}_{\mu\nu\rho} + \tilde{R}^a{}_{\rho\mu\nu} + \tilde{R}^a{}_{\nu\rho\mu} \quad -(4)$$

Eq. (3) is:

$$D_\mu \tilde{T}^{a\mu\nu} = \tilde{R}^a{}_\mu{}^{\mu\nu} \quad -(5)$$

and eq. (4) is

$$D_\mu T^{a\mu\nu} = R^a{}_\mu{}^{\mu\nu} \quad -(6)$$

In the Einstein field equation:

$$T^{a\mu\nu} = \tilde{T}^{a\mu\nu} = 0 \quad -(7)$$

so:

$$\tilde{R}^a{}_\mu{}^{\mu\nu} = 0 \quad -(8)$$

$$R^a{}_\mu{}^{\mu\nu} = 0 \quad -(9)$$

Thus:

$$\tilde{R}^\kappa{}_\mu{}^{\mu\nu} = q^\kappa{}_a \tilde{R}^a{}_\mu{}^{\mu\nu} = 0 \quad -(10)$$

$$R^\kappa{}_\mu{}^{\mu\nu} = q^\kappa{}_a R^a{}_\mu{}^{\mu\nu} = 0 \quad -(11)$$

2)

Eq. (10) is:

$$R^{\kappa}{}_{\mu\nu\rho} + R^{\kappa}{}_{\rho\mu\nu} + R^{\kappa}{}_{\nu\rho\mu} = 0 \quad -(12)$$

which in the old physics is known as the "first Bianchi identity". Eq. (12) is true if and only if:

$$\Gamma^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\nu\mu} \quad -(13)$$

in which case:

$$T^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu} \quad -(14)$$
$$= 0.$$

Eq. (11) was never considered in the old physics. It was found by computer algebra that the Einstein field equation produces, in general:

$$\boxed{R^{\kappa}{}_{\mu}{}^{\mu\nu} \neq 0.} \quad -(15)$$

The Einstein field equation is refuted because:

1) Eq. (15) contradicts basic geometry, eq. (6).
2) Eq. (13) contradicts eq. (1), which shows that:

$$\Gamma^{\lambda}_{\mu\nu} = -\Gamma^{\lambda}_{\nu\mu} \quad -(16)$$

because:

$$[D_\mu, D_\nu] V^\rho = -[D_\nu, D_\mu] V^\rho \quad -(17).$$

The standard model collapses.

1) 136(8): Development of SU(2) Electrodynamics

In dynamics, the ECE fermion equation was developed in paper 135 and previous notes as:

$$(\sigma^0 p_0 - \underline{\sigma}\cdot\underline{p})\,\phi_1^R = mc\,\sigma^0\phi_1^L \quad -(1)$$

$$(\sigma^0 p_0 - \underline{\sigma}\cdot\underline{p})\,\phi_2^R = mc\,\sigma^0\phi_2^L \quad -(2)$$

$$(\sigma^0 p_0 + \underline{\sigma}\cdot\underline{p})\,\phi_1^L = mc\,\sigma^0\phi_1^R \quad -(3)$$

$$(\sigma^0 p_0 + \underline{\sigma}\cdot\underline{p})\,\phi_2^L = mc\,\sigma^0\phi_2^R \quad -(4)$$

Eqs. (3) and (4) are obtained by applying the parity operator to eqs (1) and (2), term by term.

Multiply eq. (1) by (3):

$$(\sigma^0 p_0 - \underline{\sigma}\cdot\underline{p})(\sigma^0 p_0 + \underline{\sigma}\cdot\underline{p})\,\phi_1^R\phi_1^L = m^2c^2\,\phi_1^R\phi_1^L \quad -(5)$$

Use:

$$(\underline{\sigma}\cdot\underline{p})(\underline{\sigma}\cdot\underline{p}) = \begin{bmatrix} p_Z & p_X - ip_Y \\ p_X + ip_Y & -p_Z \end{bmatrix}\begin{bmatrix} p_Z & p_X - ip_Y \\ p_X + ip_Y & -p_Z \end{bmatrix} \quad -(6)$$

$$= (p_X^2 + p_Y^2 + p_Z^2)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= p^2\sigma^0 = p^2\sigma^{0\,2} \quad -(7)$$

So eq. (5) is:

$$(\sigma^{0\,2}p_0^2 - \sigma^{0\,2}p^2 - m^2c^2\sigma^{0\,2})\,\phi_1^R\phi_1^L = 0 \quad -(8)$$

i.e. $$(p^\mu p_\mu - m^2 c^2)\,\sigma^{02} q_1^R q_1^L = 0 \quad - (9)$$

where: $$\sigma^{02} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad - (10)$$

So $$(p^\mu p_\mu - m^2 c^2)\, q_1^R q_1^L = 0 \quad - (11)$$

Possible solutions are

$$(p^\mu p_\mu - m^2 c^2)\, q_1^R = 0 \quad - (12)$$

$$(p^\mu p_\mu - m^2 c^2)\, q_1^L = 0 \quad - (13)$$

Similarly:

$$(p^\mu p_\mu - m^2 c^2)\, q_2^R = 0 \quad - (14)$$

$$(p^\mu p_\mu - m^2 c^2)\, q_2^L = 0 \quad - (15)$$

Thus:

$$(p^\mu p_\mu - m^2 c^2) \begin{bmatrix} q_1^R & q_2^R \\ q_1^L & q_2^L \end{bmatrix} = 0 \quad - (16)$$

The Einstein energy equation

$$p^\mu p_\mu = m^2 c^2 \quad - (17)$$

and is a solution of eq. (16). The tetrad is

$$q^a_\mu = \begin{bmatrix} q_1^R & q_2^R \\ q_1^L & q_2^L \end{bmatrix} \quad - (18)$$

Now use:

$$p^\mu = i\hbar\,\partial^\mu \quad -(19)$$

to find:

$$\boxed{(\Box + \kappa^2)\, q^a_\mu = 0} \quad -(20)$$

where

$$\Box = \partial^\mu \partial_\mu \quad -(21)$$

$$\kappa = mc/\hbar \quad -(22)$$

Reversing the procedure, the wave equation (20) may be factorized into eqs. (1) – (4). This is a fundamental factorization of the d'Alembertian operator itself:

$$\boxed{\sigma^{0\,2}\,\Box = \left(\frac{\sigma^0}{c}\frac{\partial}{\partial t} + \underline{\sigma}\cdot\underline{\nabla}\right)\left(\frac{\sigma^0}{c}\frac{\partial}{\partial t} - \underline{\sigma}\cdot\underline{\nabla}\right)} \quad -(23)$$

The wave equation (20) is the free fermion limit of the ECE wave equation

$$(\Box + kT)\, q^a_\mu = 0 \quad -(24)$$

Therefore the SU(2) formulation of classical electrodynamics is:

4)

$$(\square + \kappa^2) A^a_\mu = 0 \quad -(25)$$

where:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(26)$$

which is the fundamental hypothesis of ECE.

Eq. (25) factorizes into:

$$i\sigma^\mu \partial_\mu A_1^R = \kappa \sigma^0 A_1^L \quad -(27)$$

$$i\sigma^\mu \partial_\mu A_2^R = \kappa \sigma^0 A_2^L \quad -(28)$$

and parity reversed equations. Here:

$$\kappa = mc / \hbar \quad -(29)$$

where m is the photon mass.

The Limit $m \rightarrow 0$

In this limit:

$$\square A^a_\mu = 0 \quad -(30)$$

i.e.

$$\square \begin{bmatrix} A_1^R & A_2^R \\ A_1^L & A_2^L \end{bmatrix} = 0 \quad -(31)$$

Eq. (31) is the d'Alembert wave equation if we define the four potential components as the right and left circularly polarized components, and their complex conjugates.

$$A_1^R = \underline{A}_1^R \cdot \underline{\sigma} \quad -(32)$$

and so on. Here:

$$\underline{A}_1^R = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{i\phi} \quad -(33)$$

$$\underline{A}_2^R = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right)e^{-i\phi} \quad -(34)$$

$$\underline{A}_1^L = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right)e^{i\phi} \quad -(35)$$

$$\underline{A}_2^L = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{-i\phi} \quad -(36)$$

So:

$$\underline{A}_1^R \cdot \underline{\sigma} = \begin{bmatrix} 0 & A_{1X}^R - iA_{1Y}^R \\ A_{1X}^R + iA_{1Y}^R & 0 \end{bmatrix} \quad -(37)$$

$$\boxed{A_1^R = \frac{A^{(0)}}{\sqrt{2}} e^{i\phi} \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}} \quad -(38)$$

Thus:

$$\Box e^{i\phi} = \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial Z^2}\right)e^{i\phi} = 0 \quad -(39)$$

QED. Here:

$$\phi = \omega t - \kappa Z \quad -(40)$$

6) Eq. (31) is also true if:

$$\Box \begin{bmatrix} \underline{A}_1^R & \underline{A}_2^R \\ \underline{A}_1^L & \underline{A}_2^L \end{bmatrix} = 0 \quad -(32)$$

Essentially this is due to the property of the phase:

$$\Box e^{i\phi} = \Box e^{-i\phi} = 0 \quad -(33)$$

For example:

$$\Box e^{i\phi} = \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial Z^2}\right) e^{i(\omega t - \kappa Z)}$$

$$= -\frac{\omega^2}{c^2} + \kappa^2 \quad -(34)$$

$$= 0$$

if

$$\kappa = \frac{\omega}{c} \quad -(35)$$

As is a feature of ECE theory, the same equations hold for dynamics and electrodynamics. In dynamics, the equation of the massless neutrino ~~(Weyl)~~ is

$$\Box q^a_\mu = 0 \quad -(36)$$

and eqs. (1) to (4) with:

$$m \to 0 \quad -(37)$$

1) 136(9): SU(3) Representation Space for the Unified Field

In ECE theory there is only one field of force, the four fundamental fields thought to exist in the standard physics are limits of the unified field. The latter can be developed in any valid representation space. It is convenient to review this procedure in this note. The method adopted is to start from the Einstein energy equation:

$$p^\mu p_\mu = m^2 c^2 \quad -(1)$$

of classical special relativity. Using the operator equivalence:

$$p^\mu = i\hbar \partial^\mu \quad -(2)$$

eq. (1) becomes

$$(\square + \kappa^2)\phi = 0 \quad -(3)$$

where ϕ is an eigenfunction and where

$$\kappa = mc/\hbar . \quad -(4)$$

Eq. (3) is a limit of the ECE wave equation of the unified field:

$$(\square + kT)\, q^a_\mu = 0 \quad -(5)$$

Conversely, the Einstein energy equation (1) may be obtained from eq. (5). Eq. (3) represents all the known wave equations of physics, for example the Dirac wave equation and the Proca wave equation.

The Dirac equation is usually written in an SU(2) representation space and in a condensed format it is:

$$(i\gamma^\mu \partial_\mu - \kappa)\psi = 0 \quad -(6)$$

2) where γ^μ is the Dirac matrix. Eq. (6) is a factorization of eq. (3) which originates in a factorization of the d'Alembertian operator itself. Therefore as Dirac himself inferred, his equation is geometrical in origin. In ECE theory the origin of all the wave equations of physics is the very fundamental tetrad postulate:

$$D_\mu q^a_\nu = 0 \quad -(7)$$

without which geometry would be meaningless. Eq. (7) may be expressed as the ECE Lemma:

$$\square q^a_\mu = R q^a_\mu \quad -(8)$$

By postulate:

$$R = -kT \quad -(9)$$

Eq. (9) links geometry and physics, giving eq. (5).

Recently it has been shown that the Dirac eq. (6) can be written as an equation in the Pauli matrices without use of the Dirac matrices:

$$\left.\begin{aligned} \sigma^\mu p_\mu q^R_1 &= mc\,\sigma^0 q^L_1 \\ \sigma^\mu p_\mu q^R_2 &= mc\,\sigma^0 q^L_2 \end{aligned}\right\} \quad -(10)$$

If we define:

$$\left.\begin{aligned} \phi^R &= [q^R_1 \;\; q^R_2] \\ \phi^L &= [q^L_1 \;\; q^L_2] \end{aligned}\right\} \quad -(11)$$

then:

$$\boxed{\sigma^\mu p_\mu \phi^R = mc\,\sigma^0 \phi^L} \quad -(12)$$

5) Here:

$$\sigma^\mu = (\sigma^0, \sigma^1, \sigma^2, \sigma^3) \quad -(13)$$

$$p_\mu = (p_0; p_1, p_2, p_3), \quad -(14)$$

So:

$$\sigma^\mu p_\mu = \sigma^0 p_0 - \underline{\sigma} \cdot \underline{p} \quad -(15)$$

where:

$$\underline{\sigma} = \sigma^1 \underline{i} + \sigma^2 \underline{j} + \sigma^3 \underline{k} \quad -(16)$$

$$\underline{p} = p_x \underline{i} + p_y \underline{j} + p_z \underline{k}. \quad -(17)$$

So eq (12) is:

$$\boxed{(\sigma^0 p_0 - \underline{\sigma} \cdot \underline{p}) \phi^R = mc\sigma^0 \phi^L} \quad -(18)$$

The Pauli matrices are:

$$\sigma^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad -(19)$$

in SU(2) representation space.

Now apply the parity operator to eq. (18), term by term. This produces:

$$\boxed{(\sigma^0 p_0 + \underline{\sigma} \cdot \underline{p}) \phi^L = mc\sigma^0 \phi^R} \quad -(20)$$

The product $p^\mu p_\mu$ has been factorized by application of parity:

4)

$$\boxed{\sigma^{0\,2} p^{\mu} p_{\mu} = \left(\sigma^{0} p_{0} + \underline{\sigma} \cdot \underline{p}\right)\left(\sigma^{0} p_{0} - \underline{\sigma} \cdot \underline{p}\right)} \quad -(21)$$

Using the fundamental operator equivalence of quantum mechanics, eq. (2), eq. (21) is a factorization of the d'Alembertian operator:

$$\boxed{\sigma^{0\,2} \, \Box = \left(\frac{\sigma^{0}}{c}\frac{\partial}{\partial t} + \underline{\sigma} \cdot \underline{\nabla}\right)\left(\frac{\sigma^{0}}{c}\frac{\partial}{\partial t} - \underline{\sigma} \cdot \underline{\nabla}\right)} \quad -(22)$$

This is the essence of the Dirac equation, or the simplest and more powerful ECE fermion equation. It is well known that the Dirac equation and ECE fermion equation give rise to important techniques such as ~~ERS +~~ ESR, NMR and MRI, so eq. (22) is much more than a mathematical exercise.

<u>SU(3) Representation Space</u>.

This rep space is well known to be used in 3-quark ~~theory~~ in elementary particle physics. Eqs. (21) and (22) may be developed in SU(3) rep space, or in any SU(n) rep space. In ECE theory this means that the wave equation may be developed

5) in any SU(n) rep space for the unified field. Therefore any field of force may be developed in any representation space. This provides an origin for any elementary particle in geometry.

In SU(3) the Pauli matrices are replaced by eight 3×3 matrices. In particle theory these are known as the Gell-Mann matrices:

$$\lambda^1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \lambda^2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \lambda^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\lambda^4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \lambda^5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}, \quad \lambda^6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$

$$\lambda^7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad -(23)$$

The Pauli matrices are well known to give the SU(2) cyclic relations:

$$\left[\frac{\sigma^1}{2}, \frac{\sigma^2}{2} \right] = i \frac{\sigma^3}{2} \quad -(24)$$

The structure factor of the Lie algebra of the SU(2) group is:

$$f_{123} = 1 \quad -(25)$$

The SU(3) matrices give:

$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2} \right] = i f_{abc} \frac{\lambda^c}{2} \quad -(26)$$

6) with:

$$\left.\begin{array}{l} f_{123} = 1, \\ f_{147} = -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = \frac{1}{2}, \end{array}\right] - (27)$$

$$f_{458} = f_{678} = \sqrt{3}/2.$$

The problem is to factorize the d'Alembertian, or $p^\mu p_\mu$, using the Gell-Mann matrices rather than the Pauli matrices. This is a mathematical problem that leads to a lot of new physics.

Define:

$$\lambda^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad -(28)$$

Thus:

$$2\lambda^0 = \lambda^{01} + \lambda^{02} + \lambda^{03} \quad -(29)$$

where:

$$\lambda^{01} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \lambda^{02} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \lambda^{03} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad -(30)$$

Define:

$$\lambda^8 = \frac{1}{\sqrt{3}}\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\right) \quad -(31)$$

$$= \frac{1}{\sqrt{3}}\left(\lambda^9 + \lambda^{10}\right)$$

Now define:

7)

$$\left.\begin{aligned}
\underline{\alpha}^1 &= \lambda^1 \underline{i} + \lambda^2 \underline{j} + \lambda^3 \underline{k} \\
\underline{\alpha}^2 &= \lambda^4 \underline{i} + \lambda^5 \underline{j} + \lambda^9 \underline{k} \\
\underline{\alpha}^3 &= \lambda^6 \underline{i} + \lambda^7 \underline{j} + \lambda^{10} \underline{k}
\end{aligned}\right\} \tag{33}$$

So:

$$\underline{\alpha}^1 \cdot \underline{\rho} = \begin{bmatrix} \rho_z & \rho_x - i\rho_y & 0 \\ \rho_x + i\rho_y & -\rho_z & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{34}$$

$$\underline{\alpha}^2 \cdot \underline{\rho} = \begin{bmatrix} \rho_z & 0 & \rho_x - i\rho_y \\ 0 & 0 & 0 \\ \rho_x + i\rho_y & 0 & -\rho_z \end{bmatrix} \tag{35}$$

$$\underline{\alpha}^3 \cdot \underline{\rho} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \rho_z & \rho_x - i\rho_y \\ 0 & \rho_x + i\rho_y & -\rho_z \end{bmatrix} \tag{36}$$

and:

$$(\underline{\alpha}^1 \cdot \underline{\rho})(\underline{\alpha}^1 \cdot \underline{\rho}) = (\rho_x^2 + \rho_y^2 + \rho_z^2) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{37}$$

$$(\underline{\alpha}^2 \cdot \underline{\rho})(\underline{\alpha}^2 \cdot \underline{\rho}) = (\rho_x^2 + \rho_y^2 + \rho_z^2) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{38}$$

$$(\underline{\alpha}^3 \cdot \underline{\rho})(\underline{\alpha}^3 \cdot \underline{\rho}) = (\rho_x^2 + \rho_y^2 + \rho_z^2) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{39}$$

8)

Therefore:

$$(\underline{\alpha}^1\cdot\underline{p})(\underline{\alpha}^1\cdot\underline{p})+(\underline{\alpha}^2\cdot\underline{p})(\underline{\alpha}^2\cdot\underline{p})+(\underline{\alpha}^3\cdot\underline{p})(\underline{\alpha}^3\cdot\underline{p})$$

$$=2\left(p_x^2+p_y^2+p_Z^2\right)\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=2p^2\lambda^0 \quad -(40)$$

where:

$$p^2=p_x^2+p_y^2+p_Z^2 \quad -(41)$$

It follows that $p^\mu p_\mu$ may be factorized in three ways.

$$\lambda^{32}p^\mu p_\mu=(\lambda^3p-\underline{\alpha}^1\cdot\underline{p})(\lambda^3p+\underline{\alpha}^1\cdot\underline{p}) \quad -(42)$$

$$\lambda^{92}p^\mu p_\mu=(\lambda^9p-\underline{\alpha}^2\cdot\underline{p})(\lambda p+\underline{\alpha}^2\cdot\underline{p}) \quad -(43)$$

$$\lambda^{102}p^\mu p_\mu=(\lambda^{10}p-\underline{\alpha}^3\cdot\underline{p})(\lambda p+\underline{\alpha}^3\cdot\underline{p}) \quad -(44)$$

Adding:

$$(\lambda^{32}+\lambda^{92}+\lambda^{102})p^\mu p_\mu \quad -(45)$$

$$=(\lambda^{32}+\lambda^{92}+\lambda^{102})p^2$$

$$-\left((\underline{\alpha}^1\cdot\underline{p})(\underline{\alpha}^1\cdot\underline{p})+(\underline{\alpha}^2\cdot\underline{p})(\underline{\alpha}^2\cdot\underline{p})+(\underline{\alpha}^3\cdot\underline{p})(\underline{\alpha}^3\cdot\underline{p})\right)$$

9)

i.e.:

$$2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} p^{\mu} p_{\mu} = 2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\left(p^{2} - \underline{p}\cdot\underline{p}\right) \quad -(46)$$

Q.E.D.

The factorization of d'Alembertian is accomplished, finally, by using:

$$p^{\mu} p_{\mu} = -\hbar^{2}\Box \quad -(47)$$

and

$$p^{\mu} = i\hbar\,\partial^{\mu} \quad -(48)$$

so

$$\left(p, \underline{p}\right) = i\hbar\left(\frac{1}{c}\frac{\partial}{\partial t}, -\underline{\nabla}\right). \quad -(49)$$

Therefore:

$$p = \frac{i\hbar}{c}\frac{\partial}{\partial t} \quad -(50)$$

$$\underline{p} = -i\hbar\underline{\nabla} \quad -(51)$$

Eq. (42), for example, becomes

$$-\hbar^{2}\lambda^{32}\Box = \left(\frac{i\hbar\lambda^{3}}{c}\frac{\partial}{\partial t} + i\hbar\,\underline{\alpha}^{1}\cdot\underline{\nabla}\right)\left(\frac{i\hbar\lambda^{3}}{c}\frac{\partial}{\partial t} - i\hbar\,\underline{\alpha}^{1}\cdot\underline{\nabla}\right) \quad -(52)$$

$$\lambda^{32}\Box = \left(\frac{\lambda^{3}}{c}\frac{\partial}{\partial t} + \underline{\alpha}^{1}\cdot\underline{\nabla}\right)\left(\frac{\lambda^{3}}{c}\frac{\partial}{\partial t} - \underline{\alpha}^{1}\cdot\underline{\nabla}\right) \quad -(53)$$

10) Therefore in the SU(3) representation space, the d'Alembertian operator may be factorized in ~~the~~ three ways:

$$\lambda^{32}\,\Box = \left(\frac{\lambda^3}{c}\frac{\partial}{\partial t} + \underline{\alpha}^1\cdot\underline{\nabla}\right)\left(\frac{\lambda^3}{c}\frac{\partial}{\partial t} - \underline{\alpha}^1\cdot\underline{\nabla}\right) \quad -(54)$$

$$\lambda^{92}\,\Box = \left(\frac{\lambda^9}{c}\frac{\partial}{\partial t} + \underline{\alpha}^2\cdot\underline{\nabla}\right)\left(\frac{\lambda^9}{c}\frac{\partial}{\partial t} - \underline{\alpha}^2\cdot\underline{\nabla}\right) \quad -(55)$$

$$\lambda^{102}\,\Box = \left(\frac{\lambda^{10}}{c}\frac{\partial}{\partial t} + \underline{\alpha}^3\cdot\underline{\nabla}\right)\left(\frac{\lambda^{10}}{c}\frac{\partial}{\partial t} - \underline{\alpha}^3\cdot\underline{\nabla}\right) \quad -(56)$$

Compare these results with eq (32) in the SU(2) representation space:

$$\sigma^{02}\,\Box = \left(\frac{\sigma^0}{c}\frac{\partial}{\partial t} + \underline{\sigma}\cdot\underline{\nabla}\right)\left(\frac{\sigma^0}{c}\frac{\partial}{\partial t} - \underline{\sigma}\cdot\underline{\nabla}\right) \quad -(57)$$

It is known that eq (57) gives rise to:

$$\left.\begin{aligned}(\sigma^0 p_0 - \underline{\sigma}\cdot\underline{p})\,\phi^R &= mc\sigma^0\phi^L \\ (\sigma^0 p_0 + \underline{\sigma}\cdot\underline{p})\,\phi^L &= mc\sigma^0\phi^R\end{aligned}\right] \quad -(58)$$

i.e.

$$\left.\begin{aligned}\left(\frac{\sigma^0}{c}\frac{\partial}{\partial t} + \underline{\sigma}\cdot\underline{\nabla}\right)\phi^R &= mc\sigma^0\phi^L \\ \left(\frac{\sigma^0}{c}\frac{\partial}{\partial t} - \underline{\sigma}\cdot\underline{\nabla}\right)\phi^L &= mc\sigma^0\phi^R\end{aligned}\right] \quad -(59)$$

Therefore eqs. (54) to (56) will give

rise to new equations of the ECE field in SU(3) rep. space. There are six new equations:

$$\left(\frac{\lambda^3}{c} \frac{\partial}{\partial t} + \underline{\alpha}^1 \cdot \underline{\nabla} \right) \phi^R = mc\sigma^0 \phi^L \quad -(60)$$

$$\left(\frac{\lambda^3}{c} \frac{\partial}{\partial t} - \underline{\alpha}^1 \cdot \underline{\nabla} \right) \phi^L = mc\sigma^0 \phi^R \quad -(61)$$

and similarly from eqs. (55) and (56).

Here ϕ^R and ϕ^L must be three-spinors:

$$\phi^R = \left[\phi_1^R \; \phi_2^R \; \phi_3^R \right] \quad -(62)$$

$$\phi^L = \left[\phi_1^L \; \phi_2^L \; \phi_3^L \right] \quad -(63)$$

The electromagnetic field is given by:

$$A^R = A^{(0)} \phi^R \quad -(64)$$

$$A^L = A^{(0)} \phi^L \quad -(65)$$

and the gravitational field is given by:

$$\Phi^R = \Phi^{(0)} \phi^R \quad -(66)$$

$$\Phi^L = \Phi^{(0)} \phi^L \quad -(67)$$

Many new types of spin resonance are predicted by these equations, which unify boson and fermion theory

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12)

In condensed notation, the SU(3) equations of the unified field are:

$$\alpha^{\mu 1} p_\mu \phi^R = mc\, \alpha^{01} \phi^L \quad (68)$$

$$\alpha^{\mu 2} p_\mu \phi^R = mc\, \alpha^{02} \phi^L \quad (69)$$

$$\alpha^{\mu 3} p_\mu \phi^R = mc\, \alpha^{03} \phi^L \quad (70)$$

where

$$\alpha^{\mu 1} = \left(\alpha^{01}, \underline{\alpha}^{1} \right) \quad (71)$$

etc.

and

$$\alpha^{01} = \lambda^{01} \quad (72)$$

There are ~~three~~ parity reversed equations from eqns. (68) to (70), giving six equations in ~~three~~ pairs.

1)

136(10): Effective Photon Charge and Electromagnetic Momentum

: As described in Jackson, third edition, pp. 261 ff., the standard interpretation is that the total electromagnetic momentum of the field in a volume V is:

$$\underline{p}\,(\text{field}) = \epsilon_0 \int_V \underline{E} \times \underline{B}\, dV \qquad -(1)$$

For one photon: $p = \hbar \kappa$ $\quad -(2)$

The volume V is the volume of radiation. For a plane wave:

$$\underline{E} = \frac{E^{(0)}}{\sqrt{2}} \left(\underline{i} - i\underline{j} \right) \exp\left(i(\omega t - \kappa Z)\right) \qquad -(3)$$

$$\underline{B} = \frac{B^{(0)}}{\sqrt{2}} \left(i\underline{i} + \underline{j} \right) \exp\left(i(\omega t - \kappa Z)\right) \qquad -(4)$$

Therefore

$$\underline{E} \times \underline{B} = \frac{E^{(0)}B^{(0)}}{2} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -i & 0 \\ i & 1 & 0 \end{vmatrix} e^{2i\phi} \qquad -(5)$$

$$= ?0$$

Averaging over many cycles

$$\langle e^{2i\phi} \rangle = 0 \qquad -(6)$$

Therefore to obtain a finite momentum density for the plane wave, the definition (1) given by Jackson must be modified. This is done by using the conjugate product:

$$\underline{p}(\text{field}) = \epsilon_0 \int_V \underline{E} \times \underline{B}^* \, dV \quad -(7)$$

In the notation of ECE theory:

$$\boxed{\underline{p}^{(3)}(\text{field}) = \epsilon_0 \int \underline{E}^{(1)} \times \underline{B}^{(2)} \, dV} \quad -(8)$$

Here:

$$\underline{E}^{(1)} = \frac{E^{(0)}}{\sqrt{2}} \left(\underline{i} - i \underline{j} \right) e^{i\phi} \quad -(9)$$

$$\underline{B}^{(2)} = \frac{B^{(0)}}{\sqrt{2}} \left(-i \underline{i} + \underline{j} \right) e^{-i\phi} \quad -(10)$$

So:

$$\underline{E}^{(1)} \times \underline{B}^{(2)} = \frac{E^{(0)} B^{(0)}}{2} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -i & 0 \\ -i & 1 & 0 \end{vmatrix} \quad -(11)$$

$$\boxed{\underline{E}^{(1)} \times \underline{B}^{(2)} = E^{(0)} B^{(0)} \underline{k}} \quad -(12)$$

In this case the electromagnetic field has the correct linear momentum as observed experimentally in the Beth experiment (Princeton, 1936) and in the Compton and photoelectric effects.

The effective charge of the photon is defined by a longitudinally directed potential $\underline{A}^{(3)}$ as follows:

3) $$\boxed{\underline{p}^{(3)} = e\underline{A}^{(3)} = -\frac{i\epsilon_0}{c}\int \omega^2 \underline{A}^{(1)} \times \underline{A}^{(2)} \, dV} \quad -(13)$$

Thus: $$\underline{p}^{(3)} = e\underline{A}^{(3)} = \frac{\epsilon_0 \omega^2}{c} A^{(0)2} V \underline{k} \quad -(14)$$

i.e. $$\boxed{e = \frac{\epsilon_0 \omega^2}{c} V A^{(0)}} \quad -(15)$$

as first derived in "The Enigmatic Photon" (on the Omnia Opera of www.aias.us). The effective gyromagnetic ratio of the photon

then: $$g = \frac{e}{m} \quad -(16)$$

where m is the photon mass. Eq. (13) is a variation on the B Cyclic Theorem

$$i\underline{B}^{(3)*} = \underline{B}^{(1)} \times \underline{B}^{(2)} \quad -(17)$$

et cyclicum

which is Lorentz covariant and reduces to the O(3) symmetry laws of the complex circular frame:

$$i\underline{e}^{(3)*} = \underline{e}^{(1)} \times \underline{e}^{(2)} . \quad -(18)$$