Note 391(9): Final Version

The ECE2 hamiltonian is:

$$H_0 = (\gamma - 1)mc^2 - \frac{mMG}{r} \quad -(1)$$

where

$$\gamma = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} = 1 + \frac{1}{2}\frac{v_N^2}{c^2} + \frac{3}{8}\frac{v_N^4}{c^4} + \frac{5}{16}\frac{v_N^6}{c^6} + \dots \quad -(2)$$

so

$$H_0 = \frac{1}{2}mv_N^2 - \frac{mMG}{r} + \frac{m}{c^2}\left(\frac{3}{8}v_N^4 + \frac{5}{16}\frac{v_N^6}{c^4} + \dots\right) \quad -(3)$$

$$= \text{constant.}$$

The Newtonian hamiltonian is:

$$H_N = \frac{1}{2}mv_N^2 - \frac{mMG}{r_N} = -\frac{mMG}{2a} \quad -(4)$$

where

$$\frac{1}{r_N} = \frac{1}{\alpha}\left(1 + \epsilon\cos\phi\right) \quad -(5)$$

The relativistic hamiltonian is:

$$H_0 = \frac{1}{2}mv_N^2 + \frac{m}{c^2}\left(\frac{3}{8}v_N^4 + \frac{5}{16}\frac{v_N^6}{c^4} + \dots\right) - \frac{mMG}{r} \quad -(6)$$

where:

$$\frac{1}{r} = \frac{1}{\alpha}\left(1 + \epsilon\cos(\phi + \Delta\phi)\right) \quad -(7)$$

At perihelion:

$$\phi = 2\pi, \quad -(8)$$

and the experimentally observed precession is

$$\Delta\phi = \frac{6\pi MG}{\alpha c^2} \quad -(9)$$

So at perihelia:

$$\frac{1}{r} = \frac{1}{\alpha}\left(1+\epsilon\cos\left(2\pi\left(1+x\right)\right)\right) \quad -(10)$$

where

$$x = \frac{3MG}{\alpha c^2} \quad -(11)$$

At the perihelia:

$$v_N^2 = \frac{MG}{\alpha}\left(1+\epsilon\right)^2 \quad -(12)$$

So

$$H_0 = (\gamma-1)mc^2 - \frac{mMG}{\alpha}\left(1+\epsilon\cos\left(2\pi\left(1+x\right)\right)\right) \quad -(13)$$

where

$$\gamma = \left(1 - \frac{MG}{\alpha c^2}\left(1+\epsilon\right)^2\right)^{-1/2} \quad -(14)$$

1) Calculate H_0 for exact agreement from eqs. (13) and (14).

2) Calculate H_N at the perihelion:

$$H_N = -\frac{mMG}{2a} \quad -(15)$$

where the semi major axis is:

$$a = \frac{\alpha}{1-\epsilon^2} \quad -(16)$$

It has been assumed that the precessing orbit is:

$$r = \frac{\alpha}{1+\epsilon\cos(\phi+\Delta\phi)} \quad -(17)$$