(7): Some More Details of the Calculation of the Perihelion Precession from ECE2 Theory

The starting hamiltonian is:

$$H = \gamma m c^2 - \frac{mMG}{r} \quad -(1)$$

where

$$\gamma = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} \quad -(2)$$

Therefore:

$$H_0 = H - mc^2 = (\gamma - 1) m c^2 - \frac{mMG}{r} \quad -(3)$$

in which

$$T = (\gamma - 1) m c^2 \quad -(4)$$

is the relativistic kinetic energy:

$$T \xrightarrow[v_N \ll c]{} \frac{1}{2} m v_N^2 \quad -(5)$$

The Newtonian orbital velocity is:

$$v_N^2 = MG\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(6)$$

where the semi major axis is:

$$a = \frac{\alpha}{1 - \epsilon^2} \quad -(7)$$

where α is the ~~semi~~ ~~major~~ half right latitude and where ϵ is the eccentricity. If the precession is small (as in the solar system), the orbit is, to an excellent approximation:

$$r = \frac{\alpha}{1 + \epsilon \cos \phi} \quad -(8)$$

The perihelion, the distance of closest approach, is defined by:

$$\phi = 2\pi \quad -(9)$$

So at closest approach:

$$R_0 = \frac{\alpha}{1+\epsilon} \quad -(10)$$

The Newtonian velocity at closest approach is:

$$v_N^2 = mG\left(\frac{2}{R_0} - \frac{(1-\epsilon^2)}{\alpha}\right) \quad -(11)$$

$$= \frac{mG}{\alpha}\left(2(1+\epsilon) - 1 + \epsilon^2\right)$$

$$= \frac{mG}{\alpha}\left(1 + 2\epsilon + \epsilon^2\right)$$

$$= \frac{mG}{\alpha}(1+\epsilon)^2$$

So at the perihelion, the Lorentz factor is

$$\gamma = \left(1 - \frac{mG}{\alpha c^2}(1+\epsilon)^2\right)^{-1/2} \quad -(12)$$

and is found by measuring α, ϵ and mG/c^2. It can be found experimentally and cannot be varied.

Assume that the effect of using the relativistic kinetic energy is to produce the orbit:

$$r = \frac{\alpha}{1+\epsilon\cos(\phi+\Delta\phi)} \quad -(13)$$

so

$$H_0 = (\gamma)mc^2 - \frac{mMG}{\alpha}\left(1+\epsilon\cos(\phi+\Delta\phi)\right) \quad -(14)$$

is a constant of motion. In eq. (14):

$$\cos(\phi + \Delta\phi) = \cos\phi \cos\Delta\phi - \sin\phi \sin\Delta\phi \quad -(15)$$

At the perihelion:

$$\phi = 2\pi \quad -(16)$$

so

$$\cos(2\pi + \Delta\phi) = \cos\Delta\phi \quad -(17)$$

Therefore:

$$H_0 = (\gamma - 1) mc^2 - \frac{mMG}{\alpha}\left(1 + \epsilon\cos\Delta\phi\right) \quad -(18)$$

so

$$mMG\frac{\epsilon}{\alpha}\cos\Delta\phi = (\gamma - 1) mc^2 - \frac{mMG}{\alpha} - H_0 \quad -(19)$$

in which γ is defined by eq. (12).

Therefore $\cos\Delta\phi$ may be found, given H_0.

Assume that H_0 is, to an excellent approximation:

$$H_0 = \frac{1}{2} m v_N^2 - \frac{mMG}{r} \quad -(20)$$

From eqs. (6) and (20):

$$H_0 = \frac{1}{2} mMG\left(\frac{2}{r} - \frac{1}{a}\right) - \frac{mMG}{r}$$

$$= -\frac{mMG}{2a}, \quad -(21)$$

well known result of orbit theory.

From eqs. (19) and (21):

$$mMG\frac{\epsilon}{\alpha}\cos\Delta\phi = (\gamma - 1) mc^2 - \frac{mMG}{\alpha} + \frac{mMG}{2a} \quad -(22)$$

so

$$\frac{\epsilon}{\alpha}\cos\Delta\phi = \frac{c^2}{MG}(\gamma - 1) + \frac{1}{2a} - \frac{1}{\alpha} \quad -(23)$$

It follows that:

$$\boxed{\cos\Delta\phi = \frac{1}{\epsilon}\frac{\alpha c^2}{mG}(\gamma-1) + \frac{\alpha}{\epsilon}\left(\frac{1}{2a}-\frac{1}{\epsilon}\right)} \quad -(24)$$

In the classical limit:

$$mMG\frac{\epsilon}{\alpha}\cos\Delta\phi = \frac{1}{2}mv_N^2 - \frac{mMG}{\alpha} + \frac{mMG}{2a}$$

$$= \frac{1}{2}\frac{mMG}{\alpha}(1+2\epsilon+\epsilon^2) - \frac{mMG}{\alpha} + \frac{mMG}{2}\left(\frac{1-\epsilon^2}{\alpha}\right)$$

$$= \frac{1}{2}\frac{mMG}{\alpha}(1+2\epsilon+\epsilon^2) - \frac{mMG}{2\alpha} - \frac{\epsilon^2}{2\alpha}mMG$$

$$= \frac{\epsilon}{\alpha}mMG \quad -(25)$$

so

$$\cos\Delta\phi = 1, \quad \Delta\phi = 0 \quad -(26)$$

Q.E.D., a self consistent result, because in the classical limit there is no precession.

Comparison with Marion and Thornton

They produce the secular equation:

$$u_{secular} = \frac{1}{\alpha}\left(1+\epsilon\cos(\phi(1-x))\right) \quad -(27)$$

where

$$x = \frac{3MG}{c^2\alpha} \quad -(28)$$

They increase the argument by 2π, so

$$\phi(1-x) = 2\pi \quad -(29)$$

and

$$\phi = \frac{2\pi}{1-x} \sim 2\pi(1+x) \quad -(30)$$

) So

$$\phi \rightarrow 2\pi + 2\pi x \quad -(31)$$

In the ECE2 theory:

$$\phi \rightarrow \phi + \Delta\phi \quad -(32)$$

so there is no need to multiply $\Delta\phi$ by 2π.

Fitting Eq. (19) to Experimental Data

Eq. (19) is:

$$m M G \frac{\epsilon}{\alpha} \cos\Delta\phi = \left(\left(1 - \frac{v_N^2}{c^2} \right)^{-1/2} - 1 \right) m c^2 - \frac{m M G}{\alpha} - H_0 \quad -(33)$$

where

$$v_N^2 = \frac{MG}{\alpha} (1+\epsilon)^2 \quad -(34)$$

i.e.:

$$H_0 = \left(\left(1 - \frac{MG}{\alpha c^2} (1+\epsilon)^2 \right)^{-1/2} - 1 \right) m c^2 + \frac{m M G}{\alpha} \left(1 + \epsilon \cos\Delta\phi \right) \quad -(35)$$

In order to obtain the observed $\Delta\phi$, vary H_0, i.e. adjust H_0 for exact agreement.

This procedure is equivalent to assuming a constant background (or aether) potential U_0 in the universe:

$$H = \gamma m c^2 - \frac{m M G}{r} + U_0 \quad -(36)$$