391(5): Calculation of the Precession of the Perihelion of Mercury

The equation to be solved is (7.24) of Marion and Thornton, the Binet equation with Einstein's effective potential:

$$\frac{d^2u}{d\phi^2} + u = \frac{m^2MG}{L^2} + \frac{3MG}{c^2}u^2 \quad - (1)$$

where the half right latitude is:

$$\alpha = \frac{L^2}{m^2MG} \quad - (2)$$

The Newtonian limit is:

$$\frac{d^2u}{d\phi^2} + u = \frac{1}{\alpha} \quad - (3)$$

and this gives a conic section orbit such as an ellipse.

Dr. Horst Eckardt has solved eqs. (1) and (3) numerically. An ellipse was found numerically for (3), and for (1) with the usual small second term on the RHS of eq. (1), a precessing orbit was found numerically. However as the second term becomes large, the orbit was found numerically to become wildly unphysical.

The Einstein theory is totally incorrect, because its geometry omits torsion.

Marion and Thornton solve eq. (1) with a dubious analytical method which claims that the effect of the relativistic term in eq. (1) is to displace the perihelion for each revolution of 2π by:

$$\Delta = 2\pi x \quad - (4)$$

where

$$x = \frac{3MG}{c^2\alpha} \quad - (5)$$

This method is claimed to result in what they

2) call a "secular equation":

$$u_{secular} \sim \frac{1}{\alpha}\left(1+\epsilon\cos\left((1-x)\phi\right)\right) \quad -(6)$$

(Eq. (7.83) of Marion and Thornton, 3rd edition). Note that eq. (6) is similar to the x theory of previous UFT papers. An increase of 2π means that:

$$(1-x)\phi = 2\pi \quad -(7)$$

so

$$\phi = \frac{2\pi}{1-x} \sim 2\pi(1+x) \quad -(8)$$

if

$$x << 1 \quad -(9)$$

Therefore the perihelion is displaced in a revolution of 2π by:

$$\Delta = 2\pi x = \frac{6\pi MG}{c^2\alpha} \quad -(10)$$

The experimental result given by Marion and Thornton is

$$\Delta = 43.11 \pm 0.45'' \quad -(11)$$

per earth century (100 Julian years).

In eq. (10) M is the mass of the sun, c the vacuum speed of light, and α is the half right latitude of Mercury. G is Newton's constant. There are uncertainties in all these quantities. The Wikipedia entry on Mercury gives:

$$G = 6.67408 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2} \quad -(12)$$

$$\alpha = 5.7909050 \times 10^{10}\ m \quad -(13)$$

Using:

$$M = 1.989 \times 10^{30}\ kg \quad -(14)$$

$$c = 2.99792\ m\ s^{-1} \quad -(15)$$

it is found that:

$$x = \frac{3MG}{c^2 \alpha} = \frac{3 \times 1.989 \times 6.6741}{8.98752 \times 5.7909050} \times 10^{-7} \quad -(16)$$

$$= 7.6518 \times 10^{-8}$$

Now use:

$$1'' = 4.84814 \times 10^{-6} \text{ radians.} \quad -(17)$$

It follows that:

$$2\pi x = 0.09915'' \quad -(18)$$

For 100 revolutions, each of 2π radians:

$$\Delta = 9.915'' \quad -(19)$$

This is the displacement for one hundred Mercury years.

According to the Wikipedia site on the planet Mercury, one Mercury year is 87.969 Earth days = 0.240846 Earth years. So:

$$\Delta = \frac{9.915}{0.240846} = 41.17'' \quad -(20)$$

per Earth century.

This is within the observed $43.11 \pm 0.45''$ per Earth century in Table 7-2 of Marion and Thornton.