

358(4): Gravitational Field from a Constant Spacetime Angular Momentum

Consider the spacetime or rather angular momentum:

$$\underline{L}_F = m \underline{r}_F \times \underline{v}_F \quad - (1)$$

where m is the mass of a material object and \underline{r}_F and \underline{v}_F refer to fluid spacetime. Here \underline{v}_F is the velocity field of the fluid spacetime and \underline{r}_F a position vector of fluid spacetime. If a planar orbit is considered, \underline{r}_F is the vector along the radius of the planar orbit.

Multiply both sides of eq. (1) by $\underline{r}_F \times$:

$$\begin{aligned} \underline{r}_F \times \underline{L}_F &= m \underline{r}_F \times (\underline{r}_F \times \underline{v}_F) \quad - (2) \\ &= m (\underline{r}_F (\underline{r}_F \cdot \underline{v}_F) - \underline{v}_F (\underline{r}_F \cdot \underline{r}_F)) \end{aligned}$$

The orbital velocity \underline{v}_F is $\perp \underline{r}_F$, so:

$$\underline{r}_F \cdot \underline{v}_F = 0 \quad - (3)$$

It follows that the velocity field of a spacetime with constant angular momentum \underline{L}_F is:

$$\underline{v}_F = \frac{1}{m r_F^2} \underline{L}_F \times \underline{r}_F \quad - (4)$$

Now align \underline{L}_F to the \underline{k} axis:

$$\underline{L}_F = L_{Fz} \underline{k} \quad - (5)$$

so:

$$\begin{aligned}
 \underline{V}_F &= \frac{1}{m r_F^2} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & L_{Fz} \\ r_{Fx} & r_{Fy} & 0 \end{vmatrix} \\
 &= \frac{1}{m r_F^2} \left(-L_{Fz} r_{Fy} \underline{i} + L_{Fz} r_{Fx} \underline{j} \right) \\
 &= \frac{L_{Fz}}{m r_F^2} \left(-Y \underline{i} + X \underline{j} \right) \quad - (6)
 \end{aligned}$$

The material gravitomagnetic field generated by the spacetime velocity field \underline{V}_F is the material vorticity:

$$\begin{aligned}
 \underline{\Omega}(\text{material}) &= \underline{\omega}(\text{material}) \\
 &= \underline{\nabla} \times \underline{V}_F
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{L_{Fz}}{m r_F^2} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & 0 \\ -Y & X & 0 \end{vmatrix} \quad - (7) \\
 &= \frac{2 L_{Fz}}{m r_F^2} \underline{k} = \frac{2}{m r_F^2} \underline{L}_F
 \end{aligned}$$

The material's gravitomagnetic field is directly proportional to the spacetime angular momentum.

The material's gravitational field is:

$$\underline{g}(\text{material}) = \left(\underline{V}_F \cdot \underline{\nabla} \right) \underline{V}_F \quad - (8)$$

Here:

$$\underline{v}_F \cdot \underline{\nabla} = \frac{L_F Z}{m r_F^2} (-Y \underline{i} + X \underline{j}) \cdot \left(\frac{\partial}{\partial X} \underline{i} + \frac{\partial}{\partial Y} \underline{j} + \frac{\partial}{\partial Z} \underline{k} \right)$$
$$= \frac{L_F Z}{m r_F^2} \left(-Y \frac{\partial}{\partial X} + X \frac{\partial}{\partial Y} \right) \quad - (9)$$

So:

$$\underline{g}(\text{matter}) = \frac{L_F Z^2}{m^2 r_F^4} \left(-Y \frac{\partial}{\partial X} + X \frac{\partial}{\partial Y} \right) (-Y \underline{i} + X \underline{j})$$
$$= \frac{L_F Z^2}{m^2 r_F^4} \left(\left(Y \frac{\partial Y}{\partial X} - X \right) \underline{i} + \left(X \frac{\partial X}{\partial Y} - Y \right) \underline{j} \right) \quad - (10)$$

Now assume that:

$$\frac{\partial Y}{\partial X} = \frac{\partial X}{\partial Y} = 0 \quad - (11)$$

It follows that:

$$\underline{g}(\text{matter}) = - \frac{L_F Z^2}{m^2 r_F^4} \underline{r} \quad - (12)$$

where

$$\underline{r}_F = X \underline{i} + Y \underline{j} \quad - (13)$$

Finally use:

$$\underline{r}_F = r_F \underline{e}_r \quad - (14)$$

where \underline{e}_r is the radial unit vector.

It follows that:

$$\underline{g}(\text{matter}) = - \frac{L_F^2}{n^2 r_F^3} \underline{e}_r - (15)$$

This is an inverse cubed dependence which gives
the orbit of a whirlpool galaxy.

The whirlpool galaxy is therefore due
to a fluid spacetime with constant angular
momentum
