189(2): Energy Equation and Equation of Orbits

: These are unaffected by the use of a coordinate independent metric. In a spherical spacetime the methods of note 179(4) can be used. The infinitesimal line element is:

$$ds^2 = c^2 d\tau^2 = m(r,t)c^2 dt^2 - n(r,t)\,dr^2 - r^2 d\phi^2 \quad -(1)$$

in a plane:

$$dZ^2 = 0. \quad -(2)$$

It follows that the energy equation of the spherical spacetime is

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(3)$$

where

$$E = m(r,t)\, m c^2 \frac{dt}{d\tau} \quad -(4)$$

is the total energy, where

$$p_r = n(r,t)\, m \frac{dr}{d\tau} \quad -(5)$$

is the linear momentum, and where:

$$L = m r^2 \frac{d\phi}{d\tau} \quad -(6)$$

where, is the angular momentum, and

$$p^2 = p_r^2 + \frac{L^2}{r^2}. \quad -(7)$$

In the limit of flat spacetime:

$$m(r,t) \longrightarrow 1 \quad -(8)$$

$$n(r,t) \longrightarrow 1 \quad -(9)$$

$$\left.\begin{aligned} ds^2 = c^2 d\tau^2 &= c^2 dt^2 - dr^2 - r^2 d\phi^2 \\ &= c^2 dt^2 - dX^2 - dY^2 \end{aligned}\right] -(10)$$

However, the coordinate free metric is:

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad -(11)$$

in any coordinate system. It follows from eq. (11) that all elements of torsion and curvature vanish and that

$$\boxed{D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu} = 0} \quad -(12)$$

The Evans identity is obeyed rigorously in the coordinate independent metric system for any system of coordinates.

In the old system, the metric (11) differs from one coordinate system to another, and this can cause problems of interpretation and problems of mathematics. For reasons which are not clear, the Minkowski metric in the cylindrical polar system was written in the twentieth century as:

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{bmatrix} \quad -(13)$$

and as shown in CEFE p. 112, non-zero Christoffel connections are produced by eq (13):

$$\Gamma^1_{22} = -r \quad \text{etc.} \quad -(14)$$

This is a <u>self - contradiction</u> in the <u>old theory</u>.