# 226(4): Momentum Transfer in a Low Energy Nuclear Reaction.

This theory develops the method of note 226(3). Consider an atom of initial four momentum $p^\mu$ interacting with a matter wave of momentum $\hbar\kappa^\mu$. Using the minimal prescription:

$$p^\mu \rightarrow p^\mu + \hbar\kappa^\mu \quad -(1)$$

so:

$$E \rightarrow E + \hbar\omega, \quad \underline{p} \rightarrow \underline{p} + \hbar\underline{\kappa}. \quad -(2)$$

The fermion equation for this interaction is:

$$\left(E + \hbar\omega + c\underline{\sigma}\cdot(\underline{p} + \hbar\underline{\kappa})\right)\phi^L = mc^2\phi^R \quad -(3)$$

$$\left(E + \hbar\omega - c\underline{\sigma}\cdot(\underline{p} + \hbar\underline{\kappa})\right)\phi^R = mc^2\phi^L \quad -(4)$$

Therefore:

$$\left((E+\hbar\omega)^2 - c^2\underline{\sigma}\cdot(\underline{p}+\hbar\underline{\kappa})\underline{\sigma}\cdot(\underline{p}+\hbar\underline{\kappa})\right)\phi^R = m^2c^4\phi^R \quad -(5)$$

Here m is the measured mass of the atom. The R factor of the incoming matter wave is defined by:

$$R = \kappa^\mu\kappa_\mu = \frac{\omega^2}{c^2} - \kappa^2 \quad -(6)$$

2)

If $m_1$ is the measured mass of the incoming matter wave, then:

$$\hbar^2\omega^2 = c^2\kappa^2\hbar^2 + m_1^2c^4 \quad -(7)$$

So:
$$\omega^2 = c^2\kappa^2 + \left(\frac{m_1c^2}{\hbar}\right)^2 \quad -(8)$$

and
$$\boxed{\frac{\omega^2}{c^2} - \kappa^2 = \left(\frac{m_1c}{\hbar}\right)^2 = R} \quad -(9)$$

Therefore from eq. (5):

$$c^2\underline{\sigma}\cdot(\underline{p}+\hbar\underline{\kappa})\,\underline{\sigma}\cdot(\underline{p}+\hbar\underline{\kappa})\phi^R$$
$$= \left((E+\hbar\omega)^2 - m^2c^4\right)\phi^R. \quad -(10)$$

Now consider the matter wave to be defined by:

$$E_1 = \hbar\omega = \gamma_1 mc^2 \quad -(11)$$
$$\underline{p_1} = \hbar\underline{\kappa} = \gamma_1 m\underline{v} \quad -(12)$$

where
$$\gamma_1 = \left(1 - \frac{v_1^2}{c^2}\right)^{-1/2}. \quad -(13)$$

where $\underline{v_1}$ is its velocity. Then:

$$c^2\underline{\sigma}\cdot(\underline{p}+\hbar\underline{\kappa})\,\underline{\sigma}\cdot(\underline{p}+\hbar\underline{\kappa})\phi^R \quad -(14)$$
$$= \left((\gamma+\gamma_1)^2m^2c^4 - m^2c^4\right)\phi^R$$

3) where
$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(15)$$

Therefore:
$$\underline{\sigma} \cdot \left(\underline{p} + \hbar \underline{\kappa}\right) \underline{\sigma} \cdot \left(\underline{p} + \hbar \underline{\kappa}\right) \phi^R$$
$$= m^2 c^2 \left( (\gamma + \gamma_1)^2 - 1 \right) \phi^R \quad -(16)$$

For real valued $\underline{p} + \hbar \underline{\kappa}$:
$$\left(\underline{p} + \hbar \underline{\kappa}\right) \cdot \left(\underline{p} + \hbar \underline{\kappa}\right) \phi^R \quad -(17)$$
$$= m^2 c^2 \left( (\gamma + \gamma_1)^2 - 1 \right) \phi^R$$

At this point eqn. (17) may be transformed into an operator equation using:
$$p^\mu = i \hbar \partial^\mu, \quad -(18)$$
so
$$\underline{p} = - i \hbar \underline{\nabla} \quad -(19)$$

That leads to a Schroedinger type equation:
$$\left(- i \hbar \underline{\nabla} + \hbar \underline{\kappa}\right) \cdot \left(- i \hbar \underline{\nabla} + \hbar \underline{\kappa}\right) \phi^R$$
$$= m^2 c^2 \left( (\gamma + \gamma_1)^2 - 1 \right) \phi^R \quad -(20)$$

4)

i.e.

$$-\hbar^2\nabla^2\phi^R = \left(m^2c^2\left((\gamma+\gamma_1)^2-1\right)-\hbar^2\kappa^2\right)\phi^R + i\hbar^2\left(\underline{\nabla}\cdot(\underline{\kappa}\phi^R)+\underline{\kappa}\cdot\underline{\nabla}\phi^R\right)$$

$$= \left(m^2c^2\left((\gamma+\gamma_1)^2-1\right)-\hbar^2\kappa^2\right)\phi^R + i\hbar^2\left((\underline{\nabla}\cdot\underline{\kappa})\phi^R+\underline{\kappa}\cdot\underline{\nabla}\phi^R+\underline{\kappa}\cdot\underline{\nabla}\phi^R\right)$$

$$= \left(m^2c^2\left((\gamma+\gamma_1)^2-1\right)-\hbar^2\kappa^2\right)\phi^R + i\hbar^2\underline{\kappa}\cdot\underline{\nabla}\phi^R \quad -(21)$$

The quantized momentum after interaction of the atom with the matter wave is:

$$\boxed{\begin{aligned}\hat{p}^2\phi^R &= -\hbar^2\nabla^2\phi^R\\ &= \left(m^2c^2\left((\gamma+\gamma_1)^2-1\right)-\hbar^2\kappa^2\right)\phi^R + i\hbar^2\underline{\kappa}\cdot\underline{\nabla}\phi^R\end{aligned}} \quad -(22)$$

This gives <u>momentum</u> <u>levels</u>. It is possible to develop a theory where transitions and

5) resonance can occur by absorption of a quantum of momentum from spacetime itself:

$$\underline{p}\,(\text{spacetime}) = \hbar\, \underline{\kappa}\,(\text{spacetime}) \quad -(23)$$

This resonance causes the low energy nuclear reaction.

If it is assumed for the sake of simplicity that:

$$\mathrm{Re}\left(i\hbar^2\left(\underline{\nabla}\,\phi^R\right)\cdot\underline{\kappa}\right) = 0 \quad -(24)$$

then:

$$\nabla^2\phi^R = -\left(\left(\frac{mc}{\hbar}\right)^2\left((\gamma+\gamma_1)^2 - 1\right) - \kappa^2\right)\phi^R \quad -(25)$$

i.e

$$\boxed{\nabla^2\phi^R = -R_{int}\,\phi^R} \quad -(26)$$

where

$$R_{int} = \left(\frac{mc}{\hbar}\right)^2\left((\gamma+\gamma_1)^2 - 1\right) - \kappa^2 \quad -(27)$$

For motion along the Z axis:

$$\frac{\partial^2\phi^R}{\partial Z^2} = -R_{int}\,\phi^R \quad -(28)$$

) and

$$\phi^R = \exp\left(-iR_{int}^{1/2} Z\right) \quad (29)$$

This can be normalized to:

$$\phi^R = \phi^{(0)} \exp\left(-iR_{int}^{1/2} Z\right) \quad -(30)$$

where:

$$R_{int}^{1/2} = \left(\frac{mc}{\hbar}\left(\left(\gamma+\gamma_1\right)^2 - 1\right) - \kappa^2\right)^{1/2} \quad -(31)$$

The de Moivre Theorem gives:

$$\exp\left(-iR_{int}^{1/2} Z\right) = \cos\left(R_{int}^{1/2} Z\right) - i\sin\left(R_{int}^{1/2} Z\right) \quad -(32)$$

where:

$$\cos\left(R_{int}^{1/2} Z\right) = \cos\left(R_{int}^{1/2} Z + 2\pi n\right) \quad -(33)$$

$$\sin\left(R_{int}^{1/2} Z\right) = \sin\left(R_{int}^{1/2} Z + 2\pi n\right) \quad -(34)$$

so

$$\phi^R = \phi^{(0)} \exp\left(-i\left(R_{int}^{1/2} Z + 2\pi n\right)\right) \quad -(35)$$

where $n$ is an integer, so $\phi^R$ is

 quantized. Here:

$$n = 0, 1, 2, \ldots \qquad (36)$$

This means that the interaction wave function during a low energy nuclear reaction is quantized. The system may acquire extra ~~energy~~ momentum by the absorption of a quantum of momentum from spacetime.

Finally this process may be worked out in terms of energy by using from eq. (22):

$$\hat{H}\phi^R = -\frac{\hbar^2\nabla^2}{m}\phi^R$$

$$= \left(mc^2\left((\gamma+\gamma_1)^2 - 1\right) - \frac{\hbar^2\kappa^2}{m}\right)\phi^R + i\frac{\hbar^2}{m}\underline{\kappa}\cdot\underline{\nabla}\,\phi^R \qquad (37)$$

So the interacting system acquires extra energy by the absorption of a quantum of energy from spacetime.