

1) 162(7) : Test of Eq. (8) of Note 162(6) with H Atom

The energy levels of the H atom are given by:

$$E = - \frac{E_R}{n^2}, \quad n = 1, 2, \dots \quad - (1)$$

where

$$E_R = - 2.2 \times 10^{-18} \text{ J} \quad - (2)$$

$$\text{So } E_1 = - 2.2 \times 10^{-18} \text{ J} \quad - (3)$$

$$E_2 = - 0.55 \times 10^{-18} \text{ J} \quad - (4)$$

If the rest mass of the electron is accepted to be:

$$m = 9.10953 \times 10^{-31} \text{ kg} \quad - (5)$$

Then

$$E_0 = mc^2 = 8.1872 \times 10^{-14} \text{ J} \quad - (6)$$

Therefore:

$$\cos \theta = \frac{E_1 E_2 - E_0^2}{(E_1^2 - E_0^2)^{1/2} (E_2^2 - E_0^2)^{1/2}} \\ \sim -1 \quad - (7)$$

In eqs. (13) - (16) of note 162(6), eq. (7) gives:

$$a = 2, \quad b = (E_1 - E_2)^2, \quad c' = 2E_1^2 E_2^2 \quad - (8)$$

$$\text{So: } E_0^2 = \frac{1}{4} \left[- (E_1 - E_2)^2 \pm \left((E_1 - E_2)^4 - 8E_1^2 E_2^2 \right)^{1/2} \right] \quad - (9)$$

$$\text{which: } (E_1 - E_2)^4 = 7.41 \times 10^{-72} \text{ J}^4 \\ 8E_1^2 E_2^2 = 11.71 \times 10^{-72} \text{ J}^4$$

So the result (9) is wildly self-inconsistent.

2) The theory of atomic absorption fails completely if conservation of momentum is considered. Conservation of momentum is:

$$\underline{p}' + \hbar \underline{k} = \underline{p}'' \quad (10)$$

where \underline{p}' is the initial momentum of the electron in orbital 1, and \underline{p}'' the final momentum of the electron in orbital 2. We have:

$$\underline{p}' = \gamma' m \underline{v}' \quad (11)$$

$$\underline{p}'' = \gamma'' m \underline{v}'' \quad (12)$$

Therefore

$$\begin{aligned} c p'^2 &= \gamma'^2 m^2 c^4 \frac{v'^2}{c^2} \\ &= \gamma'^2 m^2 c^4 \left(1 - \frac{1}{\gamma'^2}\right) \\ &= \gamma'^2 m^2 c^4 - m^2 c^4 \\ &= E'^2 - E_0^2 \quad (13) \end{aligned}$$

So:

$$E'^2 = c^2 p'^2 + E_0^2 \quad (14)$$

$$E''^2 = c^2 p''^2 + E_0^2 \quad (15)$$

This gives eq. (7), QED from rule 162(6), with $E_1 = E'$, $E_2 = E''$.