

207(8): Evaluation of  $\frac{df}{dt}$

Consider the following functional relations.

If  $f = f(r(t), \theta(t))$  then  $\frac{df}{dt} = \frac{\partial f}{\partial r} \frac{dr}{dt} + \frac{\partial f}{\partial \theta} \frac{d\theta}{dt}$  — (1)

If  $f = f(t(\theta), r(\theta))$  then  $\frac{df}{d\theta} = \frac{\partial f}{\partial r} \frac{dr}{d\theta} + \frac{\partial f}{\partial t} \frac{dt}{d\theta}$  — (2)

If  $f = f(\theta(r), t(r))$  then  $\frac{df}{dr} = \frac{\partial f}{\partial \theta} \frac{d\theta}{dr} + \frac{\partial f}{\partial t} \frac{dt}{dr}$  — (3)

As a note 207(7):  $\frac{df}{dt} = \frac{\partial f}{\partial r} \frac{dr}{dt}$  — (4)

using eqns. (1) and (3).

From eqs. (1) and (2):

$$\begin{aligned} \frac{df}{dt} &= \frac{dr}{dt} \frac{d\theta}{dr} \left( \frac{df}{d\theta} - \frac{\partial f}{\partial t} \frac{dt}{d\theta} \right) + \frac{\partial f}{\partial \theta} \frac{d\theta}{dt} \\ &= \frac{df}{dt} - \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \theta} \frac{d\theta}{dt} \end{aligned} \quad \text{--- (5)}$$

so  $\frac{\partial f}{\partial t} = \frac{\partial f}{\partial \theta} \frac{d\theta}{dt}$  — (6)

From eqs. (4) and (2):

$$\begin{aligned} \frac{df}{dt} &= \frac{d\theta}{dr} \frac{dr}{dt} \left( \frac{df}{d\theta} - \frac{\partial f}{\partial t} \frac{dt}{d\theta} \right) \\ &= \frac{df}{dt} - \frac{\partial f}{\partial t} \end{aligned} \quad \text{--- (7)}$$

So

$$\boxed{\frac{df}{dt} = \frac{1}{2} \frac{df}{dt}} \quad \text{--- (8)}$$

From eqs. (6) and (1): --- (9)

$$\frac{df}{dt} = \frac{d\theta}{dt} \frac{dr}{d\theta} \left( \frac{df}{dr} - \frac{df}{dt} \frac{dt}{dr} \right) = \frac{df}{dt} - \frac{df}{dt}$$

So

$$\boxed{\frac{df}{dt} = \frac{1}{2} \frac{df}{dt}} \quad \text{--- (10)}$$

Therefore

$$\begin{aligned} \boxed{\frac{df}{dt} = \frac{1}{2} \frac{df}{d\theta} \frac{d\theta}{dt}} \\ = \frac{1}{2} \omega \frac{df}{d\theta} \end{aligned} \quad \text{--- (11)}$$

The fraction  $df/d\theta$  can be found from the orbit, and the angular velocity can be found either from direct observation or the analysis of AFT 206

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