391(4): Integration of the Binet Equations for Einsteinian and ECE2 Gravitation

In the Einstein theory, the Binet equation is:

$$\frac{d^2u}{d\phi^2} + u = \frac{1}{\alpha} + \frac{3MG}{c^2} \cdot \frac{1}{r^2} \quad -(1)$$

$$= \frac{1}{\alpha} + \alpha \Delta \phi u^2$$

where α is the half right latitude and:

$$\Delta \phi = \frac{3MG}{c^2 \alpha} \quad -(2)$$

is the orbital precession. Here:

$$u = \frac{1}{r} \quad -(3)$$

Therefore

$$\frac{d^2u}{d\phi^2} = \frac{1}{\alpha} - u + \alpha \Delta \phi u^2 \quad -(4)$$

$$= f(u)$$

In the theory of differential equations this is an example of the autonomous equation (http://eqworld.ipmnet.ru)

Its solution is

$$C_2 \pm \phi = \int \left(C_1 + 2 \int f(u) \, du \right)^{-1/2} du \quad -(5)$$

where C_1 and C_2 are constants of integration.

Therefore the Einsteinian Binet equation (1) can be solved analytically. The autonomous equation is:

$$\boxed{\frac{d^2u}{d\phi^2} = f(u)} \quad -(5a)$$

Note that:

$$\int f(u)\,du = \frac{u}{\alpha} - \frac{u^2}{2} + \alpha \Delta\phi \frac{u^3}{3} \quad -(6)$$

So:

$$C_2 \pm \phi = \int \left(C_1 + \frac{2u}{\alpha} - u^2 + \frac{2\alpha}{3}\Delta\phi u^3 \right)^{-1/2} du \quad -(7)$$

If

$$\Delta\phi = 0 \quad -(8)$$

the integration (7) gives the conic section:

$$u = \frac{1}{r} = \frac{1}{\alpha}(1+\epsilon\cos\phi) \quad -(8)$$

i.e.

$$\cos\phi = \frac{1}{\epsilon}\left(u - \frac{1}{\alpha}\right) \quad -(9)$$

$$\phi = \cos^{-1}\left(\frac{1}{\epsilon}\left(u - \frac{1}{\alpha}\right)\right) \quad -(10)$$

It is well known that eq. (10) is obtained from:

$$\phi(r) = \int \frac{L}{r^2} \frac{dr}{\left(2m\left(E - U - \frac{L^2}{2mr^2}\right)\right)^{1/2}} \quad -(11)$$

From eq. (3): $\frac{du}{dr} = -\frac{1}{r^2} \quad -(12)$

So

$$\phi = -\int \frac{L\,du}{\left(2m\left(E - U - \frac{L^2}{2m}u^2\right)\right)^{1/2}} \quad -(13)$$

where

$$U = -nMGu \quad -(14)$$

So eq. (13) is:

$$\phi = -\int \left(\frac{2nE}{L^2} + \frac{2}{\alpha} u - u^2 \right)^{-1/2} du \quad -(15)$$

where

$$\alpha = \frac{L^2}{n^2 MG} \quad -(16)$$

Eqs (7) and (15) are the same if:

$$C_1 = \frac{2nE}{L^2}, \quad C_2 = 0 \quad -(17)$$

and the negative sign is taken on the right hand side of eq. (7). So the orbit of the Einstein theory is given by

$$\boxed{\phi = -\int \left(\frac{2nE}{L^2} + \frac{2u}{\alpha} - u^2 + \frac{2\alpha\Delta\phi}{3} u^3 \right)^{-1/2} du} \quad -(18)$$

in which

$$\Delta\phi = \frac{3MG}{c^2\alpha} \quad -(19)$$

as observed experimentally.

In the solar system, $\Delta\phi$ is very tiny.

Therefore:

$$\phi = -\int \left(\frac{2mE}{L^2} + \frac{2u}{\alpha} - u^2 + \frac{2MG}{c^2} u^3 \right)^{-1/2} du \quad -(20)$$

This integral has no analytical solution and in previous UFT papers numerical integration was applied. It was found that the orbit is very unstable and that Einstein's attempts to evaluate the integral contained errors. It may now be possible to evaluate it in a new way, using recent improvements in integration techniques.

Einstein's method was to factorize eq. (20):

$$\phi = -\int \frac{du}{\left((u-u_1)(u-u_2)(u-u_3)\right)^{1/2}} \quad -(21)$$

but this is obviously unstable, because the integral becomes singular at:

$$u = u_1, \quad u = u_2, \quad \text{or } u = u_3 \quad -(22)$$

The methods used by Maria and Thomas have also been severely criticized in previous papers.

From eq. (18) it can be seen immediately that ϕ depends on ϕ and varies at each point in the orbit. The Einstein theory therefore gives a very intricate and complicated orbit, and not a precessing ellipse.

To emphasize this, it is possible to evaluate eq. (18) using a binomial expansion by writing:

5)

$$\frac{2mE}{L^2}+\frac{2u}{\alpha}-u^2+\frac{2mG}{c^2}u^3$$

$$=\left(\frac{2mE}{L^2}+\frac{2u}{\alpha}-u^2\right)\left(1+\frac{2mG/c^2}{\frac{2mE}{L^2}+\frac{2u}{\alpha}-u^2}\right) \quad -(22)$$

So:

$$\phi=-\int\left(\frac{2mE}{L^2}+\frac{2u}{\alpha}-u^2\right)^{-1/2}\left(1+\frac{2mG/c^2}{\frac{2mE}{L^2}+\frac{2u}{\alpha}-u^2}\right)^{-1/2}du \quad -(23)$$

Now assume that:

$$\frac{2mG}{c^2} \ll \frac{2mE}{L^2}+\frac{2u}{\alpha}-u^2 \quad -(24)$$

$$-(25)$$

so

$$\phi=-\int\left(\frac{2mE}{L^2}+\frac{2u}{\alpha}-u^2\right)^{-1/2}\left(1-\frac{mG/c^2}{\frac{2mE}{L^2}+\frac{2u}{\alpha}-u^2}\right)du \quad -(25a)$$

$$\boxed{\phi \doteq -\int\frac{du}{\left(\frac{2mE}{L^2}+\frac{2u}{\alpha}-u^2\right)^{1/2}}+\frac{mG}{c^2}\int\frac{du}{\left(\frac{2mE}{L^2}+\frac{2u}{\alpha}-u^2\right)^{3/2}}}$$

This gives a combination of an elliptical orbit from the first integral and another type of orbit from the second integral

The integral in (25a) according to the Wolfram integrator is:

6) $$\int (ax^2+bx+c)^{-3/2}\,dx = \frac{2(2ax+b)}{(b^2-4ac)(x(ax+b+c))^{1/2}}$$
$$+ \text{constant} \qquad -(26)$$

To evaluate eq (25a):

$$\frac{mG}{c^2}\int \frac{du}{\left(\frac{2mE}{L^2}+\frac{2u}{\alpha}-u^2\right)^{3/2}} \qquad -(27)$$

$$= \frac{2mG}{c^2}\left[\frac{(2/\alpha-2u)}{\left(\frac{4}{\alpha^2}+\frac{8mE}{L^2}\right)\left(u\left(\frac{2}{\alpha}+\frac{2mE}{L^2}-u\right)\right)^{1/2}}\right]$$

where $$u = \frac{1}{r}$$

The Einstein orbit is given by Eqs. (25a) and (27) in the approximation (24). This can be graphed as ϕ versus r. At the point:

$$u = \frac{1}{r} = \frac{2}{\alpha}+\frac{2mE}{L^2} \qquad -(28)$$

it becomes singular. This refutes the Einstein theory in yet another way.

In the next note this method will be applied to the ECE 2 Binet equation.