DERIVATION OF O(3) ELECTRODYNAMICS FROM THE GAUSS LAW APPLIED TO MAGNETISM AND THE FARADAY LAW OF INDUCTION.

In the Evans field theory, the Gauss law of magnetism and the Faraday law of induction are derived from the Bianchi identity of differential geometry:

$$D \wedge T^a = R^a{}_b \wedge q^b \qquad -(1)$$

which can be rewritten as:

$$d \wedge T^a = R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b . \qquad -(2)$$

The Bianchi identity is converted to the homogeneous Evans field equation (HE) using:

$$A^a = A^{(0)} q^a \qquad -(3)$$

$$F^a = A^{(0)} T^a \qquad -(4)$$

so eqn (2) becomes the HE:

$$\boxed{\begin{aligned} d \wedge F^a &= R^a{}_b \wedge A^b - \omega^a{}_b \wedge F^b \\ &= \mu_0 j^a \end{aligned}} . \qquad -(5)$$

Eqn (5) defines the homogeneous current:

$$j^a = \frac{1}{\mu_0} \left(R^a{}_b \wedge A^b - \omega^a{}_b \wedge F^b \right) \qquad -(6)$$

2)

In general the homogeneous current is non-zero. However it is known experimentally to be very small within contemporary instrumental limits: because experimentally,

$$d \wedge F^a = 0. \quad -(7)$$

Eqn (7) defines the Gauss Law applied to magnetism:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad -(8)$$

and the Faraday law of induction:

$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} = \underline{0}. \quad -(9)$$

The index a appearing in eqns (8) and (9) comes from general relativity and indicates states of polarization, notably circular polarization, first observed experimentally by Arago in 1811. Arago was the first to observe the two transverse states of circular polarization, each denoted by either:

$$a = (1) \text{ or } \cancel{\text{and}} \ (2). \quad -(10)$$

In the mid nineteen sixties, it was observed exptlly that circularly polarized radiation magnetizes all material matter, a phenomenon of non-linear optics known as the inverse Faraday effect. In 1992 it was inferred by Evans ~~that~~ this effect is due to the Evans spin field $\underline{B}^{(3)}$, and therefore

3) to a third state of polarization:

$$a = (3) \quad - (11)$$

caused by the rotation and translation of ~~the~~ the electromagnetic field in free space. The Gauss Law (8) and Faraday induction law (9) hold for $a = (3)$, but it is observed experimentally ~~that~~ that:

$$\underline{E}^{(3)} = \underline{0}. \quad - (12)$$

Therefore:

$$\underline{\nabla} \cdot \underline{B}^{(3)} = 0 \quad - (13)$$

$$\frac{\partial \underline{B}^{(3)}}{\partial t} = \underline{0}. \quad - (14)$$

The fundamental reason for this is ~~that~~ the spin of the e/m field produces an angular momentum (first observed experimentally by Beth in 1936). Angular momentum and magnetic field are axial vectors, and so $\underline{B}^{(3)}$ is directly proportional to angular momentum. The electric field is a polar vector and is not produced by spin. There is no electric analogue of the inverse Faraday effect. Similarly, a static magnetic field produces the forward (original) Faraday effect, but a static electric field does not. The original Faraday effect is the rotation of the plane of linearly polarized light by a static magnetic field.

4) In the Maxwell Heaviside (MH) field theory of special relativity the index a is not defined, because in the MH theory there is only one frame, that of the Minkowski spacetime (or manifold). The index a is that of the tangent bundle of the general 4-D manifold (Evans spacetime). Therefore in MH theory:

$$d \wedge F = 0 \quad -(15)$$

and the Evans spin field:

$$\underline{B}^{(3)} = -ig\,\underline{A}^{(1)} \times \underline{A}^{(2)}, \quad -(16)$$

$$g = \frac{\kappa}{A^{(0)}}, \quad -(17)$$

is not defined. In eqn (17) κ is the free space wavenumber magnitude in inverse metres. The inverse Faraday effect is the magnetization:

$$\underline{M}^{(3)} = \frac{1}{\mu_0} g' \,\underline{A}^{(1)} \times \underline{A}^{(2)} \quad -(18)$$

where g' is a molecular property, in the simplest case a property of one electron. The $\underline{M}^{(3)}$ in eqn (18) cannot be defined in MH theory without the introduction of the conjugate product $\underline{A}^{(1)} \times \underline{A}^{(2)}$ of non-linear optics. This is an ad hoc or empirical procedure in MH field theory.

5) In the Evans field theory however, the spin field $\underline{B}^{(3)}$ and conjugate product $\underline{A}^{(1)} \times \underline{A}^{(2)}$ are deduced from differential geometry and from the experimental fact:

$$j^a \sim 0. \qquad -(19)$$

Eqn. (19) and (6) imply:

$$R^a{}_b \wedge A^b = \omega^a{}_b \wedge F^b \qquad -(20)$$

to high precision. In other words the Gauss and Faraday induction laws appear to be true within contemporary instrumental limits. The reason for this is eqn. (20), a constraint of differential geometry. Using the Maurer Cartan structure equations:

$$T^a = D \wedge q^a \qquad -(21)$$

$$R^a{}_b = D \wedge \omega^a{}_b \qquad -(22)$$

eqn (20) becomes the following constraint in differential geometry:

$$\cancel{R^a{}_b \wedge (D}$$

$$\boxed{(D \wedge \omega^a{}_b) \wedge q^b = \omega^a{}_b \wedge (D \wedge q^b)} \qquad -(23)$$

a constraint implied experimentally, as discussed.

6)

A solution of eqn. (23) is:

$$\omega^a{}_b = \kappa \epsilon^a{}_{bc} q^c \quad -(24)$$

where $\epsilon^a{}_{bc}$ is the Levi-Civita symbol in the tangent bundle spacetime. The latter is a Minkowski spacetime, so eqn (24) can be written as:

$$\omega_{ab} = \kappa \epsilon_{abc} q^c. \quad -(25)$$

Eqn. (25) states that the spin connection is the antisymmetric tensor dual to the axial vector q^c within a factor κ. The latter is introduced by dimensionality. Thus eqn. (25) defines κ in the Evans field theory.

It follows from eqn (25) that the covariant derivative defining the torsion form in the first Maurer Cartan structure equation can be written as:

$$\begin{aligned} T^a &= d \wedge q^a + \omega^a{}_b \wedge q^b \\ &= d \wedge q^a + \kappa q^b \wedge q^c \end{aligned} \quad -(26)$$

$$\Rightarrow \boxed{F^a = d \wedge A^a + g A^b \wedge A^c} \quad -(27)$$

7)

In the complex circular basis:

$$F^{(1)*} = d \wedge A^{(1)*} - ig A^{(2)} \wedge A^{(3)}$$
$$F^{(2)*} = d \wedge A^{(2)*} - ig A^{(3)} \wedge A^{(1)}$$
$$F^{(3)*} = d \wedge A^{(3)*} - ig A^{(1)} \wedge A^{(2)}$$
$$— (28)$$

and this defines O(3) electrodynamics, developed by Evans from 1992 to 2003.

It is seen that O(3) electrodynamics has been inferred from the Evans field ~~theory~~ given the experimental constraint of ~~the~~ Gauss and Faraday induction laws.

It follows ~~that~~:

$$d \wedge F^{(1)} = 0 \quad — (29)$$
$$d \wedge F^{(2)} = 0 \quad — (30)$$
$$d \wedge F^{(3)} = 0 \quad — (31)$$

as observed experimentally. The conjugate product has been inferred in eqn. (28) from differential geometry, and it follows ~~that the~~ spin field and inverse Faraday effect have also been inferred, a major advance from MH.