1)

189(10): Summary of Equations

The metrical function $m(r,t)$ is:

$$m(r,t) = \exp\left(2\exp\left(-\frac{r}{3R(t)}\right)\right). \quad -(1)$$

In the limit

$$r \to \infty, \quad -(2)$$

$$m(r,t) \to 1. \quad -(3)$$

The function is defined as:

$$m(r,t)) = m(r, R(t)) \quad -(4)$$

so

$$\frac{dR(t)}{dr} = 0. \quad -(5)$$

The solar system is defined by experimental data as being describable by a precessing ellipse:

$$r = \frac{\alpha}{1+\epsilon\cos(y\theta)}. \quad -(6)$$

The equation of orbits is

$$\frac{1}{r}\frac{dr}{dt} = \frac{1}{R(t)}\frac{dR(t)}{dt} \quad -(7)$$

and in the Newtonian limit:

$$y \to 1 \quad -(8)$$

is described by:

$$\frac{1}{R(t)}\frac{dR(t)}{dt} = \left(\frac{L\epsilon}{\mu\alpha^2}\right)\left(1+\epsilon\cos\theta(t)\right)\sin\theta(t) \quad -(9)$$

Therefore $R(t)$ can be found by computer

2) algebra, thus giving the time dependent part of $n(r,t)$.

In the Newtonian limit:

$$n(r,t) \rightarrow 1, \quad -(10)$$

so the Newtonian limit is given by:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 \quad -(11)$$

This equation gives:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(12)$$

and the relativistic kinetic energy,

$$T = (\gamma - 1) mc^2 \quad -(13)$$

$$\rightarrow \frac{1}{2} mv^2$$

for

$$v << c. \quad -(14)$$

The precessing ellipse (6) is obtained from:

$$ds^2 = \left(1 - \frac{r_0}{r}\right) c^2 dt^2 - \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 - r^2 d\theta^2 \quad -(15)$$

and in the solar system:

$$y \rightarrow 1 \quad -(16)$$

and

$$\frac{r_0}{r} \rightarrow 0. \quad -(17)$$

$$-(18)$$

In this limit:

$$\exp\left(2 \exp\left(-\frac{r}{3R(t)}\right)\right) \sim 1 - \frac{r_0}{r}$$

to an excellent approximation.

For the precessing ellipse:

$$\frac{d\theta}{dt} = \frac{L}{m\alpha^2}\left(1 + \epsilon\cos(y\theta)\right)^2 \quad -(19)$$

$$\frac{dr}{d\theta} = \frac{y\alpha\epsilon\sin(y\theta)}{\left(1 + \epsilon \cancel{\sin} \cos(y\theta)\right)^2} \quad -(20)$$

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \left(\frac{Ly\epsilon}{m\alpha}\right)\sin(y\theta) \quad -(21)$$

$$\boxed{\begin{aligned}\frac{1}{r}\frac{dr}{dt} &= \left(\frac{Ly\epsilon}{m\alpha^2}\right)\left(1 + \epsilon\cos(y\theta)\right)\sin(y\theta) \\ &= \frac{1}{R(t)}\frac{dR(t)}{dt} \quad -(22)\end{aligned}}$$

The metrical function $m(r, t)$ is found by using $R(t)$ from eq. (22) in eq. (1). It would also be very interesting to plot this and compare it with the function:

$$m(\text{Schwarzschild}) = 1 - \frac{r_0}{r} \quad -(23)$$