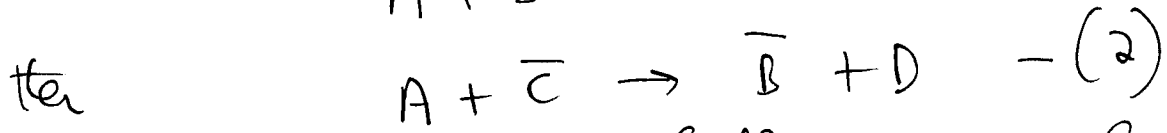
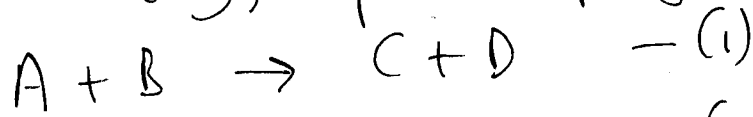
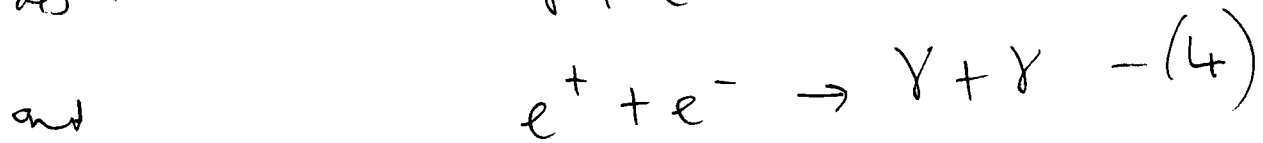


247(8) : General Theory of Particle Scattering and Annihilation and Comparison with Dirac Formalism

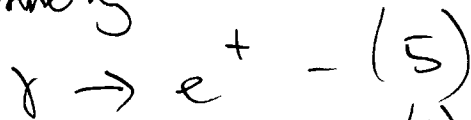
As in note 247(7), in particle physics if:



by cross over symmetry. So the same general theory should describe:

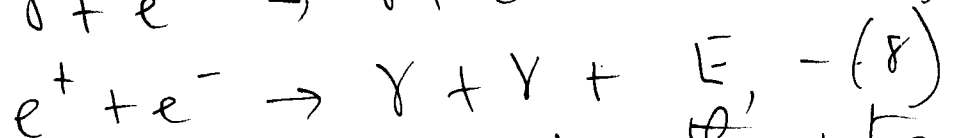
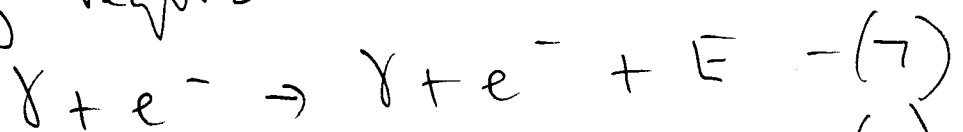


by cross over symmetry:



assuming that a photon is its own anti-particle. Eq. (3) is Compton scattering and eq. (4) is positron electron annihilation.

As argued in previous notes a more complete and correct theory requires:



However, for the purpose of development in this note we consider eqs. (3) and (4).

The equations of conservation of energy and momentum describing the processes (1) and (2) are:

$$\omega + x_2 = \omega' + \omega'' - (9)$$

$$\underline{p} = \underline{p}' + \underline{p}'' - (10)$$

where $x_2 = mc^2 / h - (11)$

and m is the mass of the initially static particle. Therefore:

$$\omega'' = x_2 + \omega - \omega' - (12)$$

and
$$\underline{p}'' = \underline{p} - \underline{p}' - (13)$$

Defining:
$$E'' = h\omega'' - (14)$$

then:
$$E''^2 = c^2 p''^2 + m^2 c^4 - (15)$$

and
$$E'' = E - E' + mc^2 - (16)$$

where
$$E' = h\omega', \quad E = h\omega. - (17)$$

From these equations:

$$(E - E' + mc^2)^2 = c^2 (\underline{p} - \underline{p}')^2 + m^2 c^4 - (18)$$

It follows that:

$$E - E' + mc^2 = \frac{c^2 (\underline{p} - \underline{p}')^2}{E - E' + mc^2} + \frac{m^2 c^4}{E - E' + mc^2} - (19)$$

This is similar to the formula obtained from the Dirac equation developed into the Klein equation.

The momenta are defined by:

$$\underline{p} = \hbar \underline{k}, \quad \underline{p}' = \hbar \underline{k}' \quad - (20)$$

For the "massless photon":

$$p = \hbar k = \hbar \frac{\omega}{c}, \quad p' = \hbar \frac{\omega'}{c} \quad - (21)$$

It follows that:

$$\left(\omega - \omega' + \frac{mc^2}{\hbar} \right)^2 = \frac{c^2}{\hbar^2} (p^2 + p'^2 - 2pp' \cos \theta) + \left(\frac{mc^2}{\hbar} \right)^2 \quad - (22)$$

$$= \omega^2 + \omega'^2 - 2\omega\omega' \cos \theta + \left(\frac{mc^2}{\hbar} \right)^2$$

$$= \omega^2 + \omega'^2 - 2\omega\omega' + 2 \left(\frac{mc^2}{\hbar} \right) (\omega - \omega') \quad - (23)$$

i.e.

$$\boxed{\omega - \omega' = \left(\frac{\hbar}{mc^2} \right) \omega \omega' (1 - \cos \theta)} \quad - (24)$$

which is the Compton formula.

The Compton formula follows from the relativistic equation (15) for the scattered electron. This is:

$$\boxed{E'' = \gamma'' mc^2 = \hbar \omega''} \quad - (25)$$

Eq. (19) may be written as:

$$-E' + mc^2 = \frac{1}{m} (\underline{p} - \underline{p}')^2 \left(1 + \frac{E - E'}{mc^2} \right)^{-1} + mc^2 \left(1 + \frac{E - E'}{mc^2} \right)^{-1} \quad - (26)$$

$$4) \quad \text{If} \quad E - E' \ll mc^2 \quad (27)$$

then:

$$E - E' + mc^2 = \frac{1}{m} (\underline{p} - \underline{p}')^2 \left(1 - \frac{E - E'}{mc^2} \right) + mc^2 \left(1 - \frac{E - E'}{mc^2} \right) \quad (28)$$

i.e.

$$E - E' = \frac{1}{2m} (\underline{p} - \underline{p}')^2 \left(1 - \frac{E - E'}{mc^2} \right)$$

$$\boxed{E - E' \sim \frac{1}{2m} (\underline{p} - \underline{p}')^2} \quad (29)$$

In this limit it is clear that E and E' are kinetic energies. They are defined by:

$$E = \hbar\omega = \gamma mc^2 \quad (30)$$

$$E' = \hbar\omega' = \gamma' m_1 c^2 \quad (31)$$

where m_1 is the photon mass. It has been assumed for the sake of argument that:

$$m_1 = 0, \quad \gamma = \gamma' \rightarrow \infty \quad (32)$$

because

$$v \rightarrow c; \quad v' \rightarrow c \quad (33)$$

So eq. (29) describes the transfer of kinetic energy from the electron to the photon in the non-relativistic

Eq. (29) may be written as:

$$\hbar(\omega - \omega') = \frac{1}{2m} \left(\frac{\hbar}{c} \right)^2 (\omega^2 + \omega'^2 - 2\omega\omega' \cos \theta) \quad (34)$$

i.e. $\omega - \omega' = \frac{\hbar}{mc^2} \left(\frac{1}{2} (\omega^2 + \omega'^2) - \omega\omega' \cos \theta \right) \quad (35)$

From eqs. (24) and (35) it is seen that the Compton formula has been approximated with:

$$\omega^2 + \omega'^2 = 2\omega\omega' \quad (36)$$

Note that eq. (36) is equivalent to:

$$(\omega + \omega')^2 = \omega^2 + \omega'^2 + 2\omega\omega' \sim 4\omega\omega' \quad (37)$$

i.e. $\omega \sim \omega' \quad (38)$

From eq. (61) of HFT II the fermion eqn. produces:

$$(E - e\phi)^2 - m^2 c^4 = c^2 \underline{\sigma} \cdot (\underline{p} - e\underline{A}) \underline{\sigma} \cdot (\underline{p} - e\underline{A}) \quad (39)$$

Eqs. (18) and (39) are the same if:

$$e\phi = E' - mc^2 \quad (40)$$

and

$$\underline{p}' = e\underline{A} \quad (41)$$

Eqs. (18) and (39) are the quantized version of

eq. (15) with:

$$E'' \rightarrow E - E' + mc^2 = E - e\phi - (42)$$

and

$$\underline{p}'' \rightarrow \underline{p} - \underline{p}' = \underline{p} - e\underline{A} - (43)$$

and the interaction of the scattered electron with the scattered photon is treated with a minimal prescription. This process describes the time reversal of eq. (10), i.e.

$$\underline{p}' + \underline{p}'' \rightarrow \underline{p} - (44)$$

The time reversal of eq. (9) is:

$$E'' + E' = E + mc^2 - (45)$$

Eqs. (44) and (45) describe a moving electron interacting with a moving photon to produce a static electron and moving photon.

This theory develops particle scattering theory into a relativistic quantum theory