

196(6) : Force Law for a Precessing Ellipse

In this case the orbit is:

$$r(t) = \frac{d}{1 + \epsilon \cos(x\theta)} \quad - (1)$$

and

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta} \frac{d\theta}{dt} = \left(\frac{xL\epsilon}{md} \right) \sin(x\theta) \quad - (2)$$

Thus:

$$\begin{aligned} \underline{v} = \frac{d\underline{r}}{dt} &= \dot{r} \underline{e}_r + r \dot{\underline{e}}_r \\ &= \dot{r} \underline{e}_r + r \dot{\theta} \underline{e}_\theta \end{aligned} \quad - (3)$$

However, the non-relativistic expression for kinetic energy can no longer be used, because the

non-relativistic:

$$T = \frac{1}{2} m v^2 \quad - (4)$$

gives a static ellipse. The next that can be deduced is that:

$$\dot{r} = \frac{dr}{dt} = \frac{xL\epsilon}{d} r^2 \sin(x\theta) \omega \quad - (5)$$

i.e.

$$\boxed{\dot{r} = \dot{\theta} \left(\frac{xL\epsilon}{d} \right) r^2 \sin x\theta} \quad - (6)$$

In the solar system however, the deviations from a static ellipse are very small. So eq.

2) (4) holds to an excellent approximation. Therefore:

$$T = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\left(\frac{dr}{dt} \right)^2 + r^2 \left(\frac{d\theta}{dt} \right)^2 \right) \quad - (7)$$

The Lagrangian is:

$$\mathcal{L} = T - V \quad - (8)$$

and the constant total angular momentum is:

$$L = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = m r^2 \dot{\theta} \quad - (9)$$

The angular momentum can also be obtained from:

$$\underline{L} = \underline{r} \times \underline{p}, \quad |\underline{L}| = r p = m r^2 \dot{\theta} \quad - (10)$$

The approximation:

$$\ddot{\theta} = \left(\frac{x L \epsilon}{m d} \right) \sin(x\theta), \quad \dot{\theta} = \frac{L}{m r^2} \quad - (11)$$

It follows that:

$$\ddot{\theta} = \frac{x L \epsilon}{m d} \frac{d}{dt} (\sin(x\theta))$$

$$= \frac{x L \epsilon}{m d} \frac{d}{d\theta} (\sin(x\theta)) \frac{d\theta}{dt}$$

$$= \frac{x^2 L \epsilon}{m d} \cdot \frac{1}{r^2} \cos(x\theta) \quad - (12)$$

and $\ddot{\theta} = -\frac{2L^2 x \epsilon \sin(x\theta)}{m^2 d r^3} \quad - (13)$

Therefore: $\ddot{r} - r\dot{\theta}^2 = \left(\frac{x^2 M G \cos(x\theta)}{r^2} - \frac{L^2}{m^2 r^3} \right) \quad - (14)$

$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \quad - (15)$

Thus the acceleration is:

$\underline{a} = \frac{d\underline{v}}{dt} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta \quad - (16)$

The force is:

$\underline{F} = m\underline{a} = m \left(\frac{x^2 \epsilon M G \cos(x\theta)}{r^2} - \frac{L^2}{m^2 r^3} \right) \underline{e}_r \quad - (17)$

Finally we:

$\epsilon \cos(x\theta) = \frac{d}{r} - 1 \quad - (18)$

To an excellent approximation:

$d = \frac{L^2}{2m\epsilon} \quad - (19)$

4) So:

$$\underline{F} = \left(-x^2 \frac{mM G}{r^2} + \frac{L^2}{mr^3} (x^2 - 1) \right) \underline{e}_r \quad - (20)$$

This is the same as eq. (9) of UFT 193 given
eq. (19).

So for very small precession of the perihelion, the force law is a combination of inverse square and inverse cubed. The Einstein theory is incorrect because it gives the sum of an inverse square and inverse fourth.

A fully relativistic theory requires the use of eqs. (6), (3) and (16). These would derive a force law that can be compared with that of the field equations of EFE.
